Phys 508: HOMEWORK ASSIGNMENT No (2)

Friday February 5th 2016

DUE DATE: Monday Feb 22nd 2016.

Assignments handed in late will not receive a full mark.

This assignment is mostly to do with equations of motion and related topics. You can also be using the time before you hand it in to start working on assignment 3 (the essay/project question).

QUESTION (1): EQUATION of MOTION for SELF-COUPLED SCALAR FIELD

We are going to look at ϕ^5 theory, and as in the first assignment, look at the zero-dimensional case. Thus we consider an 'amplitude' function $A(\phi) = e^{iS(\phi)/\hbar}$ of a variable ϕ, with exponent $S(\phi) = -(m^2\phi^2/2 + \lambda_o\phi^5/5!)$. We define a generating functions $Z(J) = \int d\phi A(\phi) e^{i\phi J/\hbar}$ with 'correlators' g_n given by the power series expnasion

$$Z(J) = \sum_{n=0}^{\infty} \left(\frac{i}{\hbar}\right)^n \frac{1}{n!}\, g_n J^n$$

and cumulants c_n given by the analogous power series expansion of $W(J) = -i\hbar \ln Z(J)$, ie.,

$$W(J) = \sum_{n=0}^{\infty} \left(\frac{i}{\hbar}\right)^n \frac{1}{n!}\, c_n J^n$$

1(a) For finite λ_o, derive an expression for $Z(J)$ in terms of differential operators d/dJ acting on $Z_o(J)$; and then derive the differential equation of motion obeyed by $Z(J)$ for this theory.

1(b) Now find the equation of motion obeyed by $W(J)$ for the theory.

1(c) Now, by functional differentiation of these equations of motion with respect to J, find the first 3 in the infinite hierarchy of equations for the g_n; and then do the same for the c_n.

1(d) Now, finally, derive the general recursive expression relating the different cumulants of this theory, for arbitrary values of n.

NB: There is of course nothing stopping you from first finding the general recursive equation, and then using it to write down the results for the first 3 equations.

QUESTION (2): COUPLED SCALAR FIELDS

We now consider a system in which a self-coupled scalar field $\phi(x)$ is also coupled to another free scalar field $\chi(x)$, with the system described by the action

$$S[\phi, \chi] \;\; = \;\; \int d^D x \, \frac{1}{2} \left\{ \left[\partial_\mu \phi \partial^\mu \phi - m_0^2 \phi^2 - g\phi^4/4! \right] + \left[\partial_\mu \chi \partial^\mu \chi - M^2 \chi^2 \right] - \lambda_o \phi^2(x) \chi(x) \right\}$$

in D spacetime dimensions, with inter-field coupling vertex $-i\lambda_o/2\hbar$.

2(a) Let us first look at the ϕ-field on its own, assuming it is uncoupled from the χ-field. The 1-loop graphs for the 4-point vertex Γ_4 can be singular as a function of 3 different combinations of the incoming momenta p, p' and the momentum transfer q (the Landau, Peierls, and Cooper channels). Draw the 3 different graphs, showing their internal and external momenta, and write expressions for each of them in momentum space. Then take any one of them, and show that when $D = 4$, it scales like $ln\Lambda$, where Λ is the UV cutoff in the momentum integration.

2(b) Consider now the interacting theory. Write down the equations of motion for both $Z[J, I]$ and $W[J, I]$, where J and I are external currents coupling to ϕ and χ respectively. Then, putting $J = 0$, find a Schwinger-Dyson hierarchy of equations of motion by functionally differentiating $W[I]$ with respect to I (and then setting $I = 0$).

2(c) Draw all the graphs contributing to the ϕ-field self-energy, up to second order in both g and λ_o. Do the same for the 4-point vertex Γ_4 for the ϕ-field.

2(d) Finally, find the Bethe-Salpeter equation for the complete 4-point vertex Γ_4 for the field ϕ for this theory, and write the answer in both algebraic and diagrammatic form. You can of course try to derive this eqtn. algebraically - but it is much simpler and faster to do it by diagrammatic reasoning. Note that there is more than one irreducible vertex part involved.

END of QUESTION SHEET 2