Phys 508: HOMEWORK ASSIGNMENT No (3)

Monday 16th March, 2015

DUE DATE: Wed Mar 18th 2015.

Assignments handed in late will not receive a full mark.

QUESTION (1): LOOP EXPANSION for ϕ^6 THEORY

Consider the theory of a massive relativistic scalar field containing interactions of 6-th order only in the field, so that the action is

$$S \;=\; \int d^4x \; \left[\frac{1}{2}(\partial^\mu \phi \partial^\mu \phi - m^2\phi^2) \;-\; \frac{g_6}{6!}\phi^6 \right] \tag{1}$$

and we have a "ϕ^6-theory". We are going to look at the loop expansion for this theory.

(i) Make a table, analogous to that in the notes, showing all graphs up to $\sim O(g^2)$ for the 6-point, 8-point, and 10-point vertices in the theory; give the expressions for each one as integrals over Feynman propagators, and specify the order in $\hbar$ and the number of loops L.

(ii) Now find the equation of motion for the classical solution $\Phi_o(x)$ of this field theory (ie., the solution when $\hbar = 0$). Then, writing the full solution as $\phi(x) = \Phi_o(x) + \psi(x)$, find the action in terms of $\Phi_o(x)$ and this "quantum fluctuation field" $\psi(x)$, and derive an expression for both the classical action and the shift in this action caused by the quantum fluctuations. Show the graphs for the various vertices in this theory, and by integrating over the fluctuations, evaluate the lowest correction to the classical action from these fluctuations.

QUESTION (2): The SINE-GORDON and RELATED MODELS

The standard Sine-Gordon model has the action

$$S = \int dtdx \left(\frac{1}{2}\left[(\partial_t \theta)^2 - c_o^2(\partial_x \theta)^2 \right] \;+\; \omega_o^2(\cos\theta - 1) \right) \tag{2}$$

for the field $\theta(x,t)$ in 1+1 dimensions.

(i) Derive the 1-soliton solutions $\psi_\pm(x,t)$ for this model.

(ii) Now write the field $\theta(x,t)$ around one of these solutions as $\theta_+(x,t) = \psi_+(x) + \varphi(x,t)$ (where for simplicity we choose a static solution at the origin), and derive from this an equation of motion for $\varphi(x,t)$ up to <u>2nd order</u> in $\varphi(x,t)$ (ie., one order beyond the simple linearized theory). Then write the effective action for the φ-field which corresponds to this equation of motion; and show the bare vertices that exist in this theory, along with the values to be attached to each of them.

(iii) Finally, consider the system having an action

$$S = \int dtdx \left(\frac{1}{2} \left[(\partial_t \theta)^2 - c_o^2 (\partial_x \theta)^2 \right] \; + \; K_o^2 \cos^2 \theta \right) \tag{3}$$

again for the field $\theta(x,t)$ in 1+1 dimensions. Find the small oscillation solutions for this system, and their energy as a function of their wave-number; and find the form of the 1-soliton solutions, and draw a picture of one of these as a function of x at some time t.

QUESTION (3): GRADIENT EXPANSIONS in FIELD THEORY

We are going to do a gradient expansion for the simplest possible theory, viz., the ordinary Schrodinger eqtn. with an external field $\lambda_o J(\mathbf{r},t)$ acting on a free particle (here λ_o is a dimensionless parameter).

(i) Write down the eqtn. of motion for the 1-particle propagator $G(\mathbf{r},t;\mathbf{r}',t'|J)$ in the presence of the external field $J(\mathbf{r},t)$; and then assuming that $J(\mathbf{r},t)$ varies slowly as a function of its arguments, derive an equation of motion for the Fourier-transformed function $G_q(\mathbf{r},t;s|J)$, where $q = (\mathbf{q},\omega)$ is the Fourier transform of $x - x' = (\mathbf{r} - \mathbf{r}', t - t')$, and s is the Schwinger proper time. Write down a Dyson-type equation for the expansion of $G_q(\mathbf{r},s;|J)$ in powers of the gradients, and then write down the solution up to the lowest order in the gradient operator.

(ii) Now, taking the same result for the equation of motion of $G_q(\mathbf{r},t;s|J)$ that you just derived, rewrite it in"Fradkin" form as a proper time integral over a weighting factor $calF_q(\mathbf{r},t;s|J)$ times the free particle propagator; and writing $\mathcal{F}_q(\mathbf{r},t;s|J) = e^{i\Psi_q(\mathbf{r},t;s|J)}$, expand $\Psi_q(\mathbf{r},t;s|J) = \sum_n \psi_n \lambda_0^n$ in powers of λ_o and find the recursive equations of motion for the ψ_n in proper time. Solve explicitly for $\psi_1(\mathbf{r},t;s|J)$, and write the final form for $G_q(\mathbf{r},t|J)$ as a proper time integral.

END of QUESTION SHEET 3