Phys 508: HOMEWORK ASSIGNMENT No (1)

Friday January 23rd 2015

DUE DATE: Monday Feb 9th 2014.

Assignments handed in late will not receive a full mark.

QUESTION (1): NON-RELATIVISTIC FERMIONS

Consider the graph shown in Fig. 1 here, which is closely related to graphs that are already calculated in Appendix B.3. The internal interaction is $V(\mathbf{q})$, a function of momentum only. Calculate and expression for this graph at finite T, using the Matsubara formalism. Only do the frequency integrals, not the momentum integrals.

You can assume that the external boson line is a phonon, coupled to the fermion via a coupling constant λ_o.

QUESTION (2): COUPLED FIELDS

In Appendix B.1 it is asserted that if we apply the quandratic shift operator to a Gaussian functional, ie., if we write

$$\begin{aligned} \hat{Q}_2 F[\phi] &\equiv \hat{Q}_2 \, e^{\frac{i}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \; \phi(x_1)A(x_1,x_2)\phi(x_2) + i\int \mathrm{d}x \; J(x)\phi(x)} \\ &= e^{\frac{i}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \; \frac{\delta}{\delta\phi(x_1)} K(x_1,x_2) \frac{\delta}{\delta\phi(x_2)}} \left[e^{\frac{i}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \; \phi(x_1)A(x_1,x_2)\phi(x_2) + i\int \mathrm{d}x \; J(x)\phi(x)} \right] \end{aligned} \quad (1)$$

then we get the result

$$\begin{aligned} \hat{Q}_2 F[\phi] &= \Omega_o \exp\left\{ \frac{i}{2} \int \mathrm{d}x_1 \int \mathrm{d}x_2 \right. \\ & \left. \left[\phi(x) G^{\phi\phi}(x,x')\phi(x') \; + \; 2\phi(x) G^{\phi J}(x,x') J(x') \; - \; J(x) G^{JJ}(x,x') J(x') \right] \right\} \end{aligned} \quad (2)$$

for the shifted functional. Derive this result, finding the form of the correlators and the prefactor.

(ii) Now we are going to look at the related problem of relativistic coupled fields. We assume the action discussed in section B.3.4, viz.,

$$S[\phi,\chi] \;=\; \int d^D x \, \frac{1}{2} \left\{ \left[\partial_\mu \phi \partial^\mu \phi - m_0^2 \phi^2 \right] + \left[\partial_\mu \chi \partial^\mu \chi - M^2 \chi^2 \right] - \lambda_0 \phi^2(x)\chi(x) \right\} \quad (3)$$

with coupling vertex $-i\lambda_0/2\hbar$.

Derive expressions for the first three non-trivial graphs for the propagator $\mathcal{G}_2^{\phi\phi}(x,x')$ (hint: these are shown in the figure labelled as eqtn (132) in section B.3 of the notes), by

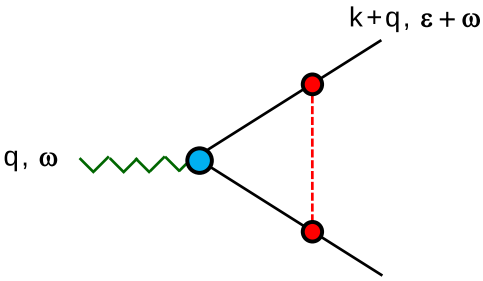

Fig 1

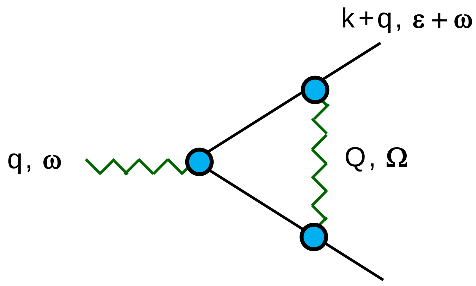

Fig 2

Figure 1: Diagrams for the Questions 1 and 3. Fig. 1 shows the graph for question 1, and Fig 2 shows the graph for question 3; the external energy and momenta are labelled for Fig 1, and Fig 3 also labels the internal phonon line.

starting from the "no loop" expression for the generating functional, given by

$$\underset{\text{(no loops)}}{\mathcal{Z}[J,0] \longrightarrow} \quad e^{\frac{i\hbar}{2}\int d^Dx \int d^Dx' \, \frac{\delta}{\delta\psi(x)} \, \Delta_F^{\chi}(x-x') \, \frac{\delta}{\delta\psi(x')}} \; e^{\frac{-i}{2\hbar}\int d^Dx \int d^Dx' \, J(x) \, \tilde{\Delta}(x,x'|\psi) \, J(x')} \Big|_{\psi=0} \tag{4}$$

QUESTION (3): ELECTRON-PHONON VERTEX

Consider the electron-phonon graph shown in Fig. 2. Calculate this graph at finite T, doing only the frequency sums, and assuming the standard electron-phonon Hamiltonian

$$H \;=\; \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \;+\; \sum_{\mathbf{q}} \hbar\Omega_{\mathbf{q}} (b^{\dagger}_{\mathbf{q}} b_{\mathbf{q}} + \frac{1}{2}) \;+\; \lambda_o \int d^3\mathbf{r} \psi^{\dagger}_{\sigma}(\mathbf{r}) \psi_{\sigma}(\mathbf{r}) \phi(\mathbf{r}) \tag{5}$$

with coupling λ_o. Again, as in question (1), the external bosonic line is a phonon, coupled to the fermion via a coupling constant λ_o.

END of QUESTION SHEET 1