PHYS 403: HOMEWORK ASSIGNMENT No. 2: SIMPLE MODELS

(Jan 26th, 2020)

HOMEWORK DUE: MONDAY, FEB 10th, 2020

To be handed in during class- Late Homework will not be accepted

QUESTION (1) MICROSYSTEM PROBABILITIES: This one is to help you again with your understanding of probability theory.

A closed box, impermeable to the passage of energy, particles, or anything else, contains 10^3 identical distinguishable particles. Each particle has 3 internal states, with energy $0, \epsilon$, and 2ϵ respectively.

1(a) What are the allowed energies of the total system, and what is the probability of finding the total system in a state of total energy E.

1(b) What are the probabilities that a given molecule will find itself in a state with one of the energies $0, \epsilon$, or 2ϵ, if we assume the the total system has energy $E = 500\epsilon$?

QUESTION (2) LATTICE INTERSTITIALS: This one is again to help you with your understanding of probability theory, and also of the canonical distibution.

Suppose we have a crystal containing N atoms, each of which at $T = 0$ occupies one of the N crytal lattice sites. However if energy is injected into the system, atoms can leave these lattice positions and go into a "dual lattice" of interstitial sites - there are N of these as well, and each of of them has energy ϵ when occupied by an atom.

2(a) Show that the total number of states in the system with energy $n\epsilon$ is

$$g(n\epsilon) \;=\; \left(\frac{N!}{n!(N-n)!}\right)^2 \tag{0.1}$$

2(b) Using the canonical distribution, find the probability that at finite temperature T the system will have a total energy $E = n\epsilon$

2(c) Using Stirling's approximation, find the mean value $< E >$ of the energy, and the mean value of n; and find the probability distribution in the immediate vicinity of this value.

QUESTION (3) TOY MODEL OF POLYMER CHAIN: Let is now apply the canonical distribution to a finite temperature toy model of a polymer chain (ie., the model discussed in the notes), but now with an added twist.

We assume a 1-dimensional chain with N links of length a, which can only lie horizontally; thus the maximum length of the chain when stretched out is Na. However we now assume that there is an orientational energy involved; each link has a "positive" and "negative" end, with links connected so that a positive end on one link connects to a negative link on the next one. We also assume that the chain is fixed at one end, at the origin. We then apply an external field, so that any given link has energy ϵ if its link is oriented with the positive end to the right, and energy $-\epsilon$ if it is oriented oppositely. The link connected to the origin has its negative end at the origin.

3(a) What is the energy of a configuration of the chain with total extension $\ell = na$, where ℓ is defined as the displacement of the end of the chain from the origin (so that $-n \leq n \leq N$, and ℓ can be positive or negative)?

3(b) What is the probability that the chain will have extension ℓ at a temperature T, and what is the entropy for such a configuration? Using Stirling's approximation, find the probability distribution for the entropy near its maximum.

END of 2ND HOMEWORK ASSIGNMENT