PHYS 403: HOMEWORK ASSIGNMENT No. 4: DEGENERATE FERMIONS

(April 6th, 2021)

DEADLINE for HOMEWORK: FRIDAY, April 16th, 2021

To be uploaded by 11.59 pm, April 16th- Late Homework will not be accepted

QUESTION (1) DEGENERATE FERMIONS - NUMBERS:

(i) Consider the sun, of mass $M_s = 2 \times 10^{30}$ kg, to be a plasma of completely ionized H atoms, of diameter 1.3×10^6 km. How many electrons are there in the sun? If the surface temperature of the sun is $6,000$K, find the power output of the sun. Finally - the temperature at the centre of the sun is 1.5×10^6K, so what is the effective radius of the region in the sun where fusion is taking place?

(ii) Suppose at the end of its life the sun collapses to a white dwarf, with the same mass, but with radius $20,000$ km, and uniform density. What will be the Fermi energy of the electrons? If the white dwarf surface temperature is 10^7K, how degenerate are the electrons?

(iii) Now suppose the sun collapses to form a neutron star, again with the same mass. Supposing a uniform density, what will be the Fermi energy of the nucleons, in both eV and in temperature units?

QUESTION (2) DEGENERATE FERMIONS - MEAN ENERGIES:

(i) Show that for a non-relativistic 3-dimensional Fermi gas at temperature $T = 0$, the mean energy per fermion is given by $\langle \epsilon \rangle = 3E_F/5$.

(ii) Now find out what is $\langle \epsilon \rangle$ for a 3-dimensional Fermi gas at temperature $T = 0$ in the extreme relativistic limit, ie., the typical momentum of the fermions satisfies $p \gg mc$.

(iii) Finally, suppose we now have a 2-dimensional Fermi gas at $T = 0$. Find the mean energy per fermion in this case, both in the non-relativistic and relativistic limits.

END of 3rd HOMEWORK ASSIGNMENT