QUANTUM MECHANICS – Uncertainty, Measurement, & Entanglement

Now we come to the bizarre implications of the Quantum rules that were introduced in the last section…

The Uncertainty Principle I

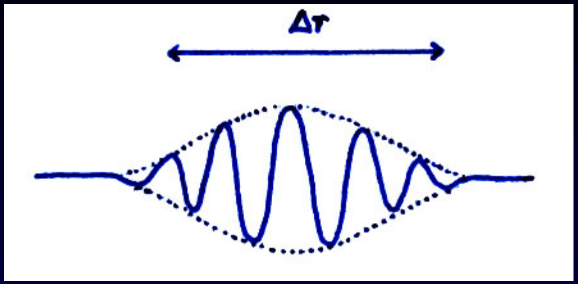
The uncertainty principle is really just a fact about waves of any kind. In the case of quantum particles it says the following:

Suppose we localize the probability wave of a particle so that it is confined to a length of magnitude Δr (often called a "wave-packet" of size Δr). We say the position is UNCERTAIN, because it can be anywhere in this region. The uncertainty principle says that the momentum p of the particle is also uncertain- it is also smeared out, over a range Δp. The crucial result is that $\Delta r \sim h/\Delta p$

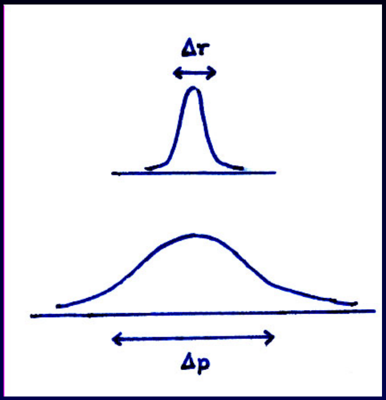
or $\Delta r \, \Delta p \sim h$ **where h is Planck's constant.**

Uncertainty Principle II

How can we understand the uncertainty principle? What it is saying is shown at right- if we want to make a wave-packet of spatial extent Δr, we can only do this by adding together waves of different λ, ie. different p.

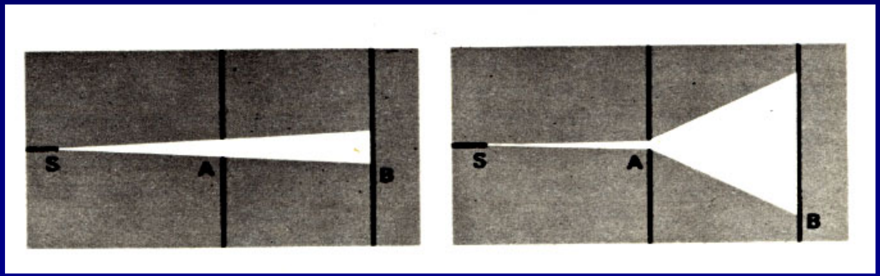


A wave-packet confined to a size Δr; the spread in wavelengths gives a spread Δp in momentum



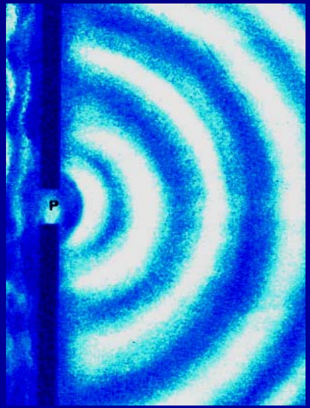
The net result is that a spread or uncertainty Δr in position means uncertainty Δp in momentum. The smaller is Δr, the larger is Δp (and vice-versa).

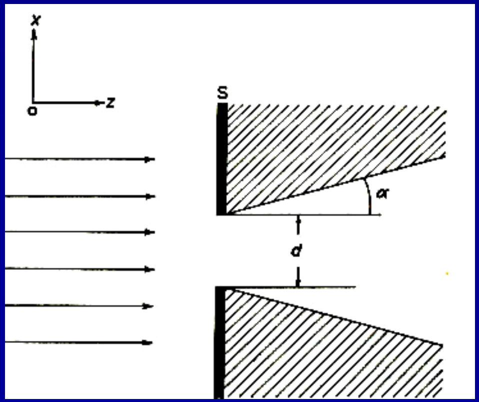
Actually we have met this already, it is merely a fact about waves. Thus diffraction effects are always larger if we confine a wave with a small hole. It is easy to show that a wave going through a hole of size d will spread an angle α by diffraction, where



$$\alpha \sim \lambda/d = h/(p.\Delta r)$$

since d is just Δr here. The diffraction gives a momentum kick proportional to α, so that $\Delta p \sim \alpha p$. We then just get the uncertainty principle back.

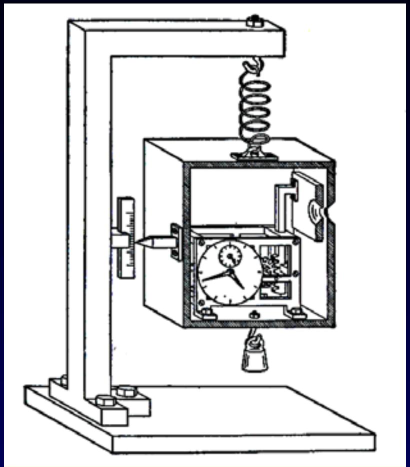




LEFT: if the wavelength is the same size or smaller than the hole, we get diffraction over all angles.
RIGHT: small wavelength gives small angle diffraction

Uncertainty Principle III

One can consider many examples of the uncertainty principle-. This was done in the early days of QM, particularly because Einstein objected strongly to it. When Bohr demonstrated that to contradict it would lead to a contradiction of QM with even General relativity (using the thought experiment at right) his opposition collapsed- the principle is now universally accepted.



Bohr's thought Experiment

Uncertainty Principle for SPIN

We saw on page 4.28 that we spin exists in discrete values or 'quantum numbers'. The uncertainty principle is very simple for spin-1/2 systems. As described on p. 4.28, an "up" spin is a superposition of left and right oriented spins. In QM one writes, eg., for an "up" spin:

$$\Psi_{+} \sim (\phi_{->} + \phi_{<-})$$

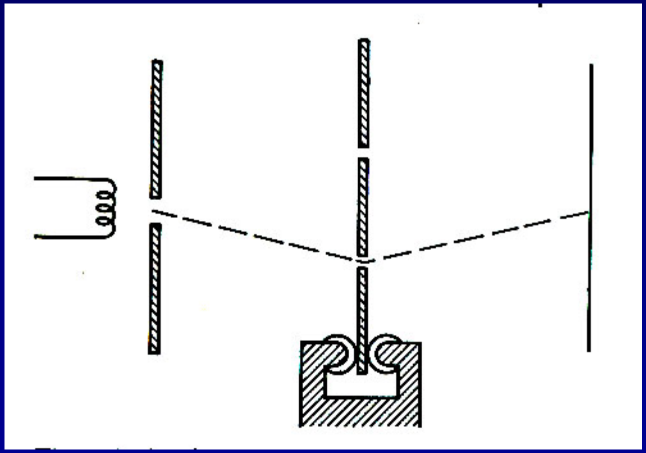
where the symbols $\rightarrow$ and $\leftarrow$ mean spins oriented "left" and "right" respectively. This means an equal probability of 50% of finding the spin up system in a left or right state.

However this has a simple interpretation- there is an uncertainty principle for spin orientations. If we fix the orientation along one axis very tightly, then it becomes indeterminate in perpendicular directions. This is the analogue of the uncertainty principle governing position and momentum.

QUANTUM MEASUREMENTS: I

One of the most difficult points in QM is the idea of the measurement. Here I give you a simple approach to this- which depends on the assumption that some BIG system is ultimately doing the measurement, and that it behaves classically.

Essentially a measurement involves an interaction between the physical system of interest and a measuring apparatus. This establishes a correlation between the state of the system before the measurement interaction, and the state of the apparatus afterwards. It is assumed that because the apparatus is classical, finding its state is then simple.



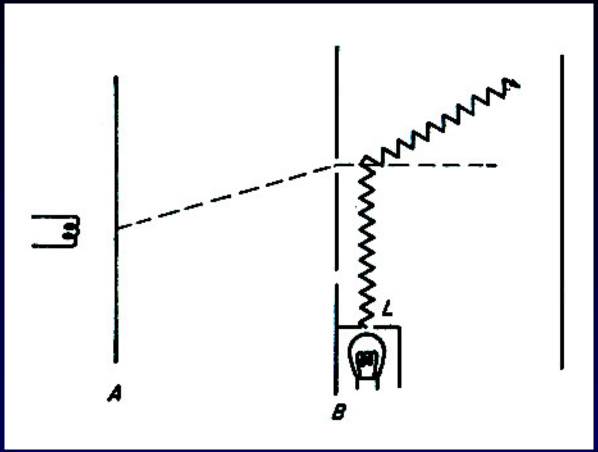
A 2-slit set-up where the plate with the slits can move up or down. Any scattering of the particle going through a slit, changing its direction, means exchange of momentum with the plate- which then recoils.

At left we see a simple example. We want to measure through which slit the particle passes. The 2-slit apparatus is set up so that it can move without friction between 2 rollers. If a particle goes through the bottom slit and back up to the screen, the plate containing the slits will recoil downwards. On the other hand it will recoil up if the particle goes through the upper slit. Then, by watching the motion of the plate containing the slits, we can see which slit the particle went through.

Actually this will destroy the interference pattern- see next slide.

QUANTUM MEASUREMENTS: II

The interaction with the measuring device also has an important connection with the uncertainty principle. Consider a slightly different way of measuring through which slit the particle passes, looking at it with photons. Two points are crucial:



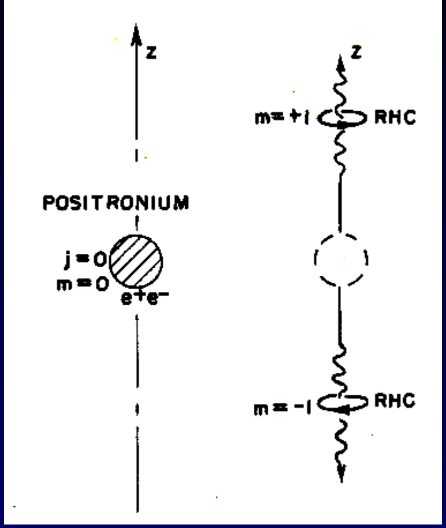
Using a microscope to see which slit the particle goes through. The particle is seen if photons scatter off it (into a microscope). But this causes the particles to recoil.

(i) To see which slit the particle goes through, we need photons of a short enough wavelength- if the wavelength is λ, we can't resolve the position at a finer scale than this (this is why light microscopes cannot resolve detail smaller than the wavelength of light).

However this light has a momentum, and in interacting with the particle it will give it a momentum kick of roughly the same size. As a result the particle acquires an uncertain momentum, so it no longer has a well-defined wavelength. If we work out the mathematical details we find the interference pattern is then smeared out completely because of this.

(ii) Actually this makes sense. If we can tell which slit the particle goes through, it follows logically that there can be no more interference pattern on the screen- interference only happens if the particle can go through both of them, without choosing a particular path.

ENTANGLEMENT between QUANTUM SYSTEMS



Positronium → 2 photons in state Ψ_{+-}

Consider a system of an electron & positron 'orbiting' each other- the overlap of their 2 wave-functions means they will eventually mutually annihilate, with emission of 2 photons. These photons must have equal & opposite momenta & spin, because the original system had zero momentum and spin, & these 2 quantities are conserved.

Such a state is shown at left. If we label the spin along the direction of photon propagation (usually called the 'helicity') by + & - , the the state shown can be written as

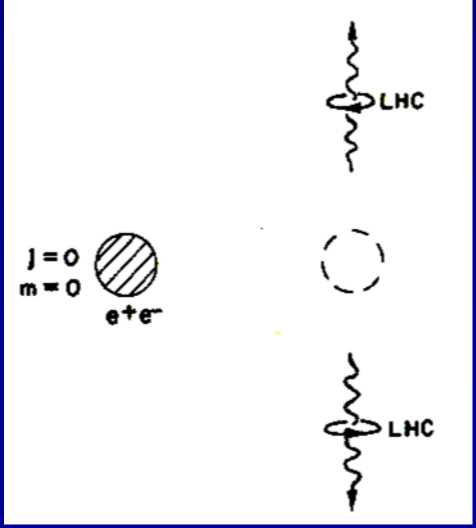
$$\Psi_{+-}(1, 2) = \phi_+(1)\ \phi_-(2)$$

where particle 1 is moving up, and particle 2 down.

However consider the state shown at right, which is:

$$\Psi_{-+}(1, 2) = \phi_-(1)\ \phi_+(2)$$

BELOW: positronium → photons in state Ψ_{-+} with photon 1 spinning clockwise along photon direction, & photon 2 anticlockwise.



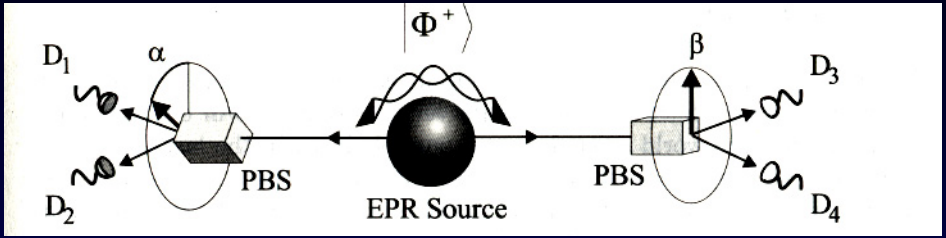
But QM uses all possible paths- so we can actually have a state like

$$\Psi \sim (\Psi_{+-} + \Psi_{-+}) = [\ \phi_+(1)\,\phi_-(2) + \phi_-(1)\,\phi_+(2)\ ]$$

Now in this state each photon has no definite spin- BUT we do have a definite quantum state! The state is such that the 2 spins must be opposite- they are "entangled".

The Einstein-Podolsky-Rosen (EPR) Paradox

!st detector, set to measure the a spin state at angle α



2nd detector, set to measure the spin state at angle β

We can now explain this paradox fairly easily. Suppose we have a state of 2 spins such that they must be opposite. We can write one such state as $\Psi = |+ -\rangle$ which is a simple notation meaning they are up/down. Another could be $|+ -\rangle$ meaning they are down/up; and we could have $(|+-\rangle + |-+\rangle)$. These are the 3 states talked about on the last slide.

Now we let the 2 spins separate by a large distance, & have 2 measuring systems to measure the spins at the 2 places. Suppose the measuring systems measure if the spins are up or down. Then if the 1^{st} measuring system finds spin 1 is up, we KNOW spin 2 will be down- & vice-versa. Apparently spin 2 must be then either up or down.

However now suppose at the very last instant we change our minds, & switch the 1^{st} apparatus to measure whether spin 1 is → or ← . Now if we find spin 1 is → we KNOW spin 2 is ← ; and vice-versa. The spins and apparatus are so far apart that no signal can travel between them in the time after we change our mind (unless it goes faster than light!). And yet by suddenly switching the 1^{st} apparatus, QM says we change the possible quantum states that spin 2 can have. This is the EPR paradox (published in 1935), which led Einstein to argue that QM was not a complete theory of Nature.

BELL's THEOREM & INEQUALITIES

JS Bell (1928-1990)

It was a relatively unknown Irish physicist who underlined the really crucial point that lay behind the ideas of EPR. What basically bothered Einstein was that QM seemed to be incompatible with any sensible notion of what 'physical reality' must be. How is it possible to say that, eg., the spin of the 2nd particle in the EPR has a is "real" (in EPR language, that it 'corresponds to an element of physical reality'), if the state that we measure can be altered by changing the disposition of some measuring system that might be light years away?

What Bell showed can be summed up in 'Bell's theorem':

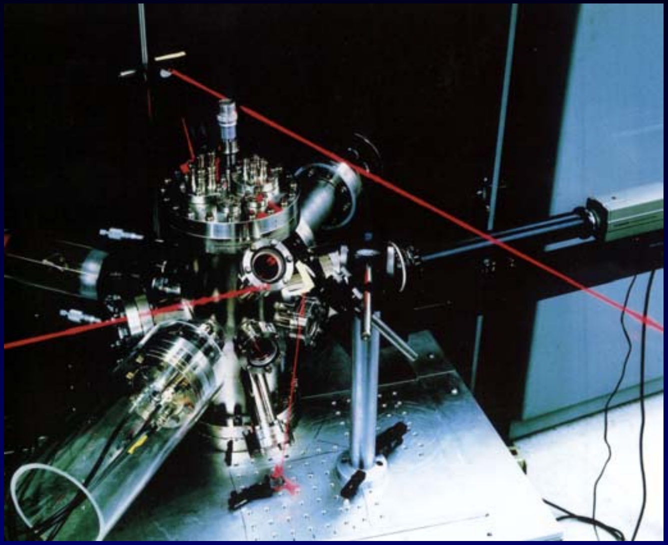
"No LOCAL HIDDEN VARIABLE THEORY of any kind can reproduce all of the experimental predictions of Quantum Mechanics" (Bell, 1964)

What does this mean? A 'hidden variable theory' tries to explain the fact that QM only predicts probabilities by arguing that there is some unknown and random disturbance acting on quantum systems. A 'local' theory is one in which the state of a system is defined locally (ie., in one point or region of spacetime), and is also assumed to be influenced locally (ie., by other objects or fields in the same region). In a non-local theory events or processes elsewhere must be involved. The crucial point is that it is impossible to have non-locality and also satisfy special relativity, if the non-locality is caused by physical interactions. Essentially what Bell's theorem is saying is that 'realism' (that the objects described by QM are objectively real) is incompatible with relativity.

Even more remarkably, Bell was able to QUANTIFY by how much the predictions of any local hidden variable theory would differ from QM- these are the famous 'Bell Inequalities'. Thus it is basically shown how one can test in an experiment (next page) whether some of our most cherished ideas about physical reality are true- this is perhaps the first time that experiment has been used to directly come to a conclusion about what are usually taken to be metaphysical questions!

ENTANGLEMENT EXPERIMENTS with PHOTONS

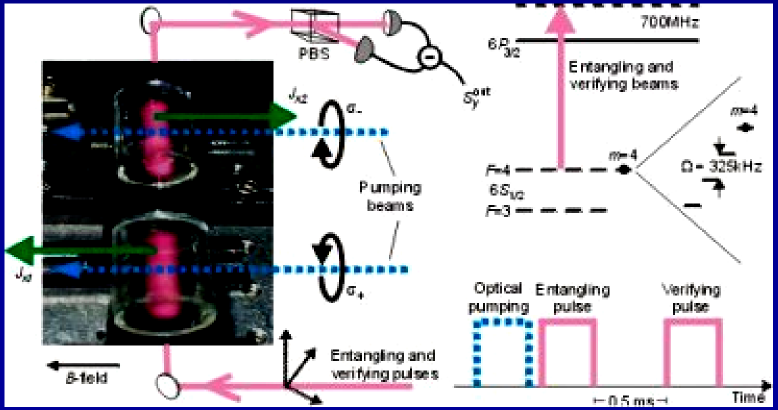
The simplest way to look for entanglement- & at the same time test Bell's inequalities- is to look at correlated pairs of photons.



The experiments look at EPR pairs of photons which must have opposite polarisation. The experiment measures the correlations between the polarisations of the 2 photons, with the angle between the 2 polarisers being varied (see figure on page 4.37). Quantum Mechanics makes definite predictions about the angular dependence of these correlations.

The 1st tests of quantum theory for entangled photons were done in the 1960's. The results indicated the validity of QM, but communication between the 2 polarisers was not eliminated.

Experiments have now been done in which the quantities measured on 2 separated but entangled systems, are varied separately & randomly- so quickly that no signal can pass between the 2 systems (page 4.37). We can compare QM with "local hidden variable theories" (in which the probabilistic results of QM arise from ignorance of underlying deterministic variables, which are 'local', ie,, which describe individual systems). The results (Aspect et al., 1982) rule out ANY such theory in favour of QM. A more recent experiment (right) has entangled huge numbers of atomic spins in 2 separate gas cylinders



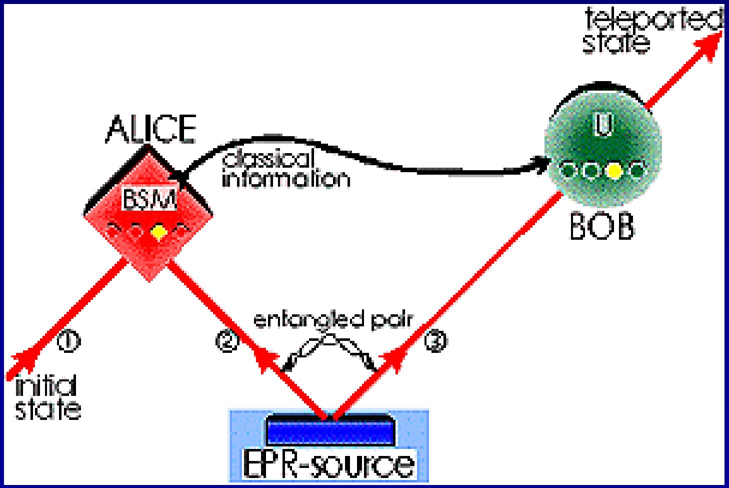
USING ENTANGLEMENT

What has been appreciated in the last 15 years is that the extraordinary property of entanglement can be used as a 'resource'. To see why this is we consider a pair of entangled spins. Now if these were classical 2-state systems (of the kind used in a computer), they would exist in ONE of 4 possible states- these being (11) (10) (01) (00) . Now the crucial difference in QM is that the general state is written in the form:

$$\Psi = \alpha_{++}|++> + \alpha_{+-}|+-> + \alpha_{-+}|-+> + \alpha_{--}|-->$$

This state is a superposition of the 4 possible states- not just one of them- with 4 different contributing Amplitudes, which can be varied. This gives us a much larger number of possible states than the 4 possible classical states- meaning that much more information can be stored or processed by the 2 quantum spins. For those of you with mathematical knowledge, the information is not just stored in the magnitude of the coefficients, but also in their PHASES:

$$\alpha_{++} = |\alpha_{++}| \exp(i\phi_{++})$$



TELEPORTATION: using an EPR source, Alice teleports a Quantum state to Bob. The Time axis is vertical, space axis horizontal

This feature is at the heart of quantum computing, to be described later. A simpler application is to what is called QUANTUM TELEPORTATION. In this scheme a pair of photons or spins is prepared in an UNKNOWN entangled state, and one of each is sent to Bob & Alice. Neither observes their spin- instead, each of them lets it interact with another one of their own. Then Alice measures the state of her new pair (thereby destroying it), & sends the result to Bob (this is classical information). Bob can then manipulate his pair, based on this info, to exactly recreate the original entangled state.