BASIC QUANTUM MECHANICS
II. ENTANGLED STATES

So far I have described states of a single quantum system - these are already very strange because of the superposition principle. But now we come to something much stranger again. Suppose we consider a set of 2 physical systems - the simplest example of these being 2 particles. One can of course consider much more complicated situations, with more than tow physical systems interacting in some way. However the key in what follows is the relationship that one may establish between two physical systems.

Let's first pause and ask - what do we really mean when we talk about two distinct physical systems? It is clear that, amongst other things, we are thinking of their autonomy - a given physical system is regarded as distinct because it has some degree of autonomy, and can be separated from the rest of the universe in some way. We make this assumption in ordinary life - two objects like, eg., 2 different plates, are considered to have an autonomous existence, with their own particular properties (colour, texture, shape, material properties, etc.). If they are not connected physically in any way, and one of them is taken away or even made to disappear, this does not affect the existence or the properties of the other. Even if they have interacted each other at some point (they could have touched while being stacked in a pile) we still suppose we can separate them and talk of them as distinct autonomous objects. Of course we can also imagine connecting them physically in a practically irreversible way (we could imagine for example, gluing them together), and at this point it might be objected that they are no longer distinct objects, even though each one has retained almost all of its distinct and individual autonomous properties, that make it what it is. Note of course that if one decided to fuse them by heating them both to extremely high temperature (so that they partially melted and melded together), then we would say that they not only now constituted a single object, but that at least some of the former plate-like properties of each individual plates had been lost.

The theory of Relativity adds a further ingredient to this discussion. For, in this theory, no physical influence whatsoever - either in the form of energy or information - can pass between 2 objects if they are separated by a sufficiently large distance. This is because such influences cannot travel faster than the speed of light. So, if the distance L between 2 objects is greater than $c_o t$, where c_o is the speed of light, and t the timescale of interest, then there can be no connection between the 2 objects at all. Suppose, eg., 2 objects are created at 2 different places separated by L. Then until a time $t = L/c_o$ has passed, no connection whatsoever between these 2 objects can exist. In this sense we can say that there is a real sense, if one accepts the theory of Relativity, in which one can talk about distinct and autonomous physical objects, which are not connected with others in any way, provided they are spatially disconnected with them.

A: ENTANGLED STATES OF 2 QUANTUM OBJECTS

What I now wish to show you is that when we try to apply what we learnt about state superpositions to a *pair* of quantum-mechanical systems, we get a result so strange that it has caused confusion ever since it was first highlighted in 1935. This result arises when the 2 systems are "*correlated*"; and so before beginning let's just stop and see what we mean by this. To do so, we imagine a 'thought experiment', wherein we create 2 particles in such a way that we force some sort of connection between them. The simplest way to do this is to have some sort of physical conservation law involved. For example,we know that the total energy of the system must be conserved, provided neither particle interacts with anything else - and we also know that their total momentum is conserved, and their total angular momentum. Imagine, for example, that the 2 particles are created by the decay (ie., 'explosion') of some object which before the decay is stationary (and so has no momentum), and is not spinning or turning in any way (so it has no angular momentum). Then the total momentum and angular momentum afterwards must both be zero. This means that the 2 particles produced must be flying away with momenta that are equal in magnitude but opposite in direction, to give zero net momentum - and if they are spinning, they must be spinning at equal rates, but in opposite directions, to give zero net angular momentum.

What we wish to do now is set up a situation in which 2 quantum systems are correlated in this kind of way, and see what happens when we allow different superpositions of such states.

Now this thought experiment can be carried out for many physical systems. We can easily imagine it for some classical system - For example, 2 elastic balls in outer space, held tightly together by some glue, can fly apart if the glue fails. In this case we have no qualms about saying that, eg., one of the balls is flying upwards and the other down, and that one of them is spinning with its spin vector up (ie., $+$), and the other down (ie., $-$). One thing we are sure of is that each one of them is in some definite physical state. Moreover these states are distinct, and so are the balls - they have their own individual properties (they might, eg., have different colours).

A.1: ENTANGLED 2-STATE SYSTEMS

The sort of thought experiment described above can of course be carried out for many physical systems. We can easily imagine it for some classical system - For example, 2 elastic balls in outer space, held tightly together by some glue, can fly apart if the glue fails. In this case we have no qualms about saying that, eg., one of the balls is flying upwards and the other down, and that one of them is spinning with its spin vector up (ie., $+$), and the other down (ie., $-$). One thing we are sure of is that each one of them is in some definite physical state. Moreover these states are distinct, and so are the balls - they have their own individual properties (they might, eg., have different colours).

Consider now the quantum-mechanical situation. The kind of classical system just described is very complicated, and so is its quantum counterpart. But we can start with a pair of 2-level systems; this makes things much simpler. Let's start with an example:

Example: the Positronium System: We start with a system for which the kind of correlation described above is easy to arrange. The name "positronium" describes a system in which an electron and a positron (the 'anti-particle' of an electron) are bound closely together by their electrostatic attraction, with zero net angular momentum. After a short time they will mutually annihilate, leaving behind 2 very high energy photons, which must then fly off in opposite directions, each with opposite spin.

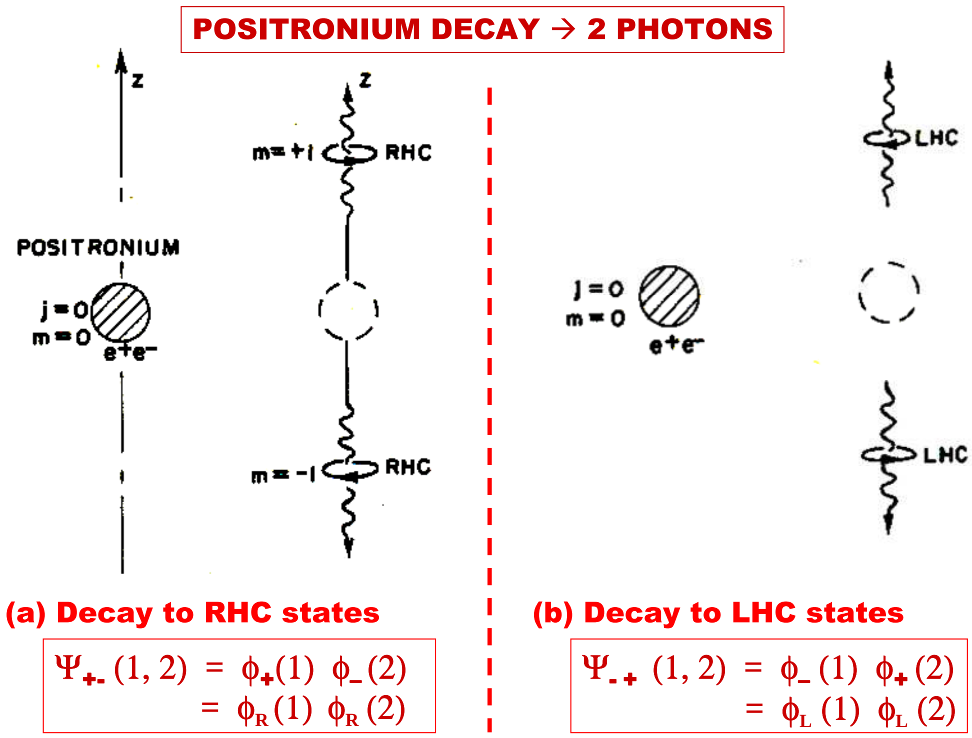

FIG. 1: The decay of positronium to produce a pair of photon states. In (a) we see the decay products with the upward moving photon having spin $m = +1$, and the downward moving photon with spin $m = -1$. This is equivalent to 2 photons with right helicity (ie., right or 'clockwise' polarization). In (b) the polarizations are now both left, corresponding to spin $m = -1$ and $m = +1$ for upwards and downwards moving photons respectively. See text for more details.

Now there are 2 possibilities here, depicted in Fig. 1. The first one is where photon 1, heading up, is in a photon state with spin $+1$ in the up direction. Then photon 2, heading down, must be in a spin state -1 in this direction. Since whether the states or 'left' or 'right' polarized depends on what is their spin *along the direction the photon is traveling* (compare Fig. **??**), this means that in fact both of these photons are right-hand polarized. Thus we can write the state of this quantum system of 2 photons as $\Psi(1,2) = \phi_R(1)\phi_R(2)$, or for short, just $|\Psi_R\rangle = |RR\rangle$. If we write it in terms of photon spins, we would call it $|\Psi_{+-}\rangle = |+-\rangle$. This state is shown in Fig 1(a).

However we can also have the opposite situation - photon 1 can have its spin *down* (ie., with spin -1, and photon 2 can have a spin $+1$; this means they are both left-polarized, and the state of the 2 systems is then $|\Psi_L\rangle = |LL\rangle$ (or, written alternatively as $|\Psi_{-+}\rangle = |-+\rangle$). This situation is depicted in Fig. 1(b).

So far so good - we can imagine either of these 2 states as legitimate classical photon states, with different circular polarizations for each photon. Thus, like the 2 plates glued together that we discussed above, they have correlated behaviour, but each has its own distinct state). If these were balls flying off, we could label their left/right polarizations by colours, eg., red and blue (or some other label like 'heads' and 'tails'); and we would be sure that each ball had its own colour (that this was an individual autonomous property of each ball).

Now let's consider what can happen in a real situation involving two such photons. You can probably see what is coming - there is nothing to stop us writing a superposition of states here, of the form

$$\begin{aligned} \Psi^+_{LR}(1,2) &= \tfrac{1}{\sqrt{2}}[\phi_L(1)\phi_L(2) + \phi_R(1)\phi_R(2)] \\ &= \tfrac{1}{\sqrt{2}}[\phi_+(1)\phi_-(2) + \phi_-(1)\phi_+(2)] \end{aligned} \tag{0.1}$$

or, to keep our notation really compact, we just write this as

$$|\Psi^+_{LR}\rangle = \frac{1}{\sqrt{2}}[|LL\rangle + |RR\rangle] \tag{0.2}$$

Now, not only is this state a perfectly legitimate physical state, it is also the one that is typically found in practise in an experiment like this - either this one, or another one like, eg., $|\Psi^-_{LR}\rangle = [|LL\rangle - |RR\rangle]/\sqrt{2}$, in which the 2 components are subtracted instead of added. But what on earth does this state really mean for the 2 photons? It seems very peculiar, because now we cannot say that each photon is in a particular state; all we can say is that each photon is in the *same* state ('right' or 'left') as the other. It seems as though the photons have completely lost their individual properties here (at least as far as their spin is concerned). And indeed this is the case - one can readily show, using the mathematical rules of quantum mechanics, that the individual photons really do not have any individual spins anymore, while in this state. The only thing that is 'real' here is that they are in the same state.

General Remarks on Entangled 2-state Systems: States of the kind we just looked at are called *entangled states* in quantum mechanics; obviously one can extend the above discussion of photons to, eg., spins. For some physicists, such entangled states are the most mysterious feature of quantum mechanics. Thus Schrodinger, one of the founders of quantum mechanics, remarked that the entanglement of 2 systems should be regarded as follows:

"*I would not call that* **one** *but rather* **the** *characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought*"

from E Schrodinger, "Math. Proc. Cambridge Phil. Soc. **31**, 555 (1935).

To see quite how peculiar it is, let us first note that if we were talking about some ostensibly classical system instead of photons or spins, we would end up with some very strange conclusions. Perhaps the most amusing examples of such states involve putative states of people. One is illustrated in the course slides - a couple who are in a superposition of states in which they are either (i) both sure that they are made for each of other, or (2) both sure that they are not made for each other. The entangled state then has them in a superposition of these two different states so that they can be sure that they both think the same, but do not know which it is! The physicist C Bennett extended this analogy to consider 2 hippies in Haight-Ashbury, in the late 1960's, who gaze into each other's eyes, both sure they are thinking the same thing, but having no idea what it is.

If we wish we can also recall our 'quantum coin states, and construct a set of entangled coin states. We can easily construct the analogy to the above entangled pair state. Suppose, for example, that we decide to entangle the states $|G\rangle$ and $|E\rangle$ that were noted previously (see part I, Fig. 1, bottom right). One possible state would then be

$$|\Psi^+_{GE}\rangle = \frac{1}{\sqrt{2}}[|GG\rangle + |EE\rangle] \tag{0.3}$$

which is illustrated here in Fig. 2. Again, in this entangled superposition, we cannot attribute any physical meaning or reality to the state of either of the two coins - only the joint state has any meaning, and is *physically real*; and its defining property is that it superposes, with equal weights, the 2 states of each coin that are the same. Moreover, this state is *exactly the same* as the state

$$|\Psi^+_{HT}\rangle = \frac{1}{\sqrt{2}}[|HH\rangle + |TT\rangle] \tag{0.4}$$

because this state also superposes, with equal weights, 2 states of each coin that are the same - this being the defining property of the state.

Thus we have defined a very strange state indeed, in which all common sense ideas of physical reality for apparently autonomous systems have been abandoned - only the joint state of the two objects is real.

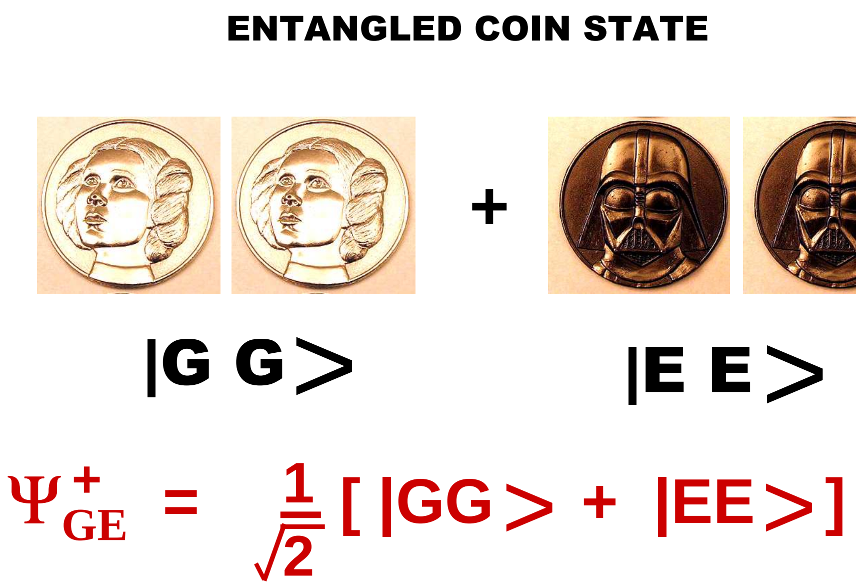

FIG. 2: Entangled coin states. Here we superpose states of a pair of coins. The states in question are G (for "good") and E (for "evil"), and the final state is a sum of $|GG\rangle$ and $|EE\rangle$. See text for more details.

These results were so strange that they caused a number of the founders of quantum mechanics to ultimately reject it. Such a situation is apparently unprecedented in the history of science. The most vocal opponent was Einstein, who had along with Planck set in motion the whole process of discovery which led to Quantum Mechanics, and had major contributions to it at every stage of the way. His philosophical attitude to the whole problem was summarized by him in a letter to Born, as follows:

"*I just want to explain what I mean when I say that we should try to hold on to physical reality. We are ... all aware of the situation regarding what will turn out to be the basic foundational concepts in physics: the point-mass or the particle is surely not among them; the field, in the Faraday-Maxwell sense, might be, but not with certainty. But that which we conceive as existing ("real") should somehow be localized in time and space. That is, the real in one part of space, A, should (in theory) somehow "exist" independently of that which is thought of as real in another part of space, B. If a physical system stretches over A and B, then what is present in B should somehow have an existence independent of what is present in A. What is actually present in B should thus not depend on the type of measurement carried out in the part of space A; it should also be independent of whether or not a measurement is made in A. If one adheres to this program, then one can hardly view the quantum-theoretical description as a complete representation of the physically real. If one attempts, nevertheless, so to view it, then one must assume that the physically real in B undergoes a sudden change because of a measurement in A. My physical instincts bristle at that suggestion. However, if one renounces the assumption that what is present in different parts of space has an independent, real existence, then I don't see at all what physics is supposed to be describing. For what is thought to be a "system" is after all, just conventional, and I do not see how else one is supposed to divide up the world objectively so that one can make statements about parts.*"

"What must be an essential feature of any future fundamental physics?"
Letter to Max Born; from "The Born-Einstein letters, 1916-55" (Macmillan, 1971)

Thus, according to Einstein, whatever else the term 'physical reality' may refer to, it definitely must refer to objects or systems that are confined to a specific region in space and time. However this clearly creates a problem in quantum mechanics, since there is no particular reason to assume the the 2 systems we are talking about are confined to the same region in spacetime - after all, they are two *separate* systems! So what is going on here?

A.2: The EPR PARADOX and "PHYSICAL REALITY"

Einstein was not just content to object to quantum mechanics on these general semi-philosophical grounds. To really bring home the full ontological content of the theory, he produced a specific 'thought experiment' designed to show just how utterly paradoxical it was. The resulting paper, written with Podolsky and Rosen, and entitled "Can Quantum-Mechanical Description of Physical Reality be considered complete?", was published in 1935, and caused uproar amongst physicists. This paper (usually called the 'EPR paper' has since become a landmark in the history of physics (and was the last really fundamentally new contribution of Einstein to physics; at this time he was 56 years old). In what follows I give a simplified presentation of Einstein's basic idea. Then I will discuss how the EPR thought experiment proposed finally became a real experiment in the 1980's, and how the theoretical contributions of Bell in the 1960's gave a fundamental importance to the experiment, which went even beyond that envisaged by Einstein.

The Einstein-Podolsky-Rosen Paradox: Let's consider again the situation discussed above, where we have a pair of 2-state systems that are entangled in what is now called an 'EPR state'. These could be 2 photons, or 2 spins, or a pair of any other 2-state objects. It is most convenient to explain this example with the use of spins, so we will start by supposing we have put a pair of spins into the state

$$|\Psi_{EPR}\rangle \;=\; \frac{1}{\sqrt{2}}[|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle] \qquad (0.5)$$

so that neither particle has any definite spin - the defining property of this state is just that the 2 spins are parallel. An interesting feature of this state is that we can also write it as, for example

$$|\Psi_{EPR}\rangle \;=\; \frac{1}{\sqrt{2}}[|\rightarrow\rightarrow\rangle + |\leftarrow\leftarrow\rangle] \qquad (0.6)$$

where we now superpose states with 'both left' and 'both right'. Again, all we know is that this state has both spins parallel - and we can actually show mathematically that it is identical to (0.5), and so we are free to use either to describe the state $|\Psi_{EPR}\rangle$ (you can easily show this for yourself - you know that the state $|\uparrow\rangle = (|\leftarrow\rangle + |\rightarrow\rangle)/\sqrt{2}$, and likewise $|\downarrow\rangle = (|\leftarrow\rangle - |\rightarrow\rangle)/\sqrt{2}$; so just substitute these expressions into (0.5), and you will get (0.6) coming out). In fact we could use any state in which we have a sum of 2 spins parallel along one direction and the same 2 spins parallel along the diametrically opposite direction - it doesn't matter which direction this is, they are all the same. As we already said - the only thing that is true about this state is that the 2 spins are parallel, with equal weights in 2 opposing directions.

Now EPR add an extra twist to all of this, as follows. Let us suppose, they say, that we set up, very far apart from each other, 2 measuring systems designed to look at these spins. These might be Stern-Gerlach systems, of the kind we saw above in Fig. 4 of Part I; in any case, they are designed to measure the component of spin in a given direction. This situation is illustrated in Fig. 3(a), where we see how 2 measuring systems M_I and M_{II} are designed to measure the spin in 2 different directions, one for each of the 2 spins (the figure shows M_I measuring the spin of spin 1 in the vertical direction, and M_{II} measuring the spin of spin 2 in the horizontal direction. A key feature of this design: M_I and M_{II} are so far apart that no signal or physical interaction of any kind can pass between them, during the operation of the experiment.

Let us begin with the simplest case, where we have both M_I and M_{II} parallel, and each is set up to measure the vertical component of spin. Suppose also that M_I is the first to measure the spin, followed immediately by M_{II}. Then, assuming only that these measuring systems do what they are designed to do, we can see that (i) M_I will find that spin 1 is either in an 'up' or 'down' state (with equal probability); and (ii) that whatever M_I finds, M_{II} must subsequently find the same. In other words, the results of the measurements will be completely correlated - each will find the same spin, whether it be up or down (and we do not know in advance which it will be, since neither spin has a definite spin state in any particular direction).

Notice therefore also that if we choose to set M_I and M_{II} to instead measure spin along the horizontal x-direction, we will also find a perfect correlation - if M_I measures 'left', so will M_{II}, and if M_I measures 'right', so will M_{II}.

So far so good. Now comes the key idea. Einstein asks - suppose we set things up so that initially M_I and M_{II} are set up to measure whether the spins are up/down; but just before the state of spin 1 has been measured to be either 'up' or 'down', we suddenly change our minds about what we are going to measure on spin 1, and instead decide to rotate M_I to measure spin 'left' or right'. Thus suddenly, at the last instant, we have set up a situation for spin 1 in which the only outcome that we will measure will be 'left' or 'right'; and moreover, once we have made this measurement, spin 1 really will be either in state $|\leftarrow\rangle$ or state $|\rightarrow\rangle$.

But now comes the crunch. For according to quantum mechanics, this means that the final state of the combined system, after M_I has measured it, is going to be either $|\leftarrow\leftarrow\rangle$ or state $|\rightarrow\rightarrow\rangle$ (ie., NOT a superposition of the two

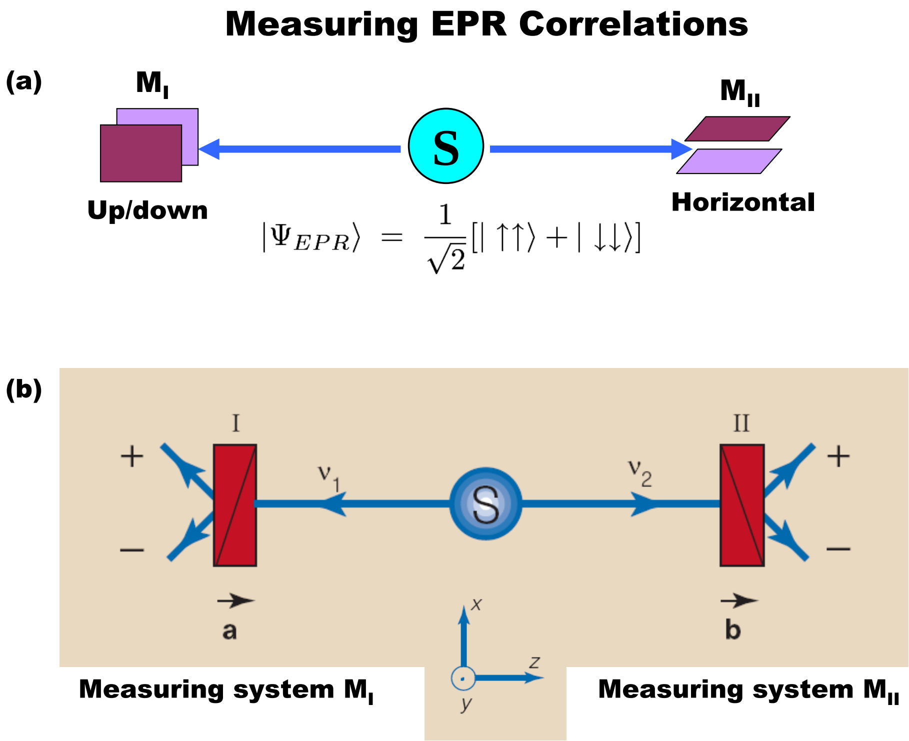

FIG. 3: The Einstein-Podolsky-Rosen thought experiment. In (a) the source S ejects 2 spins, entangled in a 'parallel spin' state. The measuring systems M_I and M_{II} can measure spin in any direction desired. In (b) we show the thought experiment for entangled photons, and the polarization measuring devices can be oriented in any direction - this allows one to check the correlations between the polarizations of the 2 photons. See text for more details.

anymore, but one or the other). Thus, *before* the measurement at M_I was made, we had to think of the state of spin 2 as being entirely undetermined. However *after* the measurement was made, we have to think of the state of spin 2 as being either $|\rightarrow\rangle$ or $|\rightarrow\rangle$.

Einstein tried to put this whole result in the following way. He argued as follows. Suppose the system was initially organized with M_I and M_{II} set up vertically, to measure whether the spin was up or down. Then, if M_I makes a measurement, it will find spin 1 to be up or down; and then we know *with certainty* what will be the state of spin 2. Then, immediately after the measurement of spin 1, even if we do not know before the measurement what is the state of spin 2, we can say with certainty afterwards what it is. Then Einstein introduced his famous idea of an "*element of reality*". In his own words:

"*If, without in any way disturbing a system, we can predict with certainty (ie., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*"

from A. Einstein, B Podolsky, N. Rosen, Phys. Rev. **47**, 777 (1935)

What is basically being said here is that if some assertion about a system is known *for sure* to be true, then this assertion (or 'fact') is telling us about something that is *real*.

Now let us return to our spins. Einstein then notes that if, say, we find spin 1 to be in state $|\uparrow\rangle$ then we *know for sure* that spin 2 must be in the same state, after the measurement of spin 1. Thus, this fact about spin 2 must refer to something real about spin 2, viz., that it is definitely in one of these states. Now, the key point. Suppose instead at the last minute we decide to change the measuring system M_I so that it is no longer measuring vertical states, but instead horizontal (ie., left/right) states. Then we will never find spin 1 to be in state $|\uparrow\rangle$ after the measurement. Instead it will be in either $|\rightarrow\rangle$ or $|\rightarrow\rangle$. It then necessarily follows, according to Quantum Mechanics, that we must find spin 2 in one or other of these states. But, no possible influence could have traveled to spin 2 in this time - so it cannot have changed in any way. It was supposed to have been in state $|\uparrow\rangle$, but now, says quantum mechanics, we can no longer say this for sure. In other words, it seems as though by changing what we do at M_I, we affect the sate of spin 2, even though no possible influence can travel between the 2 spins.

This is the famous EPR paradox - and for Einstein it was a sign that something was wrong with quantum mechanics, in fact that it was an *incomplete* theory - that it could not tell us everything about the physical system it dealt with. This attitude of his actually alienated him from the mainstream of physics, and he did not change his opinion on quantum mechanics, right up to the end of his life, 20 years later. And in fact, nothing really happened during this period that could have any real influence on the questons raised by the paradox. The EPR paradox was just that - a thought experiment designed to show a flaw in the theory - and nothing more. And so the paradox was left hanging in the air.

Experimental tests of Entanglement: By the early 1960's, the controversy set off by the EPR experiment had been largely forgotten by most physicists, who tended to be dazzled by the success of quantum theory, and largely ignorant of the issues involved. This very unfortunate attitude actually persisted up until perhaps the last decade or so, but in fact fundamental changes occurred well before then, beginning in the 1960s, and they completely changed the nature of the debate. Indeed, we are now in a completely different position - for experiments like those discussed in the EPR paper have now been done, and in fact they have completely vindicated the predictions of Quantum Mechanics for entanglement.

The story of this development is one of the most fascinating chapters in the history of 20th century science, particularly given the enormous philosophical importance of the questions involved. It begins with 2 remarkable papers by JS Bell, written in 1964, building on earlier work by D Bohm in 1952. Bohm had taken very seriously the ideas of EPR, and had then asked - can one construct an alternative theory to quantum mechanics, in which physical systems really do exist in one specific and unique state, just like in classical physics - but where this state is in some sense *hidden* from us, so that it looks as though we have to deal with superpositions and probabilities? If this were the case, one couild talk about a "*hidden variable theory*", in which the behaviour of the system was being partly determined by degrees of freedom that were inaccessible to us. And in fact, remarkably, Bohm was able to construct just such a theory. Again, this was ignored by physicists - largely because it was thought to be untestable in experiments (how can one observe 'hidden' variables?), and because they were more interested in other things.

However Bell's work radically changed things, because he found a way to test whether hidden variables could really exist. In fact, Bell cooked up a way of testing for *any* reasonable hidden variable theory - and by 'reasonable', he meant one in which the hidden variables were subject to basic things like the Theory of Relativity, so that they were "*local*" (ie., so that influences could not propagate between 2 systems faster than light, via the hidden variables). Bell realized that if one was to end up with probabilities in quantum mechanics, then somehow the values of the hidden variables were going to have to be random - this was a natural assumption anyway, since they were hidden, and so we had no way of knowing their values. He therefore asked - suppose we imagine, without even knowing what the hidden variables might be, that they take different values with some probabilities - can one, without even knowing what these probabilities are, still find ways in which such a hidden variable theory might give different results from quantum mechanics?

Incredibly, he found that the answer was *yes*: it was possible in fact to distinguish quantum mechanics from any *conceivable* theory of this kind. He actually showed that there were certain kinds of experiment that would given clear differences between Quantum Mechanics and any local hidden variable theory whatsoever. From a philosophical point of view this was really incredible - apparently it was possible to come to a really important conclusion about an ontological question - about the very nature of "*reality*" itself, not just some fact - and all this based on a single experiment. Never before had such an opportunity presented itself.

Incredibly, the attitude of the physics community to this development, particularly in North America, was one of disdain - experimenters found it very hard to get funding to do the kind of experiment discussed by Bell, which was in any case not easy. However, things turned out to be easier in France, where discussion between philosophers and physicists proceeded more easily. And this led to the famous series of experiments done by the group of A. Aspect in Paris, in the early 1980's. In this experimental set-up, which we have already seen schematically in Fig. 3(b), an entangled state of photons was used, instead of spins. A key feature of the experiment was the use of a 'randomization' technique, in which the direction that the photon polarization was being measured was constantly being flipped around between different directions, in a random way, very quickly. This was being done at each measuring system station (ie., at both M_I and M_{II}), but independently, so there was no way that the direction of M_I at any time could be known at M_{II}, and vice-versa. Thus, in this way, Aspect had set up precisely the conditions required in the EPR thought experiment - no hidden variable could simultaneously influence the 2 photons.

The experimental set up is shown very schematically in Fig. 4(a). This is similar to the 'thought experiment' scheme shown in Fig. 3, but now with a few extra bells and whistles. When each photon flies away from the source S, it eventually gets to 'beam splitters' (labeled by C_1 and $C2$ in the figure) and diverted to one or other measuring system (not both simultaneously - it has to be one or the other). Now if we consider photon 1, we see that is can go through either a measuring system which is measuring polarization along direction a, or along a'; these are different directions, and C_1 gets to choose which one the photon goes to. The same is true with photon 2, which will be measured in either direction b or b'. The function of the splitters C_1 and C_2 is to divert the outgoing photons to one or other of these measuring systems. Now the ingenious key to the experiment is that C_1 and C_2 are set up so they can flip back and forth between their 2 possible choices, *very rapidly*, and moreover, in a random fashion. In fact, the flipping is set up to happen so rapidly (hundreds of millions of times each second) that no light signal or information of any kind can pass between C_1 and C_2 during the time between flips. Moreover, the random flips at C_1 are happening *completely independently* of those at C_2. This is crucial: we do not want there to be any correlations whatsoever between the 2 measuring systems, ie., we do not want any 'hidden variables' connecting the behaviour of either the 2 photons or of the 2 measuring systems.

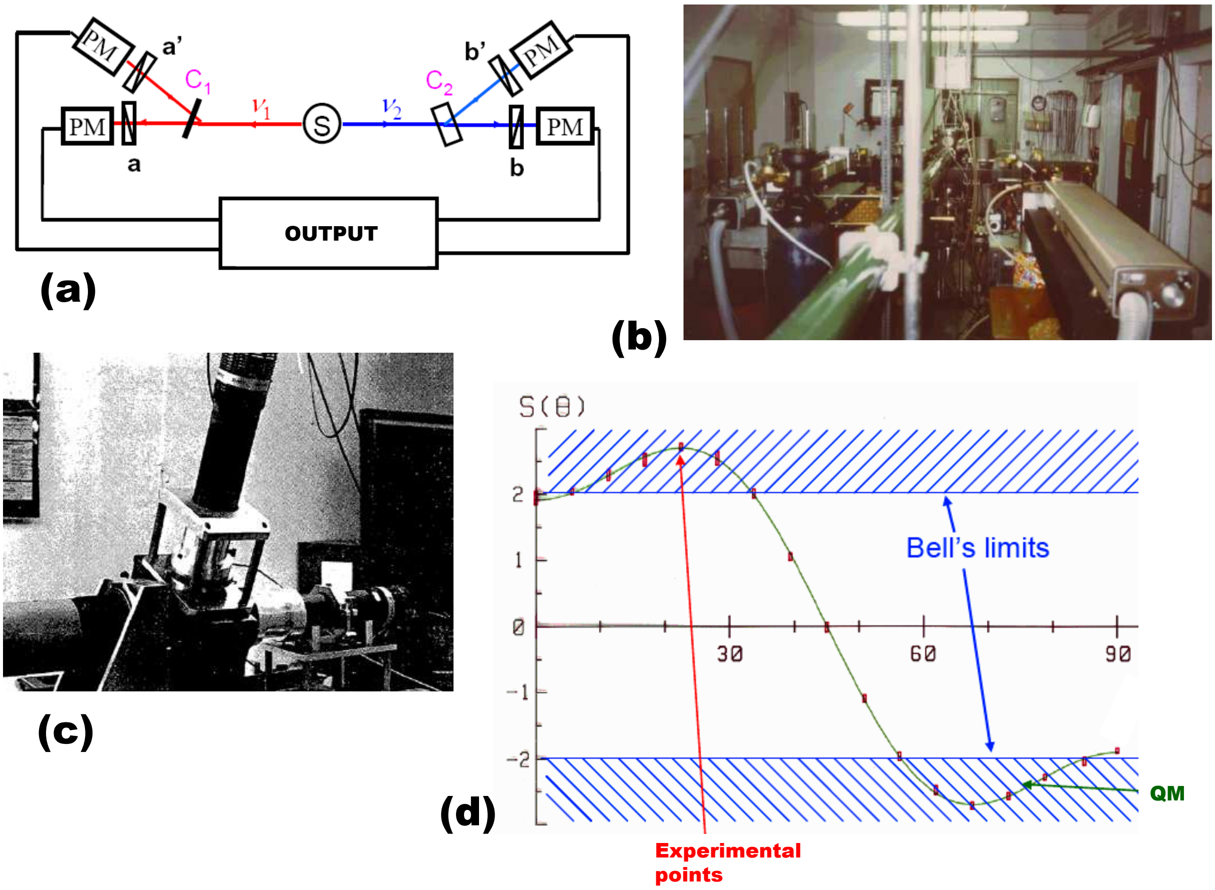

FIG. 4: The Aspect experiment. In (a) we show the design of this experiment - photons pass through "choice" beam-splitters C_1 and C_2, to have their polarizations measured along directions a or a' (for photon 1), and along b or b' (for photon 2). The signals are the amplified by photomultipliers PM, and thence to the output. In (b) we show a photo of the experiment, and in (c) a photo of one of the "choice" beam-splitters. In (d) we see the experimental results, showing how the "Bell correlation function" $S(\theta)$ varies with the angle θ between the polarization diections for the 2 photons. The curve marked "QM" shows the quantum-mechanical predictions; no local hidden variable theory can give results in the 'forbidden zone' (marked in blue). See text for more details.

This experiment was, at the time, very difficult to make work. The most difficult problems were (i) to make an efficient source of photon pairs with correlated polarizations, and (b) to make the random flipping detectors work. However (see Fig. 4(b)) eventually it was possible to make a system in which each photon had to travel 6m from the source before it reached its polarizers, so that the photons in a given correlated pair were 12m apart when the measurements were made (a distance that light takes only 4 hundred millionths of a second to cover, ie., in 4×10^{-8} secs). And the rapid and random fluctuation of the polarizers was accomplished by having the light pass through water which was being very rapidly agitated by high frequency random ultrasound (inside the box shown in Fig. 4(c)).

In 1982 the results were announced - they are shown in Fig. 4(d). As we see, they very convincingly verify the predictions of quantum mechanics. What is interesting about this figure (which shows a measure, called $S(\theta)$, of the correlations seen between the polarizations of the 2 photons, as a function of the angle between the polarizers), is that for certain angles, the results predicted by Quantum Mechanics are different from any possible hidden variable theory. Any hidden variable theory, no matter what may be the hidden variables, no matter what governs their motion, and no matter how randomly they may be distributed, must lie in the region for which $2 > S(\theta) > -2$. Outside this region is the 'forbidden zone' (shown in hatched blue in the figure). And we see that the quantum-mechanical prediction does indeed go into this forbidden zone - and so do the experimental results!

Since these famous experiments were done, many ideas have been developed to try and further understand entangled states, and also to *use* them. This has, in recent years, produced all sorts of wonderful things, like "Quantum Teleportation", and "Quantum Computing", which you can find discussed in the slides. But it has not in any way changed the enormous problem that this result raises for our common sense understanding of Nature - the problem that we do not really understand any more what it means for something to be *physically real*. Let us reiterate the main points here, just to make sure we know exactly how we are led to this impasse:

(i) The quantum state $|\psi\rangle$ of any physical system will be given in general by a "superposition" (ie., a sum) of all the possible states $|\phi_n\rangle$ of the system (in the form we saw before, viz., $|\psi\rangle = \sum_n a_n |\phi_n\rangle$). All "*possible*" states means - all states consistent with whatever constraints are put on the system (eg., spatial constraints, or constraints arising from its energy or whatever potential it is moving in). This is the fundamental principle of superposition - the quantum system will attempt to explore all of these states at once.

(ii) The quantum-mechanical state of a *pair* of particles will therefore be a sum of all possible pair states - again, consistent with whatever constraints are put on it (eg., a constraint that the 2 particles are correlated in some way).

(iii) We can set up 2 particles in such a way that all possible pair states are uniquely correlated, so that each and every state in any superposition of states must have particle 1 correlated uniquely with particle 2. For example, the 2 particles might be forced to be each in the same state, in all possible states in the superposition. For example, a state like the EPR state $|\Psi\rangle = (\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}$ is a state like this. But, as we have seen, so is the state $|\Psi\rangle = [|\rightarrow\rightarrow\rangle + |\leftarrow\leftarrow\rangle]/\sqrt{2}$; in fact this is the *same* state (recall the discussion right after eqtns. (0.5) and (0.6) above). In fact, it turns out that if we choose any superposition where we equally weight 2 states that are opposite to each other, we will get the same EPR state.

(iv) In quantum mechanics, the fact that all these different EPR states are the same means that the only thing that is true of all of them is that we have a superposition of 2 states, in each of which the spins are parallel, but where the 2 different states are opposite to each other. However we cannot say in which direction either of the particle spins is pointing - the only thing we can say is that they are both parallel. Thus in quantum mechanics, the spins do not have any individual state at all - only the state of the pair is meaningful.

(v) If now we decide to measure the state of ONE of these spins in, eg., the up/down direction we will find a definite result (say, 'up'). Then we will certainly find that the second spin is up, if we then measure it. But we could equally measure the first spin in the 'left/right' direction - and then we would find either left or right. If then we measured the second spin, we would get the same result.

(vi) Now, say EPR, let's put the 2 measuring systems very far apart, and set them up so that we are measuring to see if the spins are up or down. If we do this, then by measuring the state of spin 1 at some time, we immediately determine the state of spin 2 - it will be either up or down, depending on what we get for spin 1. *However*, suppose that at the last minute we suddenly change settings on the first measuring system, so that we are now measuring to see of spin 1 is left or right. Then we will find some result, and the state of spin 2 *must be the same* as that of spin 1 - it must be left or right. But, says Einstein, this is nonsense - right up to the time we changed the settings, the state of spin 2 was either up or down, not left or right; and now suddenly, by changing the settings on the first measuring system, we have changed what state spin 2 can be in! This must be wrong, he says, because no influence can propagate between the 2 spins fast enough to tell spin 2 that its state has changed! So, says Einstein, there is something wrong with Quantum Mechanics.

(vii) And yet we have now done the experiment, and it says that Quantum Mechanics actually is working fine - that yes, we can do an experiment no influence can possibly propagate between the 2 spins, but that nevertheless, the predictions of quantum mechanics work. And no theory in which each individual spin has its own state can explain these results.

And so, what is real here? It seems that in an EPR state (and many other pair states in which the 2 systems are completely entangled), the state of individual objects is meaningless. The only thing, according to Quantum Mechanics, that is *real* appears to be the pair state, and all that is real here is that the 2 spins are parallel - the question of which direction they are pointing in is not meaningful, and does not correspond to any physical reality, before the measurement is performed. This result is very strange - not least because it seems to assign some special role to 'measurements' (which are, after all, just some physical process, like any other).

B: IDENTICAL PARTICLES: FERMIONS and BOSONS

In the real world we do not have one or two particles but many; moreover there are many different kinds of them, and most are composite (ie., they can be further sub-divided into more 'elementary' particles). At first it seems hard to make sense of this all. However there is a fundamental feature of all particles or indeed all physical systems in Nature, whether they be composite or not. This is what is called "*Indistinguishability*" for Identical Particles, and it marks yet another fundamental feature of Quantum Mechanics that makes it quite different from anything classical. In ordinary common sense reasoning, we are perfectly happy to talk about two objects and say that 'they are the same' (for example, 2 balls, or 2 atoms, or 2 electrons - the first two being composite objects). By this we mean that in all important respects - perhaps all respects that there are - they have exactly the same properties. Thus, we would say that 2 electrons are both electrons *just because* in all respects they have identical properties (their masses, changes, etc., are all *exactly* the same). If this were not the case it is hard to see how we could call them both by the same name - these are amongst the defining properties of an electron, and if there were any difference at all, no matter how small, between 2 electrons then they could not both be examples of the same fundamental particle.

However, we would still insist on talking about them as if they were two distinct objects - even though they have identical properties. Thus we would say that even though 2 electrons may not be distinguishable by us from each other, we would nevertheless still insist that they are different (and in principle independent) objects. And this is precisely where quantum mechanics parts with common sense.

B.1: TWO INDISTINGUISHABLE PARTICLES

To see how this works, we are going to consider 2 'elementary' (ie., non-composite) particles, without yet saying what they might be - and ask how Quantum Mechanics describes them. The following will involve equations which at first you might find a little off-putting; but the appearance is a bit deceptive, and I will try to explain them in words so that you can see they are quite simple.

Quantum States for 2 Identical Particles: As always, we go back to the quantum state description. We want to describe a quantum state of 2 particles that are absolutely indistinguishable - not just in practise but also in principle. We still can imagine that the 2 particles are in different states from each other; it's just that we cannot know which particle is in which state (indeed, the question is supposed to be meaningless). Now it should be
therefore start with a state for these 2 particles which we will write as

$$\Psi_{\alpha\beta}(1,2) \;=\; |1,\alpha;\; 2,\beta\rangle \quad = \quad |\alpha\beta\rangle \tag{0.7}$$

by which we mean that "particle 1 is in state α, and particle 2 is in state β" (and in the abbreviated notation $|\alpha\beta\rangle$ it is simply assumed that the first-named state α refers to particle 1, and the second-named state β refers to particle 2). The symbols α and β could refer to one of 2 possible states, if these particles are 2 state systems; ie., "+" or "$-$", white or black, heads or tails, spin up or spin down, left or right polarization, etc. And, if these were classical particles, that would be all there was to it. There would be 4 possible states for the particles (for example, in the case where each particle could be in state "+" or "$-$", the 4 possible states would be $|++\rangle$, $|+-\rangle$, $|-+\rangle$, $|--\rangle$, where we use the abbreviated notation). And the classical system would have to be in one of these and *only one* of them.

However in quantum mechanics, we have to think differently, because we know that the system can be in a superposition of such states. Moreover, the particles in quantum mechanics have to be considered as *fundamentally indistinguishable*, ie., that there is no physical distinction at all between the two.

Now let us consider a pair of identical particles in the state $\Psi_{\alpha\beta}(1,2) \;=\; |\alpha\beta\rangle$ given above, and ask - what happens if we *swap* the particles? Then we have gone to a state $\Psi_{\alpha\beta}(2,1)$, which has to be interpreted as "particle 2 is in state α, and particle 1 is in state β"; we might just as well write it as $|\beta\alpha\rangle$. However, if indistinguishability is to be taken seriously, there can be *no distinguishable physical difference* between these two states.

You might conclude from this that we can only assume that these 2 states are identical, ie., that $|\alpha\beta\rangle = |\beta\alpha\rangle$. However, at this point there is a slight subtlety in the argument. For in quantum mechanics it not quantum states that we look at, but probabilities - this is a fundamental part of the theory. So the argument goes, it is not the wave-functions that are indistinguishable, but rather the probabilities computed from them. What this means here is that we must say that

$$P_{\alpha\beta}(1,2) = |\Psi_{\alpha\beta}(1,2)|^2 \quad = \quad P_{\alpha\beta}(2,1) = |\Psi_{\alpha\beta}(1,2)|^2 \tag{0.8}$$

ie., that it is the *squares* of the wave-functions that are the same. We can write this equation in abbreviated form as $|\Psi_{\alpha\beta}|^2 = |\Psi_{\beta\alpha}|^2$, if we like, using the abbreviated notation from before. Thus we are saying that the amplitudes of these 2 different states are related in a very simple way - their squares have to be equal.

Now in elementary school you learn that if you have 2 numbers A and B, and their squares are equal, so that $A^2 = B^2$, then there are only two possibilities - either $A = B$ or $A = -B$ (suppose eg, that $A^2 = B^2 = 4$, then clearly these 2 numbers can be either 2 or -2). Thus, it follows that we can say that there are 2 possibilities here as well - we have

$$\begin{aligned} \Psi_{\alpha\beta} &= \Psi_{\beta\alpha} \qquad (bosons) \\ \Psi_{\alpha\beta} &= -\Psi_{\beta\alpha} \qquad (fermions) \end{aligned} \tag{0.9}$$

So far this reasoning may seem like little more than a game with symbols. But now the sting in the tail. Suppose the 2 particles happen to be in the *same* state - eg., suppose they are both in the state α. Now for bosons we can just apply equation (0.9) to find that $\Psi_{\alpha\alpha} = \Psi_{\alpha\alpha}$, which is what mathematicians call a 'trivial identity' - it is obviously true. But now what happens if we do this with fermions? Then we find that $\Psi_{\alpha\alpha} = -\Psi_{\alpha\alpha}$. At first this looks like nonsense - how can a number be minus itself? However there is one number that is equal to its own negative - and that is the number *zero*. Thus for fermions we arrive at the conclusion that $\Psi_{\alpha\alpha} = 0$. What does this mean? It says that if 2 fermions are in the same state, the amplitude for this state must be zero, or in other words, such a state cannot exist - its probability must be zero. This statement is very important, so we write it out:

$$\Psi_{\alpha\alpha} = 0 \qquad (Pauli\ exclusion\ principle\ for\ Fermions) \tag{0.10}$$

or, in other words, no 2 identical fermions can be in the same state (whereas we can have as many bosons as we want in the same state).

Now this result is of enormous importance, because it actually explains the existence of all matter as we know it. Imagine a world of identical bosons. As we lower the energy, they will eventually all go into their ground state, to lower their energy - the same state for each one. Such particles could, for example, all occupy the same region of space, in some sort of localized bound state, even a very small region of space - they would then have almost no spatial extension, no matter how many of them there were. There are such particles - indeed photons are an example, and when they do 'Bose condense' together like this, we end up with the coherent light used in, eg., lasers.

However suppose we have a collection of fermionic particles. Then they cannot go into the same state - they will all have to go into different states, and so they cannot, for example, overlap in space. This means that as we put more and more of them together, they will have to occupy a bigger and bigger volume. Thus, with this one principle, we have suddenly explained one of the two fundamental properties of matter, recognized as such at the time of the Greeks - all matter has both *mass* and *extension*. We now see that the spatial extension, which is a defining property of matter, can be explained automatically if all matter is made from fermions, and indeed this is the case. All the particles that we typically consider to be part of matter (such as protons, neutrons, electrons, etc.) are fermions. On the other hand there are other particles, like photons, that we do not consider to be material - and indeed this is because they are bosons. And we see that all particles, according to this argument, have to be either fermions or bosons.

The repercussions of this simple result are quite enormous. Let's just briefly look at a simple one. Consider all the different electronic states that can exist around a nucleus in an atom (we saw these in Fig. **??**). Now if electrons were bosons, as we added more and more electrons, they simply would all go into the lowest energy bond state. But because they are fermions, each new electron has to go into a different state. We say that the electrons fill up the different electronic 'shells'. This is why an atom with, eg., 7 electrons in states around the nucleus (ie. a Nitrogen atom) looks different from one with 8 electrons (an Oxygen atom) or from one with 6 electrons (a Carbon atom); and they each have different properties from each other. Indeed, this is why we have atoms at all, each occupying some region of space, and why we have the different chemical elements in Nature, and the enormous richness of different molecules (ie., chemistry and biology).

TO BE CONTINUED.......