Phys 508: HOMEWORK ASSIGNMENT No (2)

Friday Feb 13th 2015

DUE DATE: Friday Feb 27th 2015.

Assignments handed in late will not receive a full mark.

QUESTION (1): ϕ^6 THEORY

Consider the theory of a massive relativistic scalar field containing interactions up to 6-th order in the field, so that the action is

$$S \; = \; \int d^4x \; \left[\frac{1}{2}(\partial^\mu \phi \partial^\mu \phi - m^2 \phi^2) \; - \; \frac{g_4}{4!}\phi^4 \; - \; \frac{g_6}{6!}\phi^6 \right] \tag{1}$$

which is actually a quite rich theory.

(i) Draw all graphs up to 2nd order in g_4, g_6, ie., including terms up to $\sim O(g_4^2), \sim O(g_6^2)$, and $\sim O(g_4 g_6)$, for the connected correlators $\mathcal{G}_2, \mathcal{G}_4$, and $\mathcal{G}_6$.

(ii) Consider the 2nd-order graph for the 4-point vertex $\Gamma_4(1,2,3,4)$ which contains 4 internal lines connecting the 2 interaction vertices (there is only one such graph). Write the explicit expression for this graph, in both real spacetime and in 4-momentum space.

(iii) Derive the hierarchy of Schwinger-Dyson equations for this theory. You should find the general expression, showing how you get it, and also list the equations in this hierarchy up to all equations that include $\mathcal{G}_4$.

(iv) Finally - what form will the Bethe-Salpeter equation take for this theory, for the 4-point vertex Γ_4, in terms of irreducible vertices? You should show your result graphically, and give the expression for it both real spacetime and in 4-momentum space.

QUESTION (2): COUPLED SCALAR FIELDS

Now let us consider a pair of relativistic coupled massive scalar fields. Thus we now assume the action

$$S[\phi, \chi] \;\; = \;\; \int d^4x \, \frac{1}{2} \left\{ \left[\partial_\mu \phi \partial^\mu \phi - m_0^2 \phi^2 \right] + \left[\partial_\mu \chi \partial^\mu \chi - M^2 \chi^2 \right] - \lambda_0 \phi^2(x) \chi(x) \right\} \tag{3}$$

with coupling vertex $-i\lambda_0/2\hbar$; there are no interactions in this theory, except those between these free fields.

(i) Find the lowest Schwinger-Dyson equations for this theory, giving explicit expressions for the relevant functional derivatives of the action.

(ii) Now derive the Ward identity relating the 3-point vertex $\Lambda_{21}^{\phi\chi}(p,q)$, for a ϕ excitation interacting with a χ excitation, in terms of the 2-point correlators $\mathcal{G}_2^{\phi\phi}(p)$ of the ϕ-field. You will find the comparison with the derivation for QED to be helpful.

(iii) Finally - write down and show the graphical expression for the Bethe-Salpeter equation for the 4-point vertex Γ_4 for the ϕ-field in this theory, iterated in the Cooper channel, and show all graphs for the irreducible 4-point vertex up to 3rd order in the interaction. Once you have done this, assume that the irreducible vertex is given by the lowest-order graph alone, and derive a Schrodinger equation for a pair of particles in the non-relativistic limit, in real space, assuming that the interaction is attractive. What is the real space form for this interaction?

END of QUESTION SHEET 2