PHYS 403: HOMEWORK ASSIGNMENT No. 1: PROBABILITIES, THERMODYNAMICS, and MICROSTATES

(Jan. 24th, 2021)

HOMEWORK DUE: WEDNESDAY, Feb 10th, 2021

To be uploaded by 11.59 pm, Feb 10th- Late Homework will not be accepted

QUESTION (1) PROBABILITIES for DISCRETE OUTCOMES: This question is to help you with calculating probabilities for a finite set of discrete outcomes.

Two players, A and B, are each dealt a hand of 5 cards from a randomized pack of 52 cards (the usual pack here, with aces, kings, queens, jacks, and numbers from two to ten).

1(a) What is the probability that player A will get 3 Aces in their hand of 5 cards?

1(b) Suppose now that player A throws away two cards, leaving only 3 Aces in his/her hand. The he/she is dealt two more cards. What is the probability that one of these 2 cards is the fourth Ace?

QUESTION (2) THERMODYNAMICS for a MAGNETIC SYSTEM: We consider an incompressible magnetic system, so that we can ignore any changes in volume. Assume the system has magnetization M along an external field B.

2(a) Derive the infinitesimal change dF for the Helmholtz free energy; Then an expression for the infinitesimal change dU in energy brought about by changes in the extensive variables of the system;

2(b) Now find an expression for the rate of change of S with respect to changes in M, in terms of the change dT when we make a change dB in the external field.

2(c) Now suppose we allow particles to move in and out of this system. What now do we find for dF ad dU? And what is the new relationship for the rate of change of S with respect to changes in M?

QUESTION (3) N-SPIN SYSTEM: Consider a set of N non-interacting spin-1 systems in a magnetic field, such that the energy of each individual spin is $\epsilon_j = 0$, $\epsilon_j = \Delta_o$, or $\epsilon_j = -\Delta_o$.

3(i) Find W, the number of available microstates for a system having energy U; note that the value of U ranges from $U_{min} = -N\Delta_o$ up to $U_{max} = N\Delta_o$, in discrete steps of Δ_o

3(ii) Using the relation that the entropy $S = k_B \ln W$, And assuming that $N \gg 1$, find an expression for the entropy S as a function of the energy U of the system.

END of 1ST HOMEWORK ASSIGNMENT