QUANTUM MECHANICS – Intro to Basic Features

1. QUANTUM INTERFERENCE & QUANTUM PATHS

Rather than explain the rules of quantum mechanics as they were devised, we first look at a more modern formulation (due to RP Feynman in 1948), which is in some ways easier to understand intuitively. I will then give a you some understanding of how it relates to the ideas of wave amplitude propagation, due to Schrodinger in 1925.


R.P. Feynman

E. Schrodinger

Let us recall again the mysterious feature of quantum mechanics that began to be uncovered in the work of Newton and Huyghens:
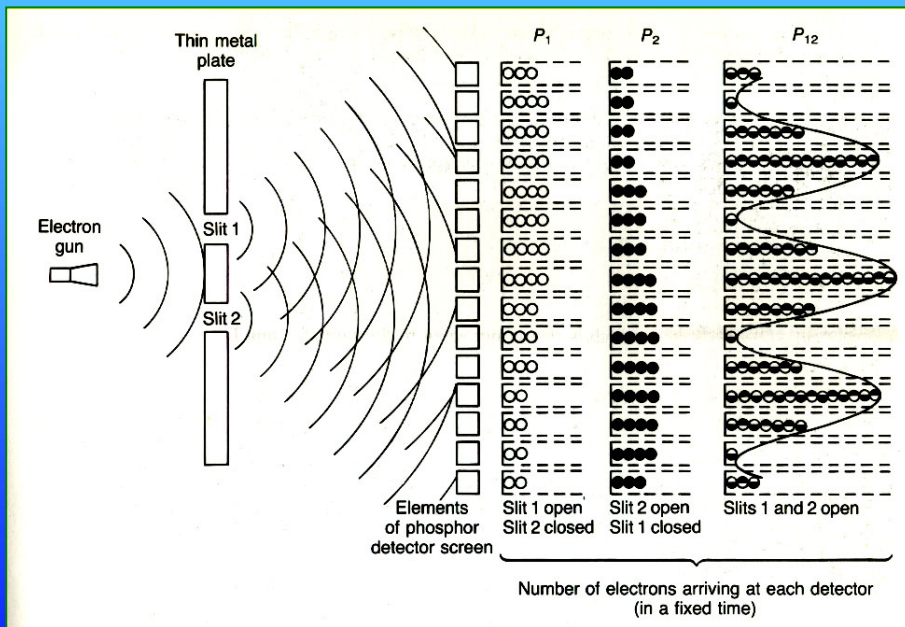
Particle – Wave DUALITY

There is NO WAY to understand the passage of quantum particles via 2 slits as 'particles'- they show wavelike interference. On the other hand we cannot understand their propagation as waves either- they arrive in "lumps".
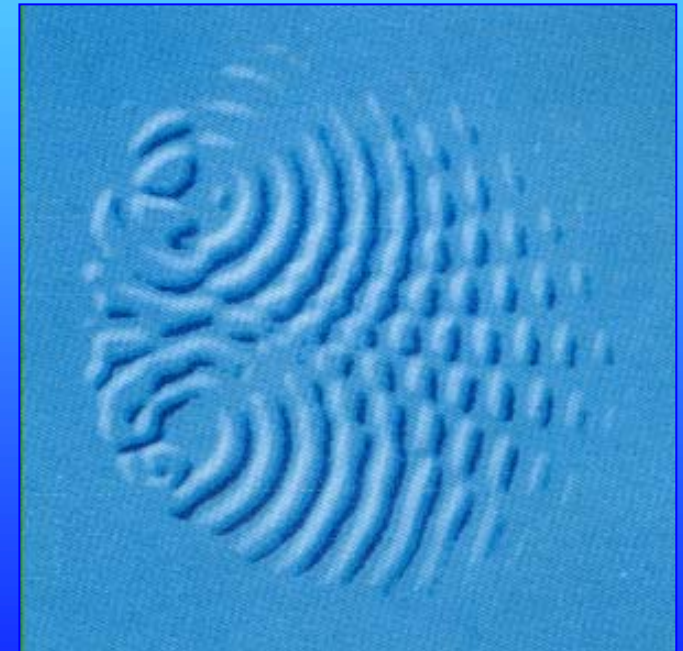
So – How can we understand the duality?
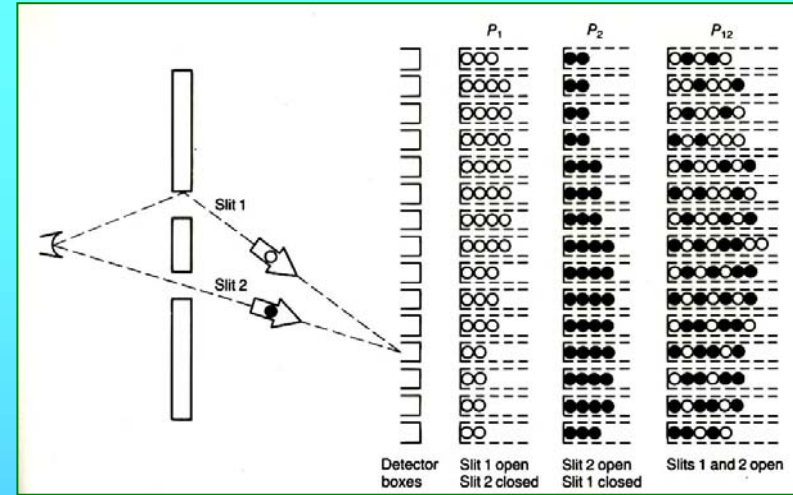
Classical probabilities for the passage of classical particles through 1 or 2 slits

Probabilities for passage of quantum particles via slits

Wave interference between 2 sources
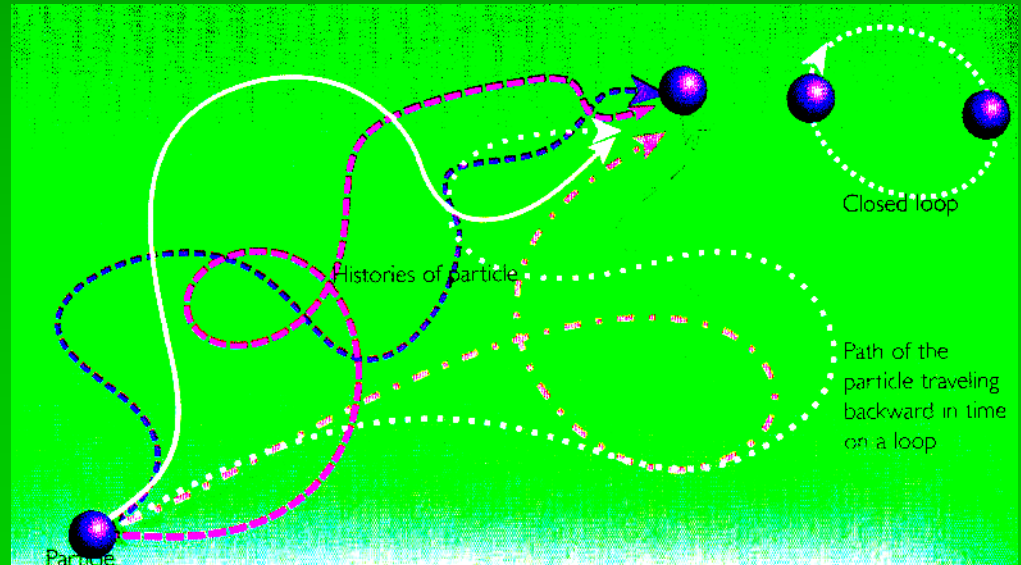
The QUANTUM RULES - Sum over PATHS

Now we come to the strange rules of quantum mechanics. Here I am going to tell you the version of them that was invented by Feynman in 1948. It goes as follows:
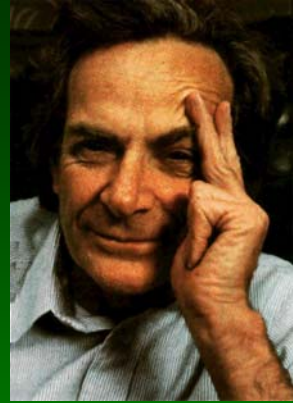
(1) First we find a quantity called the **QUANTUM AMPLITUDE** for a system to go from one quantum state ψ_1 to another state ψ_2. This amplitude we call G_{21}.

Feynman (1920-87)

(2) Now, to find out what the amplitude is we imagine that the system simultaneously tries to go over **ALL POSSIBLE PATHS** between the 2 states. We give each path a weight, as though a wave was propagating along this path. Then, we add up all the weights- **AS THOUGH THE SYSTEM REALLY WAS TRAVELLING OVER ALL THE PATHS AT THE SAME TIME.**

Then the **PROBABILITY** that the system goes from the first to the second state is just $P_{21} = |G_{21}|^2$

Closed loop

Histories of particle

Path of the particle traveling backward in time on a loop

Particle

PROPERTIES of QUANTUM PATHS

Since any path is allowed, most paths are tortuous. However most interfere destructively because they all have different phases (different total number of waves) between start & finish. However CLASSICAL PATHS hardly change phase with small change in path- all these paths then interfere constructively. The paths of large classical objects are ones where the phase is a maximum or minimum (hardly varying with slight changes).

In many quantum systems the paths involve different particles, interacting with each other. If we look closely at any interaction, we see a "sub-structure" of more complex interaction paths. This small-scale structure involves high-momentum (& hence high-energy) Processes (recall de Broglie's relation $p = h/\lambda$ which says short wavelengths have high momentum)
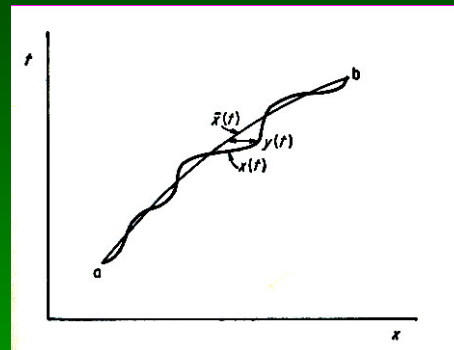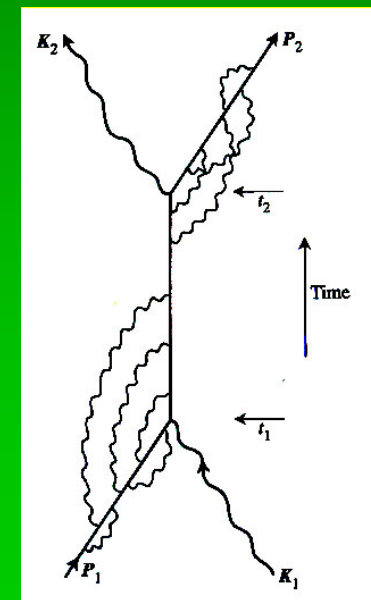
Small deviations about a path
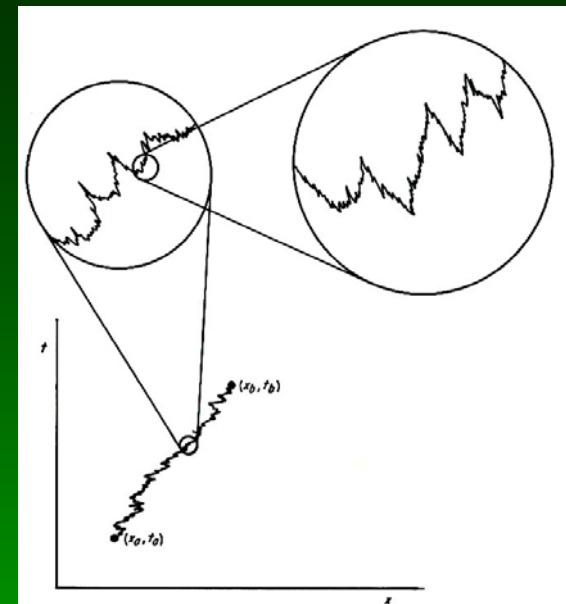
The real complexity of the path
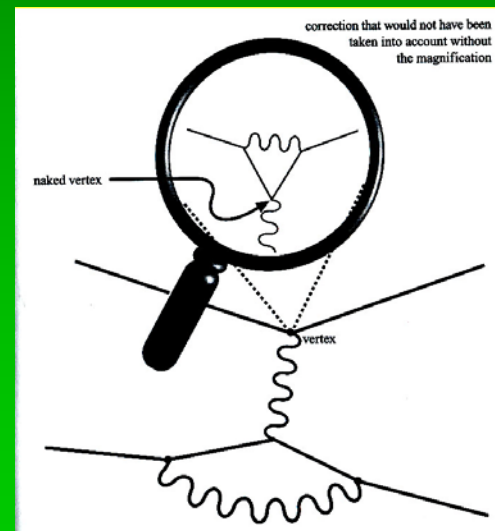
Magnifying up the path in a process (here electron-electron scattering)

Electron-photon scattering

QUANTUM PATHS: Plane Waves

The simplest kind of motion involves plane waves, traveling in some direction- corresponding classically (in the limit of short wavelength) to a particle moving in a straight line with p = h/λ.


(a)

(b)
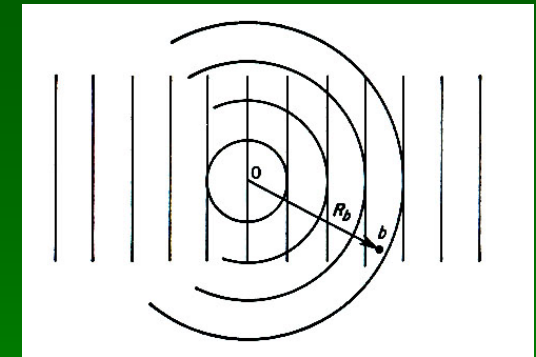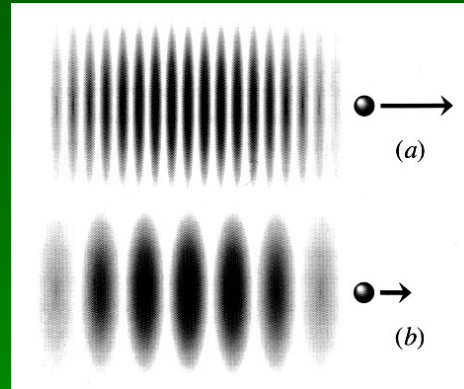
Left: plane wave states with 2 different momenta (& so different wavelengths).
Above: lane wave scatters off small object

If the plane wave interacts with some small localised object, part of it will continue on but part will be scattered- ie., re-emitted as a wave propagating away from the object. This is roughly what happens in Rutherford scattering off the nucleus.
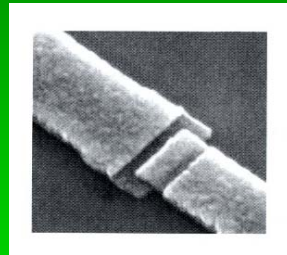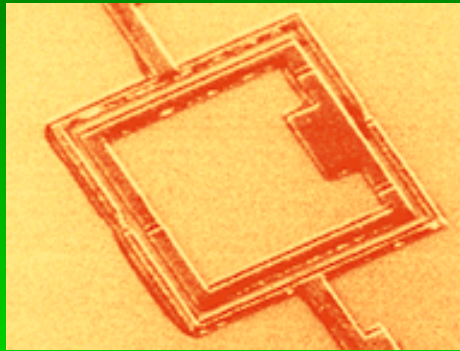
QUANTUM PATHS: Multiple Slits

One way to look in more detail at the way that different paths interfere is to have multiple slit set-ups. In this way one is giving the quantum system a "multiple-choice" set of paths to follow- but restricted by having to pass through the slits. One can then verify that (i) the final quantum amplitude to get to a point on the screen is indeed found by summing over all intermediate paths; & (ii) that the amplitude re-radiates from each slit a la Huyghens.
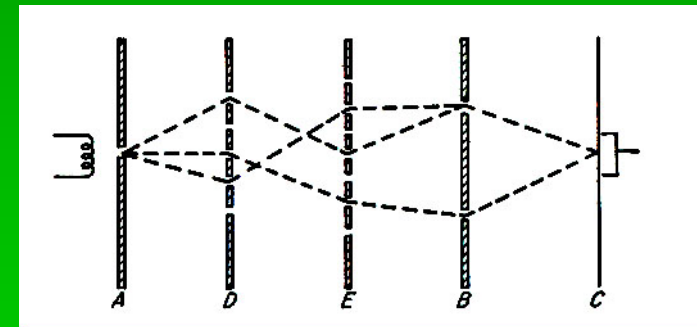

Propagation through multiple slits

# Non-local Interference- the Aharonov-Bohm effect

One fascinating effect which is impossible to understand classically is shown nicely by a set-up in which electrons go around 2 paths (allowing interference between the paths), with a region of non-zero magnetic field inside the loop. The field never touches the loop- the electrons move in a field-free region (most easily arranged with a superconducting ring).
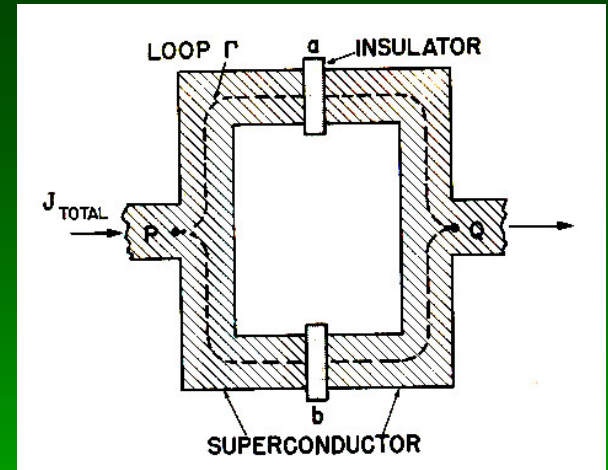
A 2-slit experiment for electrons, Passing via a superconducting loop. Photos at left.
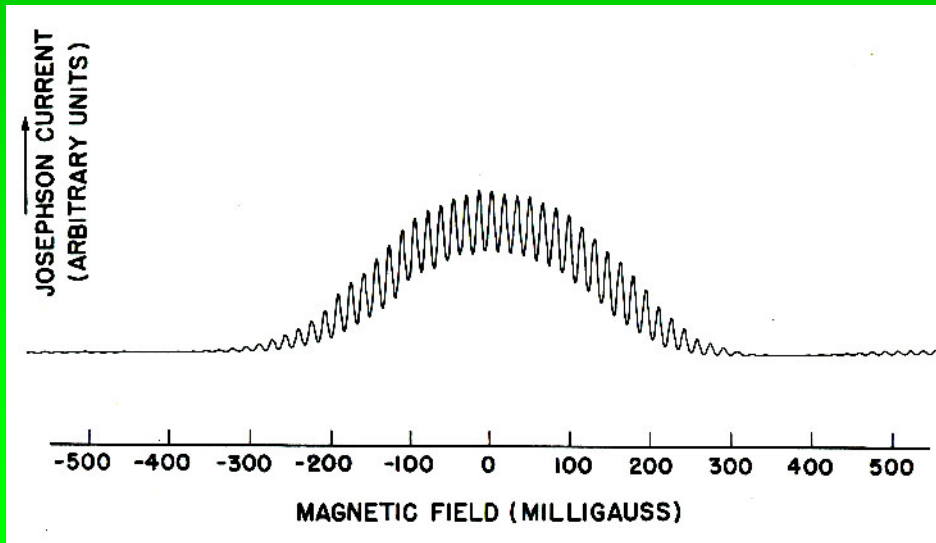
The Aharonov-Bohm prediction (1959) was that changing the field inside the ring would change the interference pattern- even though the fields were always zero along the electron paths! Even far from the field, the wave-function senses the change.

Left: the experimental results- the current shows 2-slit interference, which can be changed by varying the field.
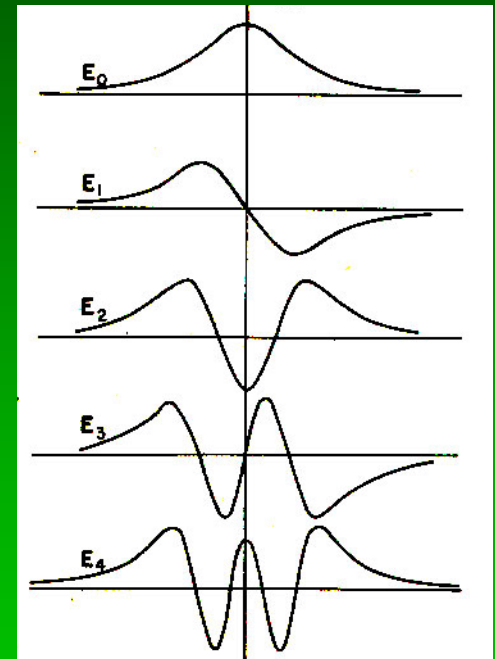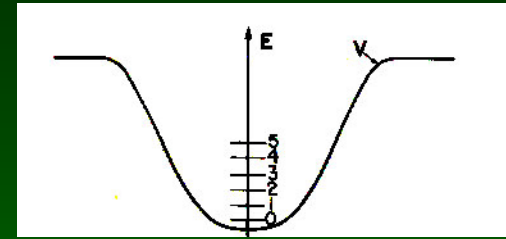
"Bound States" of Quantum Systems

As we saw with the discussion of Schrodinger's formulation of Q.M., confining a quantum system means the wave amplitudes can't propagate freely- instead they form "standing waves" (sums of waves and their reflections) inside the confining region- called "eigenstates"

The lowest-energy bound state is called the "ground state"- the higher energy states, with shorter wavelengths, are called "excited states". We go from one energy level to the next highest one by fitting in one more oscillation.

Energy levels and wave-functions in a box. Here "n" labels the no. of oscillations

Wave-functions for bound states inside the potential well shown at the top.

As we already saw with the **H** atom, the existence of discrete energy levels means that the bound system can only change its energy by certain amounts (the quantum jumps), unless the system is excited by some external energy source all the way out of the potential well. If this happens (for an atom we would say it is "ionized") then a continuum of free states is available- the particle escapes.

"SPIN" in QUANTUM MECHANICS

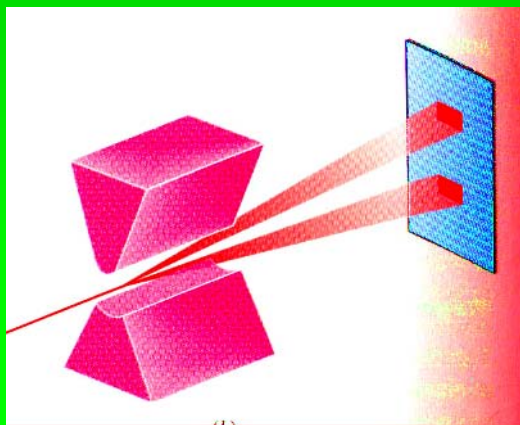We have seen that a quantum particle moves through space as a "probability wave". And de Broglie saw that rotational angular motion of objects are also described by a probability wave- the waves correspond to different discrete values of angular momentum (ie., the rotational velocity of a quantum system is also quantized).

A startling feature of quantum mechanics is that even a point particle has a property called SPIN. Actually there are 2 classes of particle in Nature. Those called FERMIONS can only have "half-integer" spin- the simplest are spin-1/2 particles, but one also has particles with spin 3/2, 5/2, etc.

Then there are BOSONS, with "integer" spin, ie., spin 1, or 2,3, etc. The angular momentum of a spin s is hs, where h is again Planck's constant.

Randomly-oriented classical spins separated according to the Z-component of their spin

What is also quantized is the value of the spin along any direction. A spin-1/2 then has spin either 'up' (ie., 1/2) or 'down' (ie., -1/2) along a vertical z-axis. But if we measure an up spin along the x- axis, we find not zero, but 'left' or 'right' with equal (50%) probability- the up spin is a quantum sum (or 'superposition') of the left & right states. This is written as
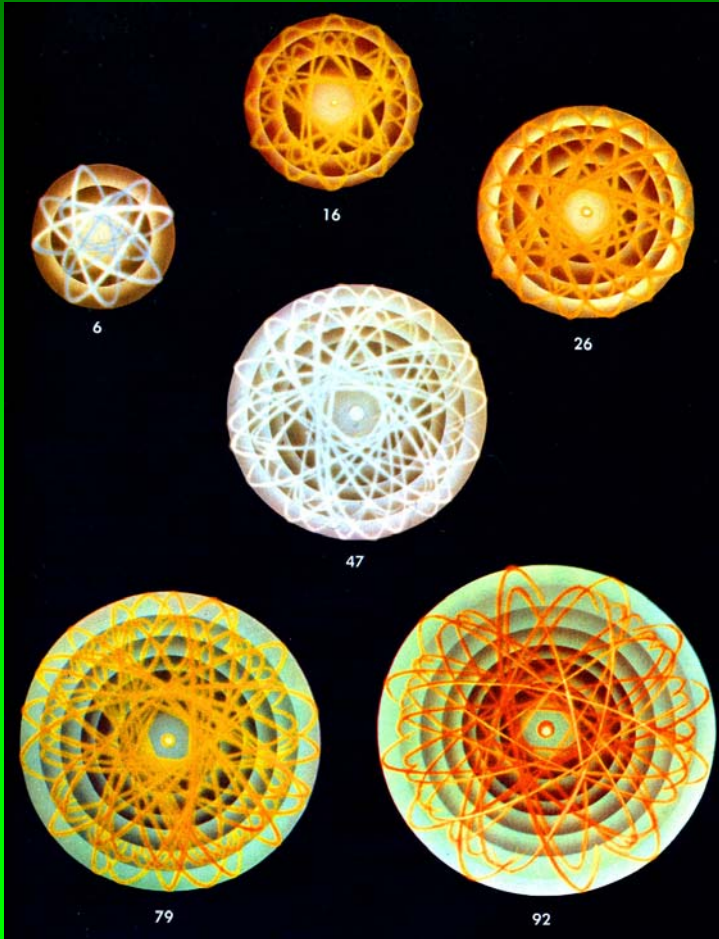
$$\Psi(+) \sim [\psi(->) + \psi(<-)]$$

Same for spin-1/2 quantum spins- only 2 values allowed.

At left we see the Stern-Gerlach apparatus- a beam of spin-1/2 particles is separated according to their spin along the z- axis.

ATOMIC STRUCTURE

The electrons fill up the different states in the -Z/r potential around the nucleus, where Z is the + charge of the nucleus (equal to the number of protons in the nucleus).

probablities for 2 different electron states in H.

The H atom, with a radius ~ 10^{-10} m, and a single proton nucleus with 1 electron around it, is the simplest. The electron states are labelled by the number n of radial oscillations, but also by oscillations as one moves around the nucleus (telling us the angular momentum).

The bound states are at the Bohr energies

$$\varepsilon_n = -R/n^2$$

below the continuum states, where R ~ 13.6 eV (electron Volts).
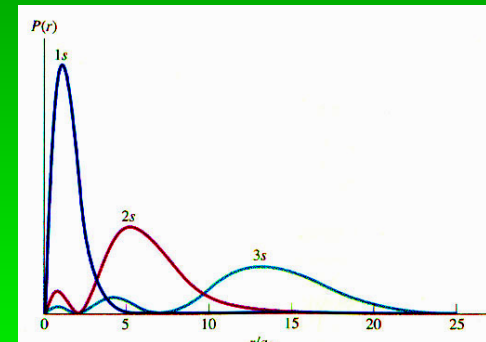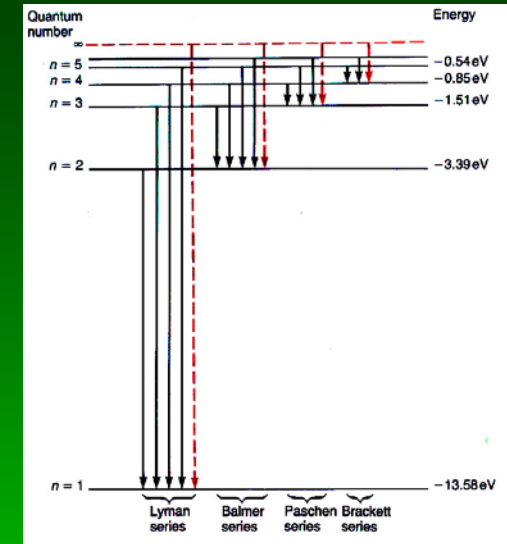
Top: allowed transitions between the different levels in a H atom- the levels have the Bohr energies. Lower: the probability for the electron in some states to be at a distance r from the nucleus

Artist impression of some atoms

QUANTUM TUNNELING

Another interesting property of waves is that they "leak" through classically "forbidden" zones- for quantum waves this is called "quantum tunneling". In the high-energy forbidden regions the waves decay, so penetration only happens if the barrier is thin.
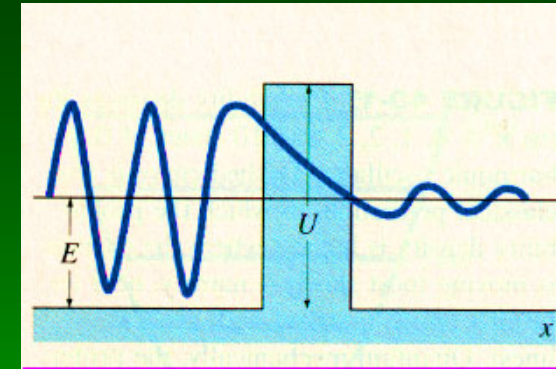
Tunneling through a barrier.
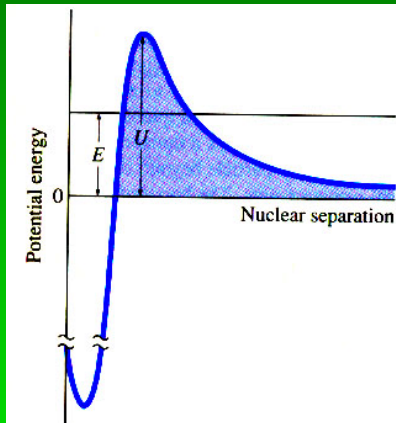
The effect of tunneling is that what were formerly stable systems (either confined by potential barriers, or by short-range attractive forces), now can decay in some way. Such tunneling transitions are crucial in Q.M.; they allow communication between states that cannot communicate classically.

The attractive potential energy well preventing nuclear fission

The first example of tunneling was discussed in 1927; it is the process by which parts of a nucleus break away from strong attractive forces- ie., radioactive decay (right).

Slow neutron

(a)

Unstable ^{236}U starts to wobble like a drop of water on a faucet just after the tap is turned off

(b)

Nucleus develops a neck before breaking in two

(c)

^{93}Rb

^{141}C

(d)

Nucleus fragments into two pieces and two or three neutrons. The neutrons are analogous to the small droplets which accompany a larger drop breaking off from the water faucet