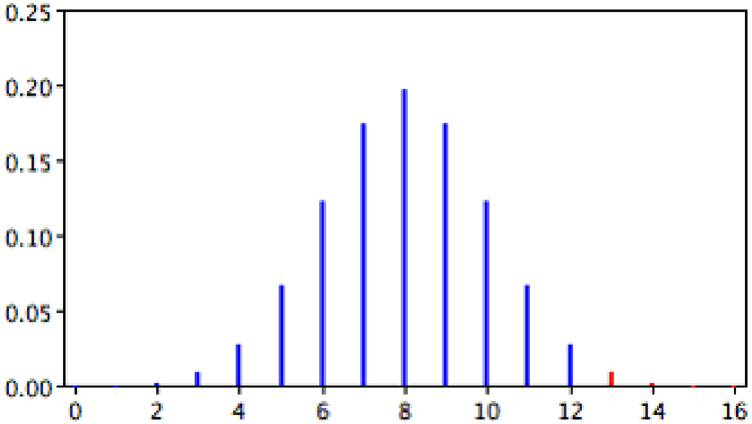
PROBABILITY & STATISTICS of COIN TOSSES

This slide is to remind you of the coin-tossing experiment we did.
In the experiment we tossed 16 coins, a total of 25 times.



1. Let's take 16 coins and toss them. The result will be that some number n will turn up on one particular side (in English we would talk about "heads" or "tails"

2. Make a record of how many heads you get (a number n between 0 and 16)

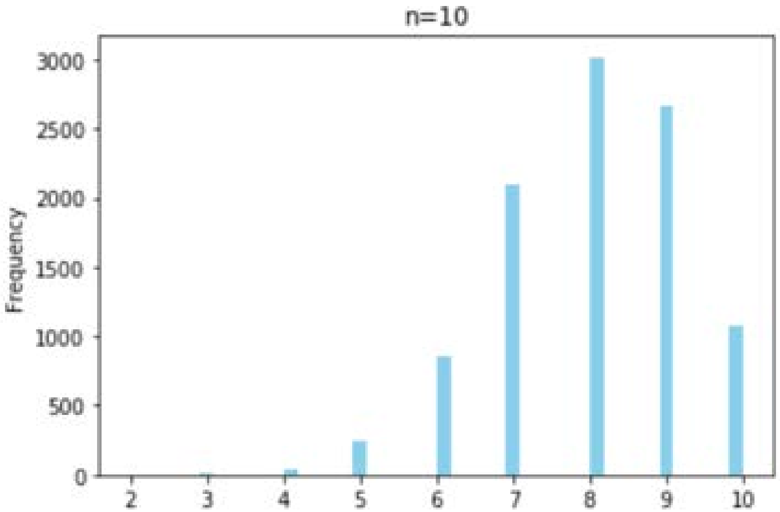
3. Now do it again – and do it for a total number of M "trials". Each time record the number of heads (in our case we simply have M = 25 different people throwing the coins, to speed things up).

4. Now plot the results as a histogram. The above graph shows which fraction of the results should give a number n, on the assumption that the probability of getting heads on each throw is ½. We expect this to show up in the statistics (ie., in the results) after we have done a very large number of trials (ie., when M is HUGE).

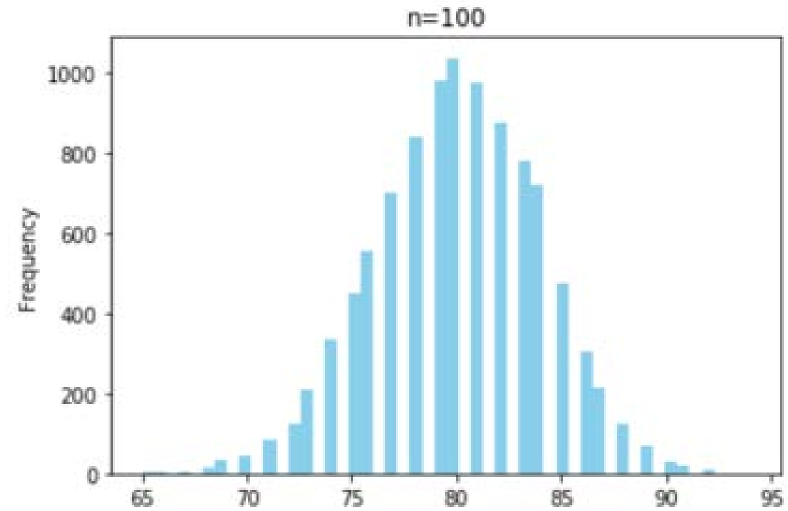
Did our graph look like this?

ANOTHER COIN TOSS EXPERIMENT……

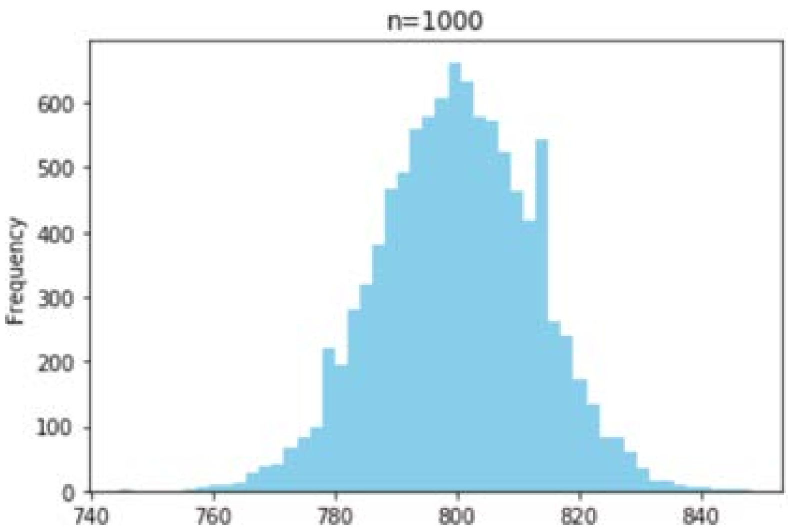
Toss 10 coins



Toss 100 coins



Toss 1000 coins



Looking at these results, we would guess that these coins are badly biased – with an 80% probability of showing heads.

This is an example of the use of the statistics of very large trials to infer probabilities

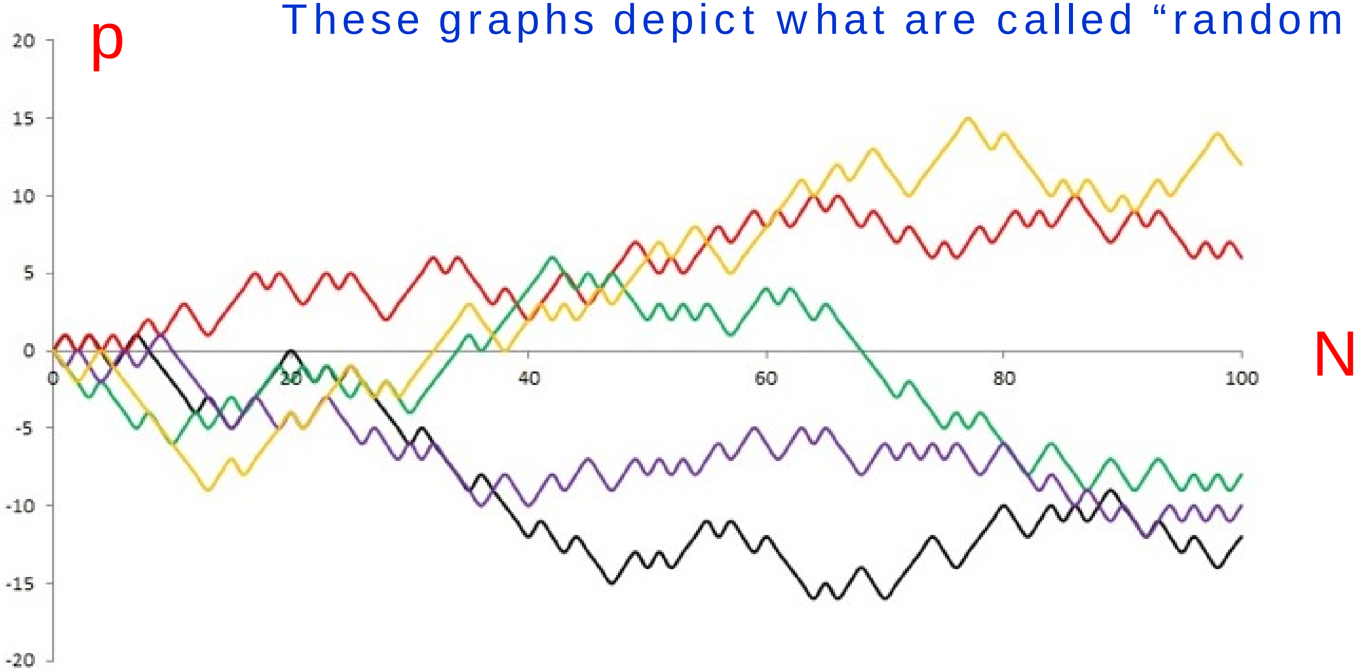
RANDOM WALKS in COIN TOSSING

Instead of just looking at the total results of the tossing of all the coins, we can also ask how the individual tosses go. In other words, we can ask about the details of a sequence of different tosses.

There are many ways to plot this graphically. Here is one – we show the evolution of 5 different trials, in each of which we toss 100 coins, one after another.

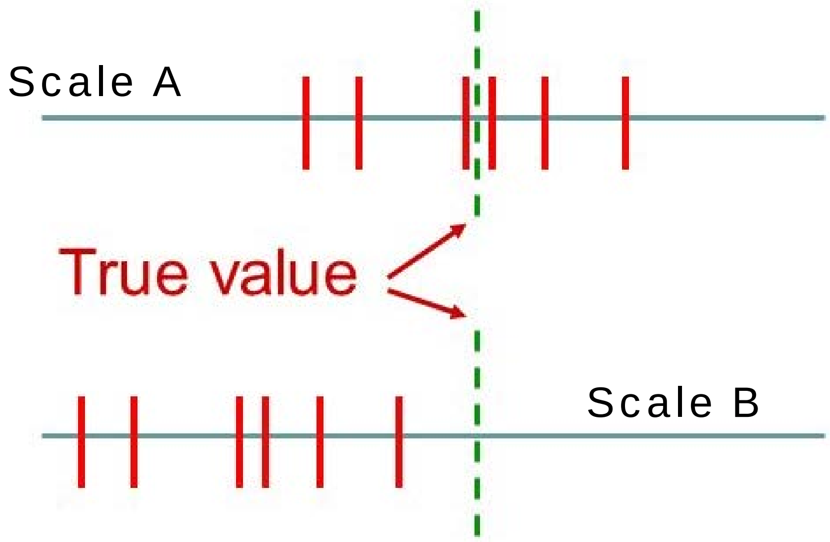
The figure shows the difference $p = n - m$ between the number n of heads and the number m of tails found, after a total number $N = n + m$ of coins has been thrown.

The 5 different trials are shown in different colours.
These graphs depict what are called “random walks”



RANDOM ERRORS vs SYSTEMATIC ERRORS

When we do an experiment over and over again, assuming that each time we are doing the experiment on the same experimental setup, we find 2 kinds of error.



Here we show 2 sets of data, each of which is measuring the weight of some object whose weight is actually known – so that we know what the reading should be.

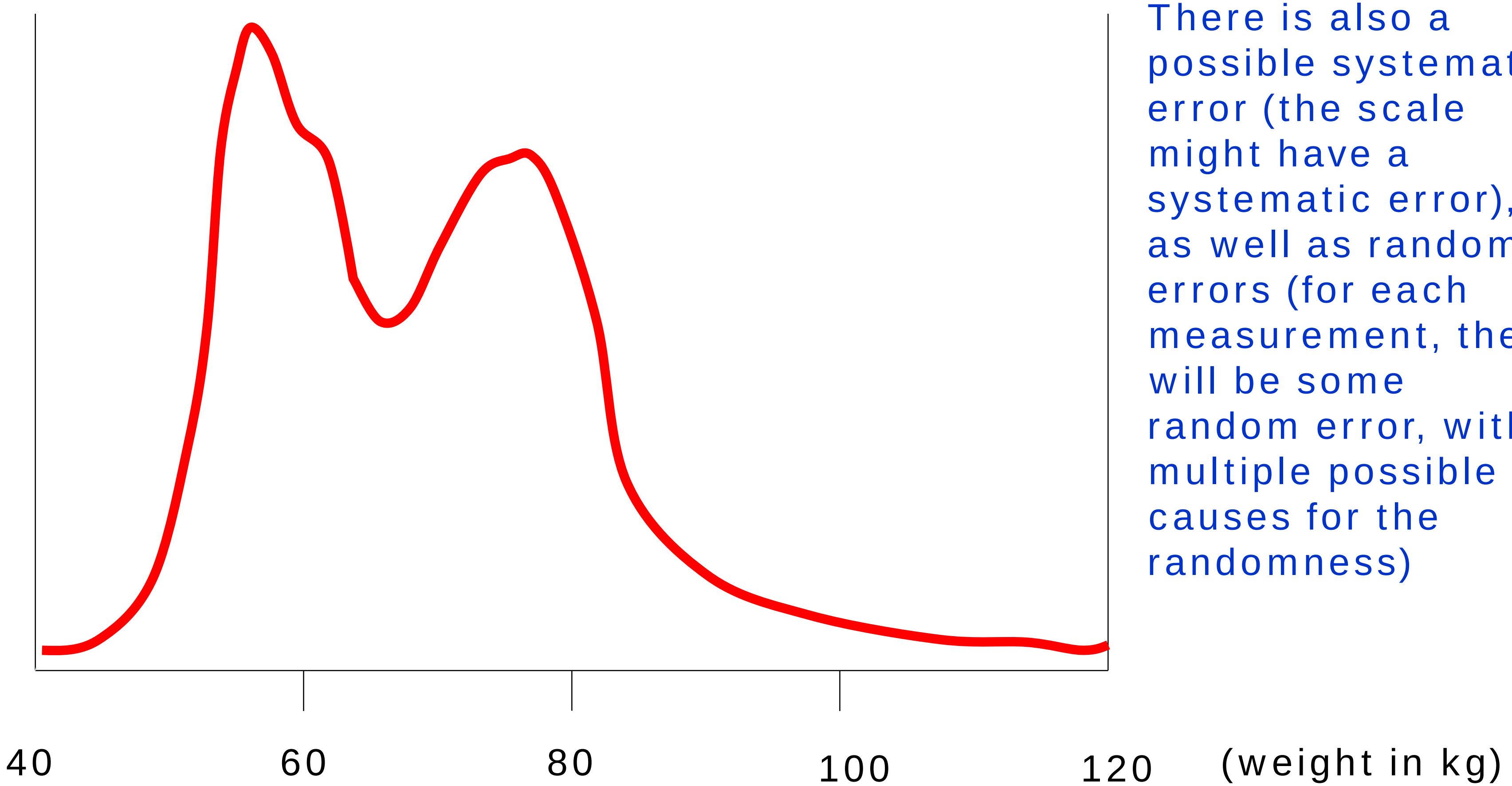
Actually what is being done here is testing to see how accurate 2 different scales are, in the measurement of weight.

For each of the two scales we take 6 readings. Note that this data is artificial (no experiment was actually done!); it was in fact computer generated.

In this case, where we KNOW the correct result, we can easily distinguish between a systematic error (for scale B) from a random error (for both scales). In real experiments we don't know the real value being measured, but we can do an initial calibration of the scale for some known weight.

DISTRIBUTION OF BODY WEIGHTS AT UBC

This is a more complicated REAL experiment – we show the results we might get on weighing all 100,000 people on campus. We see that there are multiple factors influencing these results (some of which we may not know).

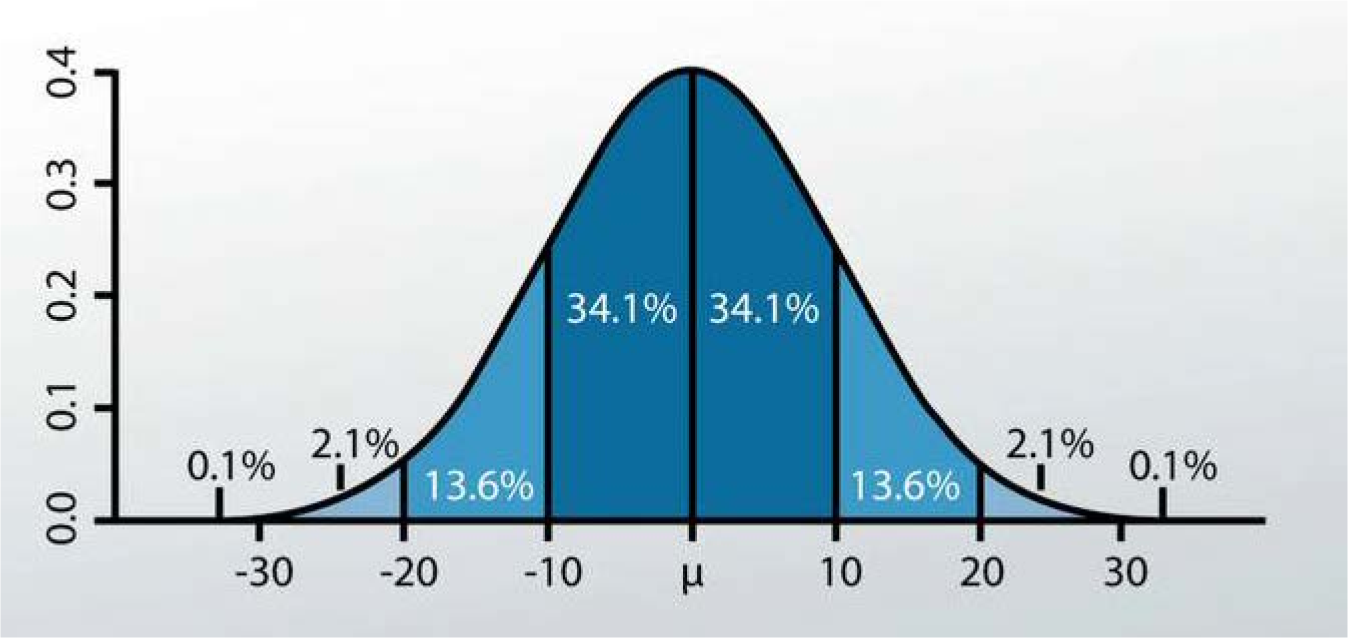


There is also a possible systematic error (the scale might have a systematic error), as well as random errors (for each measurement, there will be some random error, with multiple possible causes for the randomness)

RANDOM ERRORS

The simplest possible statistical distribution found in an experiment, caused by random errors, is the “Gaussian distribution” (sometimes called the “Bell curve”, or “normal distribution”). If there are no systematic errors, we expect that multiple random factors will give a distribution like this.

The Gaussian Distribution (Bell Curve)



https://en.wikipedia.org/wiki/Normal_distribution

https://en.wikipedia.org/wiki/The_Bell_Curve

https://study.com/academy/lesson/the-bell-curve-theory-themes-quiz.html

SIZE OF RANDOM ERRORS

Look again at the graphs for the coin tossing.
Notice how although the width of the distribution increases as we increase the number N of coins that are tossed, the FRACTIONAL WIDTH decreases.

It is known that if we toss N coins, then the "half-width" ΔN of the distribution will be proportional to the square root of N, ie., we have

$$\Delta N \sim N^{1/2}$$

So if N = 100, $\Delta N \sim 10$

If N = 1000, $\Delta N \sim 30$

However, this means that the fractional half-width is

$$\Delta N/N \sim 1/N^{1/2}$$

So if N = 100, $\Delta N/N \sim 0.1$

If N = 1000, $\Delta N/N \sim 0.03$

MORAL: Do the experiment with lots of coins! In other experiments, use as many samples or tests as possible.