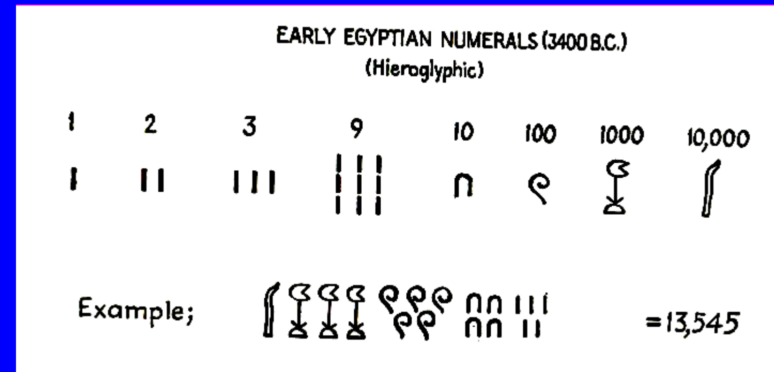
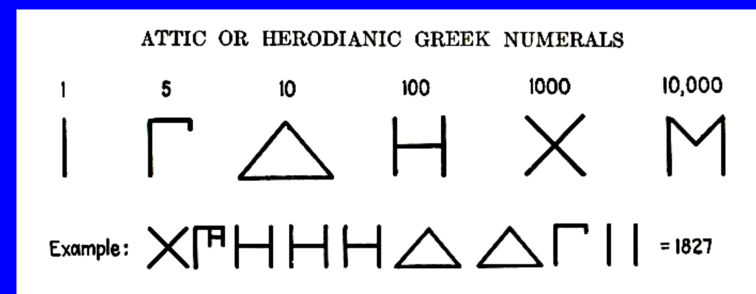
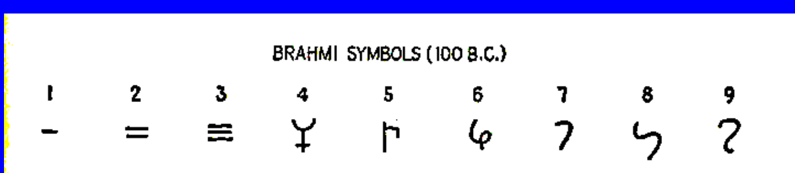
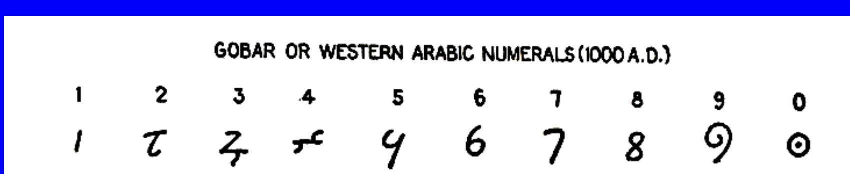
Greek Mathematics (1)

A precondition for doing any kind of mathematics is a system to represent numbers- whole civilizations have been retarded without one. The pre-Greek systems seem very primitive to us, but one sees in the old Egyptian and Greek ones the potential for development. These were already decimal systems (from hand-counting), but had not made the step to single numerals between 1 and 10.







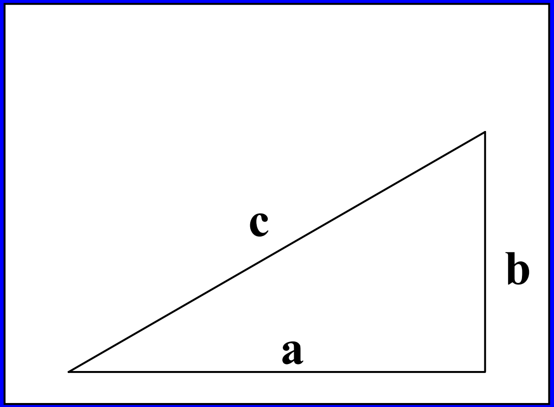


A key invention of Arab mathematics in the middle ages was the ZERO. This apparently trivial step not only permitted a simple notation for large numbers, but also allowed the construction of a proper arithmetic.

Greek Mathematics (2)

Out of the Pythagorean investigation of numbers came the first powerful results of Deductive mathematics. The famous theorem of Pythagoras concerning right-angled Triangles, viz., that

$$a^2 + b^2 = c^2$$



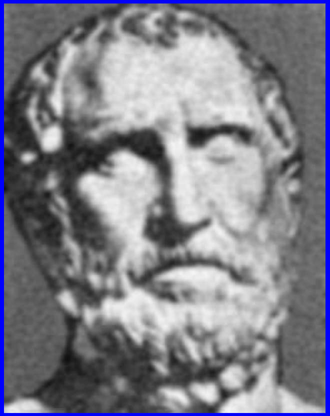
led directly to questions about the evaluation of square roots, and thence to the discovery of irrational numbers. The argument is very short. Consider, eg., the triangle with a=1, b=1, so that c is the square root of 2, ie., $c^2 = 2$. Now, IFF (this means "if and only if") we can write c as a fraction this means we can write $c = n/m$, where n & m are non-zero Integers (1,2,3,4,..), and have no common factors (so that we cannot further reduce the fraction). But Pythagoras realized this leads to a contradiction, as follows:

(a) If $c = n/m$, then $c^2 = 2 = n^2/m^2$. However this means that $n^2 = 2m^2$, which means that n^2, and thence n, must be EVEN integers (an odd number squared gives an odd number).

(b) However if n is even, we can write that $n = 2p$, where p is another integer. But then we can write $c^2 = n^2/m^2 = 2 = 4p^2/m^2$, which is the same as saying that $2p^2 = m^2$. But this means that m IS ALSO EVEN, by the same argument as before. So we have shown that both m & n are even- which contradicts the original assumption that they have no common factors. Thus, square root(2) cannot be written as a fraction!

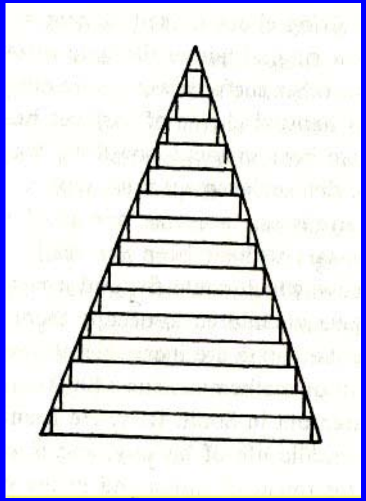
The amazing thing is that this argument nowhere exhibits any expression for the square root- but derives a completely general property of it.

Greek Mathematics (3)

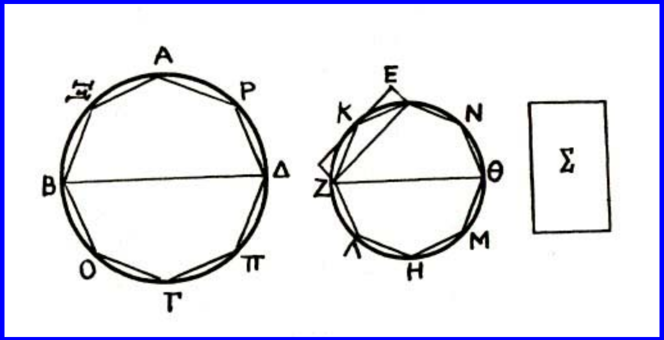


Zeno

Well before Plato, the Greeks had a very sophisticated understanding of SERIES, and UNIFORM APPROXIMATIONS. The example at right is due to Democritus; the area of the triangle is first estimated by approximating it with rectangles, and progressively approaching the triangle by making these thinner.



In this way the Greeks were led to problems such as the ones raised by Zeno, connected with what we now call 'infinitesimals'. The 2 best known of these problems are that of the Hare and the tortoise (immortalised by Aesop) and the problem of the moving arrow (see COURSE NOTES for more details). Nevertheless these methods were of great practical use- they were used to find everything from approximations to square roots, cube roots, etc., to the area different shapes, or the volume of solid bodies. This work was driven both by curiosity, and by a pragmatic interest in applications- likely the first interest was foremost, so that enormous leaps in understanding were made.



An example of considerable interest was the circle- with both the circumference and area approximated by a succession of triangles.

This led to an investigation of the number π, which perplexed the Greeks (and was not properly understood for another 2000 yrs).

Greek Mathematics (4)

Archimedes (287-212 BC)

Although Athenian culture declined fairly rapidly after Alexander the Great, the mathematical ideas were not lost, and 2 very important developments occurred elsewhere. The first is often associated with Archimedes, who took the first steps in what we would now call 'Mathematical Physics'. The key difference with all previous work was the attempt to quantify relationships in the physical world.

Two well known examples of this work are

(i) The understanding of levers, and other problems in simple mechanics- a famous application being the catapults and other machines devised by Archimedes to defend the city of Syracuse (the siege in the end succeeded, and Archimedes was killed by a Roman soldier in the ensuing massacre).

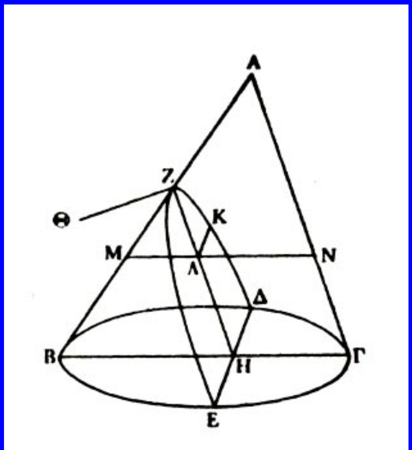
(ii) The understanding of hydrostatics, including the famous law describing the displacement of a fluid when an object is immersed in it, and the reduction in weight of the immersed object (which Archimedes, according to legend, discovered in his bath, and thereupon leapt from it crying "Eureka!").

Many of the ingredients of modern physics were already being used by Archimedes- a primitive version of calculus, precise mathematical relations describing the relationship between weights, masses, volumes, and density, an understanding of forces, etc., derived not from some imagined 'first principle', but by generalization from experience, and empirical test.

Greek Mathematics (5)

Euclid (330-275 BC)

The 2nd great advance is often associated with Euclid, whose famous 'Elements', giving a deductive description of all that was understood of geometry as it was then understood, was still in extensive use in schools in the early 20th century. Euclid attempted to derive all of geometry from a set of 5 AXIOMS; the axiomatic method has been central to mathematics ever since. The idea, described more fully in the notes and references, is to start from a set of definitions and propositions which are assumed self-evident, and which by themselves define a LOGICAL SYSTEM of relations between some set of objects. Then all proofs and other deductions follow from the axioms, without reference to any physical system they may (or may not) describe. Such chains of deduction could be very complex (as in the example at left from the later work of Appolonius, referring to the figure shown). The most famous result is connected with the "parallel axiom", in which Euclid showed the power of the axiomatic method by posing a problem which would only be solved 2500 yrs later, and lead to a revolution in physics.



(APOLLONIUS, *Conics*)

Proposition 11

Let a cone be cut by a plane through the axis, and let it be also cut by another plane cutting the base of the cone in a straight line perpendicular to the base of the axial triangle, and further let the diameter of the section be parallel to one side of the axial triangle; then if any straight line be drawn from the section of the cone parallel to the common section of the cutting plane and the base of the cone as far as the diameter of the section, its square will be equal to the rectangle bounded by the intercept made by it on the diameter in the direction of the vertex of the section and a certain other straight line; this straight line will bear the same ratio to the intercept between the angle of the cone and the vertex of the segment as the square on the base of the axial triangle bears to the rectangle bounded by the remaining two sides of the triangle; and let such a section be called a parabola.