PHYS 403: HOMEWORK ASSIGNMENT No. 3: SOME REAL WORLD SYSTEMS

(March 19th, 2021)

DEADLINE for HOMEWORK: FRIDAY, April 2nd, 2021

To be uploaded by 11.59 pm, April 2nd- Late Homework will not be accepted

QUESTION (1) HEATING A LIQUID: When a liquid is heated it eventually boils - but this does not necessarily happen at the boiling point. We would like to understand why this is.

(i): Consider a bubble of vapour forming inside a hot liquid. The energy of the bubble is a competition between the surface energy (which is positive), and the bulk energy (which is negative, because the liquid is above its boiling point). Assuming that the bulk energy goes like $U_B = U_o(T_c - T)$ per unit volume of bubble, where T_c is the liquid-gas phase transition temperature, and that the surface tension γ of the bubble is independent of temperature T, find an expression for the energy $U(R)$ of a spherical bubble as a function of its radius R, when $T > T_c$. Find the radius R_c at which this energy is a maximum.

(ii): If the bubble radius can grow past R_c, the system will boil. Estimate the probability that this will happen from a thermodynamic description of these bubbles, as a function of their radius, in the canonical distribution.

QUESTION (2) PHOTON GAS: We consider a gas of photons at equilibrium temperature T, in a box of volume V; the photons move at velocity $c \sim 3 \times 10^8\ ms^{-1}$, and have 2 internal polarization states.

(i): Show that the pressure $p(T)$ of the photon gas satisfies $3p(T) = u(T)$, where $u(T)$ is the energy density (energy per unit volume).

(ii): The chemical potential of a gas of photons in equilibrium at temperature T, in a box of volume V, is $\mu = 0$ for any temperature - explain why this is. Then, using the Maxwell relation relating entropy S, temperature T, pressure p, and volume V of the photon system, write the 2nd law of thermodynamics in differential form for the photon gas, and show that

$$u(T) \ = \ T(\partial p/\partial T)_V - p(T) \tag{0.1}$$

and then, using the result that $3p(T) = u(T)$, already found above, show that $T(du/dT) - 4u = 0$. What is the solution to this equation, and what well-known radiation law does it express?

Finally, let us imagine that the volume V of the box is changing slowly with time (so that the change is adiabatic). Again using the 2nd law of thermodynamics, show that the constant entropy of the system obeys $S \propto VT^3$.

(iii): The sun can be considered to be a black box radiator at a temperature $T = 6000\ K$. The Stefan-Boltzmann law states that the energy radiated per unit second by a black body, per unit area of the body, is

$$J(T) \ = \ J(T) \ = \ \sigma T^4 \tag{0.2}$$

where the constant $\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$. The radius of the sun is $r_o \sim 0.7 \times 10^6$ km, and its distance from the earth is $R_o \sim 1.5 \times 10^8$ km.

From this information find (a) the radiant power from the sun, in Watts per square metre, at the distance of the earth; (b) the temperature reached by a black body solar power panel on a satellite, directly facing the sun (remember that it radiates in both directions); and (c) the pressure on this panel, per square metre, from the solar radiation.

END of 3rd HOMEWORK ASSIGNMENT