PHYS 403: HOMEWORK ASSIGNMENT No. 1: MICROSTATES and PROBABILITIES

(Jan. 14th, 2020)

HOMEWORK DUE: WEDNESDAY, JAN 22nd, 2020

To be handed in during class- Late Homework will not be accepted

QUESTION (1) PROBABILITIES for DISCRETE OUTCOMES: This one is to brush up your understanding of probability theory.

You are dealt a hand of 5 cards from a randomized pack of 52 cards (the usual pack here, with aces, kings, queens, jacks, and numbers from two to ten).

1(a) What is the probability that you will get "two pairs"? This means a hand in which you have a pair of one card, a pair of another card, and any other card (an example would be 2 kings, two fours, and a seven).

1(b) What is the probability that you will get a hand with two pairs, in which one pair is a pair of aces, and the other pair is any pair you like (apart from another pair of aces of course)?

QUESTION (2) MICROSTATES for MODEL SYSTEMS: This one is to help you see how probabilistic arguments can be used on simple models for many-body systems.

1(a) Consider a system of N '3-bits' (ie., spin-1 systems), in which each spin can be in state $|1\rangle$, $|0\rangle$ or $|-1\rangle$, (where $\pm 1, 0$ refer to the spin projection along the z axis). Assume there is no applied field, so all of the 3^N states are equally probable.

What then is the probability that you will find the total system with a polarization m along the z axis, where $N \geq |m|$?

1(b) Now consider a set of N simple harmonic oscillators (SHO's), each with the same frequency ω_o. We would like to know the number of microstates which all correspond to the same macrostate, having total energy $E_n = E_o + n\hbar\omega_o$, where $E_o = n\hbar\omega_o/2$ is the zero-point energy. Show that this number is just

$$\Omega_N(n) \; = \; \frac{(N-1+n)!}{(N-1)!n!} \tag{0.1}$$

END of 1ST HOMEWORK ASSIGNMENT