ANCIENT GREEK ASTRONOMY

Greek astronomy had its roots in the much older work done in the Baylonian and Egyptian civilisations. Over a thousand years before Greek astronomy began, the Babylonians already had extensive astronomical records, with good measurements of time, and of the positions of the moon, stars and planets in the sky (from which we inherit both our systems of angular and time measurement- the 360^o circle and the time units of 24 hrs, 60 minutes, and 60 seconds). Thus the prediction of a solar eclipse by Thales relied on these earlier observations, and on the preliminary understanding of them already achieved; and Hipparchus and Ptolemy could make use of very old Babylonian and Egyptian measurements.

The Greeks of the golden age already had original ideas about some astronomical phenomena. Thus, eg., Anaxagoras was aware that the moon shines by reflected light, and gave a theory of lunar eclipses which recognised the spherical shape of the earth and its consequent shadow at the moon; he also recognised the equivalence of the morning and evening stars as one planet (Venus). However it is equally clear from reading Plato and Aristotle that they were very far from understanding the implications of what was known. This understanding came later, mainly with the 1st Alexandrian school of mathematicians. A colleague of Archimedes at the Alexandrian library (and who became one of its first directors) was Eratosthenes of Cyrene (now Aswan, in southern Egypt). By measuring the difference between shadows cast by the sun in Alexandria and Cyrene, and knowing the distance between them, he estimated the diameter of the earth - by then understood to be spherical. His answer was accurate to roughly 1 per cent. Aristarchus, a near contemporary of Eratosthenes, went much further - indeed, he advanced pretty much the complete Copernican hypothesis, arguing that the sun was the centre of the solar system, and that the planets, including the earth, revolved around it in circular orbits. He also attempted to measure the distance of the sun, obtaining a result of 180 earth diameters (the correct answer is 11,726 earth diameters).

It might be thought at this point that the Greeks were well on the road to developing modern astronomy (albeit without instruments). Unfortunately subsequent developments were characterized by increasingly accurate observations, but also by increasingly less accurate ideas to explain them. This peculiar reversal can be understood in terms of the changing intellectual climate, which was not favourable to speculative theorizing. Thus the influential Hipparchus (161-126 BC) made enormous contributions to observational astronomy - but he also rejected the Aristarchean hypothesis of a solar-centred planetary system, and instead espoused the 'epicycle theory'.

The story after this is of a gradual decline in understanding, accompanied by an increasingly sophisticated effort in observation and measurement. By the time Ptolemy produced his *Almagest* in the 2nd century AD, Alexandria had long been an integral part of the Roman empire, which had little use for speculative philosophy or what we would now call 'theoretical physics/astronomy'. The ideas of Aristarchus and his more daring colleagues had been almost forgotten, to be subsumed by an unwieldy system which, although it was capable of explaining observations up to a point, still required continuing and somewhat arbitrary elaboration. We will also return to this point in the notes on Copernicus.

It is perhaps useful to think of the history of Greek astronomy as something of a Greek tragedy (rather like the history of Greek philosophy). First there was a period of heroic efforts in the early days, which carried the corpus of theoretical understanding in a intellectual wave reaching almost as the Renaissance. This was followed by a recession of the wave, back to what eventually, in the Middle ages, became an intellectual straightjacket which stifled the development of ideas until the Renaissance.

There were 2 main reasons for this "intellectual recession". The first was simply that the tides of war and history took away just as easily as what they had given - once the Romans came to power the Greek ideas and methodology became mere curiosities, to be largely lost with the later demise of the Roman empire. Thus the records we have of the earlier period are patchy - they hint at remarkable ideas, but most of what we know is based on later commentaries.

The second reason was more subtle. The Greek work in philosophy, physics, and astronomy was too far ahead of its time, and moreover was hobbled by a crucial flaw. Often it is said that Greek science ultimately faltered because of the lack of experiments, systematic observations, and the experimental methodology that came much later. But this is only very partly true - indeed, Alexandrian observational astronomy was remarkably accurate in its later stages, and almost obsessed with data-collecting and measurement. In reality the problem was deeper, at the theoretical level. Greek physics and astronomy were, almost inevitably, slaves of the philosophical framework forced on them by the Athenian philosophers (which itself reflected ideas going back to Pythagoras, Heraclitus, and Parmenides). The prevailing orthodoxy was essentially a *geometric* one, overly influenced by considerations of symmetry and abstract mathematical form. Totally lacking was a place for *mechanics*; there were no forces, no gravity, no understanding of dynamics in the modern sense. Concepts like the "void", or vacuum, or of 'force' in the more modern sense, of particles or atoms, were largely ignored because they did not fit the standard model.

In this way the models of Aristotle in natural philosophy and cosmology, or of Ptolemy in astronomy, conformed to the orthodox Greek ideal of *what a theory should be*. The ideas of Leucippus, Democritus, and Archimedes, did not so conform - and so although they did get a hearing, their influence was much less durable. Of course the orthodoxy was partly formed by philosophers like Aristotle, and there are various reasons for its more widespread acceptance (apart from the depth and breadth of his arguments). The ideas of the Atomists, for example, probably seemed just too outlandish to many Greeks, as well as saying less about the ethical issues that concerned most of them. Even though the atomistic ideas were partially resuscitated later, by the Epicureans, they never enjoyed the same popularity in the Roman world as the philosophy of the Stoics. This partly because the ethical ideas of the Stoics were more appealing to the Romans - but it is also because the Athenian ideas (due mostly to Plato and Aristotle), of what a theory of the natural world should look like, had by then been accepted by most Romans.

This point is very important - what is accepted as a valid theoretical framework, or what are accepted as the philosophical or conceptual underpinnings of such a theoretical framework, often depend on unspoken assumptions about what *any* kind of theory should look like. We are no freer from this kind of preconception today - it is simply that our idea of what a scientific theory should look like is now very different.

(1) EARLY DEVELOPMENT of GREEK ASTRONOMY

The story of Greek astronomy is of an increasingly detailed observational picture, developed hand in hand with more sophisticated mathematical methods and a theoretical framework based on these methods and on the observations. The best way to understand this is to look at the chronological development, concentrating on the main historical figures involved. In the course of this story we also encounter the two main cosmological ideas, involving celestial spheres, and the epicycle theory (for which also see the slides).

The early history, as discussed above, is one of daring ideas, which were later dropped in favour of an earth-centred epicycle theory. This early work did not appear in a vacuum - the Greeks relied much on observations of the Egyptians and the Baylonians, stretching back well over a thousand years. However we begin our story in the Greek golden age, at the time of Plato.

EUDOXUS of CNIDUS (408-355 BC): Eudoxus was born in Cnidus (on Resadiye peninsula, on the Black Sea, now in Turkey). He studied mathematics with Archytus in Tarentum, in Italy; Archytas was a follower of Pythagorean school of mathematics. While in Italy he visited Philiston in Sicily, and studies medecine with him. The problem of 'squaring the cube' was one which interested Archytas and it would be reasonable to suppose that Eudoxus's well-known interest in that problem was stimulated by his teacher. Other topics that it is probable that he learnt about from Archytas include number theory and the theory of music.

Eudoxus made his first visit to Athens at the age of 23, in the company of the physician Theomedon with the intention of studying at Plato's Academy, which had only been recently established. Eudoxus spent

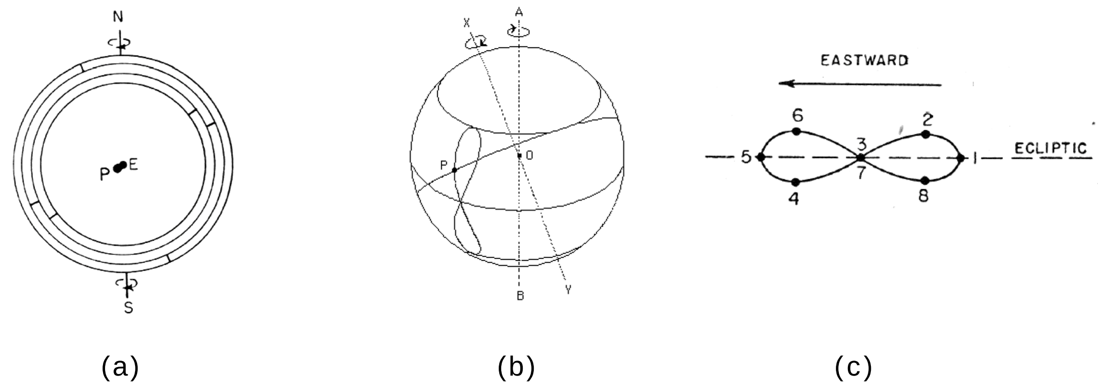


Figure 1: The construction of Eudoxus. In (a) we see his idea of concentric celestial spheres turning around the earth - the figure shows 3 spheres but Eudoxus postulated 27 of them. Each planet was entrained to move between 2 adjacent spheres. The axes of rotation of the different spheres were not parallel. In (b) we see how a planet caught between 2 spheres rotating at different rates, and with axes along different directions, would end up moving in a 'hippopede' curve. In (c) the hippopede is stretched out by the motion of the other spheres, which cause the first two spheres to turn rapidly about the earth.

several months in Athens on this visit, and attended lectures on philosophy by Plato and other philosophers at the Academy. It appears that at some time during this period Plato and Eudoxus may have had a falling out. Eudoxus was quite poor and could only afford an apartment at Piraeus, the Athenian port. According to Heath, to attend Plato's lectures, he walked the 23 km there and back, each day.

Thereafter, reputedly with the help of financial aid from friends, he went to Egypt to learn astronomy with the priests at Heliopolis, and made astronomical observations from an observatory located between Heliopolis and Cercesura. From there Eudoxus travelled to Cyzicus, in northwestern Asia Minor on the south shore of the sea of Marmara. There he established his own School - this proved in the end to be quite popular, and he had many pupils.

In around 368 BC Eudoxus made a second visit to Athens accompanied by a number of his followers. It is hard to work out exactly what his relationship with Plato and the Academy were at this time. There is some evidence to suggest that Eudoxus had little respect for Plato's analytic ability, and it is easy to see why that might be - as a mathematician his abilities went far beyond those of Plato. It is also suggested that Plato was not entirely pleased to see how successful Eudoxus's School in Marmara had become.

Eudoxus later returned to his native Cnidus and there was acclaimed by the people who elected him to an important role in the legislature. However he continued his scholarly work, writing books and lecturing on theology, astronomy and meteorology.

He had built an observatory on Cnidus and we know that from there he observed the star Canopus. The observations made at his observatory in Cnidus, as well as those made at the observatory near Heliopolis, formed the basis of two books referred to by Hipparchus. These works were the "*Mirror*" and the "*Phaenomena*", which are thought by some scholars to be revisions of the same work. Hipparchus tells us that the works concerned the rising and setting of the constellations but unfortunately these books, as with all the other works of Eudoxus, have been lost.

He had one son, Aristagoras, and three daughters, Actis, Philtis and Delphis. At the age of 53, in 355 BC, he died in Cnidos, highly honored as a lawgiver.

As we saw in the notes on Greek mathematics, Eudoxus made important contributions to the post-

Pythagorean development of mathematics, and he was the best-known member of what later was called the "Athenian school" (although, as we have seen, he spent little time in Athens). His most lasting influence came from his planetary theory, published in his book "*On velocities*". This was based on spheres, possibly following Pythagoras's belief that the sphere was the most perfect shape. The theory consisted of 27 spheres, each sphere rotating about an axis through the centre of the Earth. The axis of rotation of each sphere was not fixed but was attached to the adjacent outer sphere, and so was itself rotating with the rotation of that sphere (see course slides). This led to a very complicated set of motions. Eudoxus observed that if two spheres rotate with constant, but opposite, angular velocity, and their axes are slightly tilted with respect to each other, then a point on the equator of the inner sphere describes a figure of eight curve. This curve was called a hippopede - it is shown in Figs. 1(b) and 1(c).

Eudoxus imagined a planet as the point following this curve; he then introduced a third sphere to correspond to the general motion of the planet against the background stars while the motion round the hippopede produced the observed periodic retrograde motion. This 3-sphere subsystem was set into a 4th sphere, which yielded the daily rotation of the stars. The construction is complicated - by fiddling with the spheres, there relative orientations, and their relative angular velocities of rotation, once can approximately describe the apparent motion of the planets.

The Eudoxian planetary system was described by Aristotle in the "*Metaphysics*". It was recognized even then as a remarkable mathematical achievement. It is possible that Eudoxus used the spheres only as a computational device; however this was not the view of Aristotle, who assumed they were real. Certainly neither of them could have ever tried to test it against observational data - it would have failed the test. This is because the real motion of the planets proceeds not on circles but ellipses, and they do not move at a constant velocity along these ellipses. No construction involving spheres turning at a constant rate could have ever reproduced this.

ARISTARCHUS of SAMOS (310 - 230 BC: The career of Aristarchus came shortly after that of Euclid, and before that of Archimedes. He was a student of Strato of Lampsacus, who was head of Aristotle's Lyceum. However, it is not thought that Aristarchus studied with Strato in Athens but instead in Alexandria. Strato became head of the Lyceum at Alexandria in 287 BC and apparently Aristarchus studied with him shortly after then.

Very little is known of the life of Aristarchus. His only surviving work, entitled "*On the Sizes and Distances of the Sun and Moon*", provides the details of his remarkable geometric arguments, based on observation, whereby he concluded that

(i) the Sun was about 19 times as distant from the Earth as the Moon, and also has a diameter about 19 times that of the moon; and

(ii) That the sun lay at a distance 180 earth diameters from the earth (and hence, from (i) that the moon was at a distance of just under 10 earth diameters from us).

Now these estimates were badly wrong (in fact the distance of the sun is roughly 390 times the distance of the moon from us, and is at a mean distance of 11,726 earth diameters; and the moon is just over 30 earth diameters from us). However the fault in Aristarchus's estimates did not lie in his geometric reasoning, which was entirely correct, but in his complete lack of accurate instruments.

His method for getting his results was actually very ingenious. First, he noted what everybody knew, which was that the sun and moon appear to be almost exactly the same size in the sky. Aristarchus realized that this was pure coincidence, and that it implied that the ratio of the distances of the 2 objects had to be the same as the ratio of their diameters. It remained then to determine what this ratio was, and to determine the distance of one of the two from the earth. His method can be seen in Figs. 2(a) and 2(b), and relied on making 2 measurements - one of the angular size of the earth's shadow when it crosses the moon, and the other of the angle between the sun and the moon in the sky when the moon is exactly half-illuminated by the sun (ie., at half-moon phase). If one then knows the size of the earth, the rest follows.

He noted first that because the sun is not an infinite distance away, the shadows cast by both the moon and the earth in space would be cone-shaped. The apex of the cone in the case of the moon's shadow would be very close to the earth - but he then realized that when eclipses of the moon occur, we are simply seeing the shadow of the earth pass over the moon. Thus the earth is bigger than the moon, and if we know the size of the earth, we can deduce the size of the moon by seeing how large is the earth's shadow at the moon, compared to the moon. Aristarchus decided that the earth's shadow was roughly twice the size of the moon (it is actually more like 3 times as large), and concluded that the diameter of the earth was roughly twice that of the moon (in fact it is more like 4 times the diameter - because the earth's shadow out at the moon is 3/4 the earth's size).

To determine the ratio of the distances of the sun and moon, Aristarchus used the triangulation shown in 2(b). If the sun were an infinite distance away, the angle between the sun and moon in the sky when we observe half-moon would be 90^o. However it is actually less than this, because the sun is a finite distance away. The accuracy of such measurements is limited by the accuracy of measurement of the sun's position, of the time, and by our determination of when the moon is actually at exactly half-phase. He argued that the the angle was 87^o, which then led him to the result that the sun was 19 times further than the moon. Unfortunately, using only naked eye methods it was actually impossible for Aristarchus to get an accurate result - the sun is just too far away, and the actual angle is $89^o\ 50'$.

We have almost no evidence concerning the origin of Aristarchus's idea of a heliocentric cosmological system; what we know of his theory comes from remarks made in Archimedes "*The Sand-Reckoner*" and from a mention by Plutarch. According to Plutarch, Aristarchus followed Heraclides of Pontus in believing that the apparent daily rotation of the fixed stars was due to the rotation of the earth on its axis. Aristarchus apparently advanced pretty much the complete Copernican hypothesis - arguing that the sun was the centre of the solar system, and that the planets, including the earth, revolved around it in circular orbits. According to Archimedes this caused him a little trouble (there was an attempt to indict him by a certain Cleanthes), but nevertheless the hypothesis was adopted by his successor Seleucus. Archimedes wrote:

"*.... the 'universe' is the name given by most astronomers to the sphere, the centre of which is the centre of the earth, while its radius is equal to the straight line between the centre of the sun and the centre of the earth. This is the common account as you hear from astronomers. But Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the 'universe' just mentioned. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit, and that the the size of the sphere of fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface.*"

In fact Archimedes did not agree with Aristarchus's idea that the celestial sphere was so large. Aristarchus presumably made this assumption to explain why parallax effects were not visible in the positions of the stars.

According to Vitruvius, Aristarchus also invented a sundial in the shape of a hemispherical bowl with a pointer to cast shadows placed in the middle of the bowl.

ERATOSTHENES of CYRENE (276 - 194 BC): Eratosthenes was born in Cyrene, in North Africa (now Shahhat, Libya). By this time, after the exploits of Alexander the Great, the eastern part of North Africa had become Greek. The teachers of Eratosthenes included both the scholar Lysanias of Cyrene, and the philosopher Ariston of Chios who had studied under Zeno, the founder of the Stoic school of philosophy (not to be confused with Zeno of Elea, the much earlier colleague of Parmenides, who was responsible for 'Zeno's paradoxes). Eratosthenes also studied under the poet and scholar Callimachus, who also came from Cyrene. He then apparently spent some years studying in Athens.

Eventually Eratosthenes came to Alexandria, which by then had become the centre for activity in mathematics and astronomy. This activity centred around the great library there, which functioned as much as a research institute as a library. The library had been planned by Ptolemy I (Soter) and was really developed into the large institute that it became by his son Ptolemy II (Philadelphus), starting on a base of copies

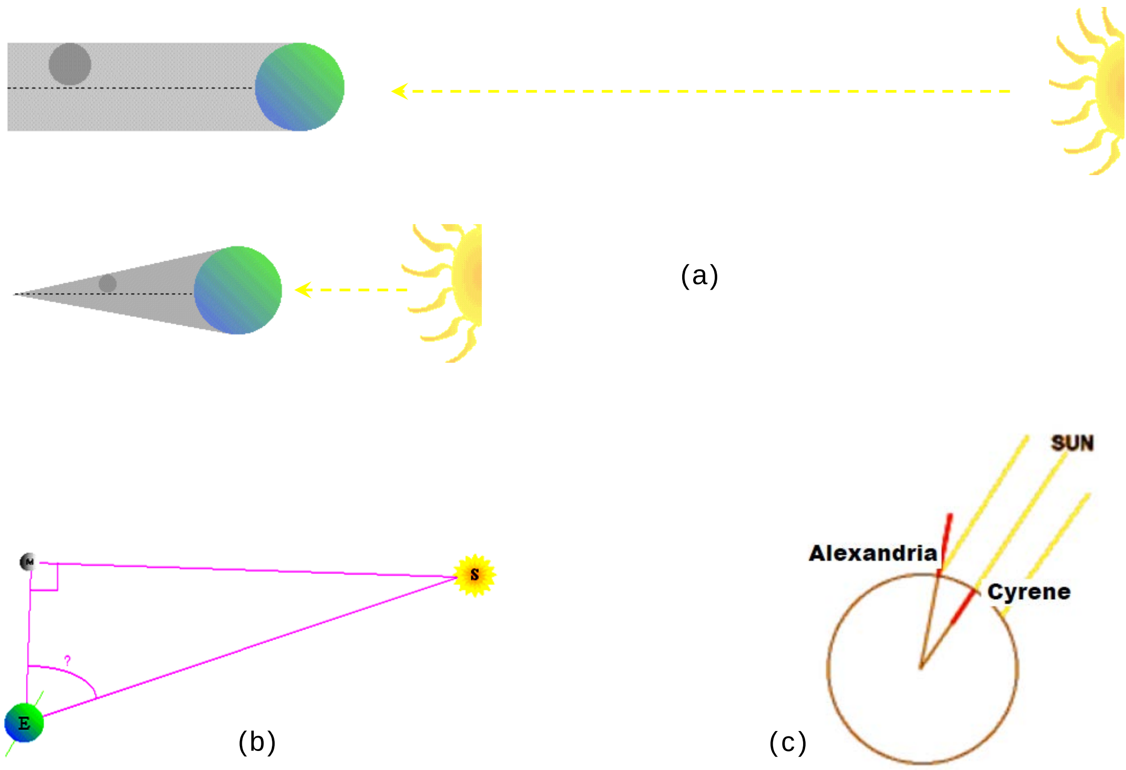


Figure 2: Some of the deductions made by the early Greeks about the size of the heavenly bodies, including the earth. In (a) we see the way shape of the earth's shadow in space depends on how far away the sun is - the rays from a very distant sun are almost parallel, so that the diameter of the earth's shadow hardly varies as we move out to the moon, but the earth's shadow rapidly shrinks to a point for a nearby sun. In (b) we see how Aristarchus tried to measure the moon's distance by measureing the angle between the sun and moon in the sky at "half moon"; and in (c) we see the construction of Eratosthenes, for the measurement of the diameter of the earth, using shadows cast by vertical posts planted in 2 different places on the earth.

of the works in the Athenian library of Aristotle. Ptolemy II appointed Callimachus, the former tutor of Eratosthenes, as the second librarian. When Ptolemy III (Euergetes) succeeded his father in 245 BC, he hired Eratosthenes to come to Alexandria as the tutor of his son Philopator. On the death of Callimachus around 236 BC BC, Eratosthenes then became the third librarian at Alexandria, in the library in a temple of the Muses called the Mouseion. In this post he was a colleague of Archimedes.

Eratosthenes apparently had the reputation of being an all-round scholar, who had mastered many different areas of knowledge. Apparently he also had the nickname "Pentathlos", denoting an all-round athlete. In any case, he spent the rest of his career in Alexandria. He is said to have became blind in old age and to have committed suicide by starving himself.

His accomplishments were remarkably varied. In the artistic sphere his works include the poem "Hermes", inspired by astronomical ideas, as well as literary works on the theatre and on ethics. He also made major contributions to geography. He accurately mapped the route of the Nile to Khartoum, showing the two Ethiopian tributaries, and suggested that lakes were the source of the river. He was the first to correctly suggest that heavy rains sometimes fell in regions near the source of the river and that these would explain the flooding lower down. He also worked out a calendar that included leap years, and laid the foundations of a systematic chronography of the world when he tried to give the dates of literary and political events from the time of the siege of Troy.

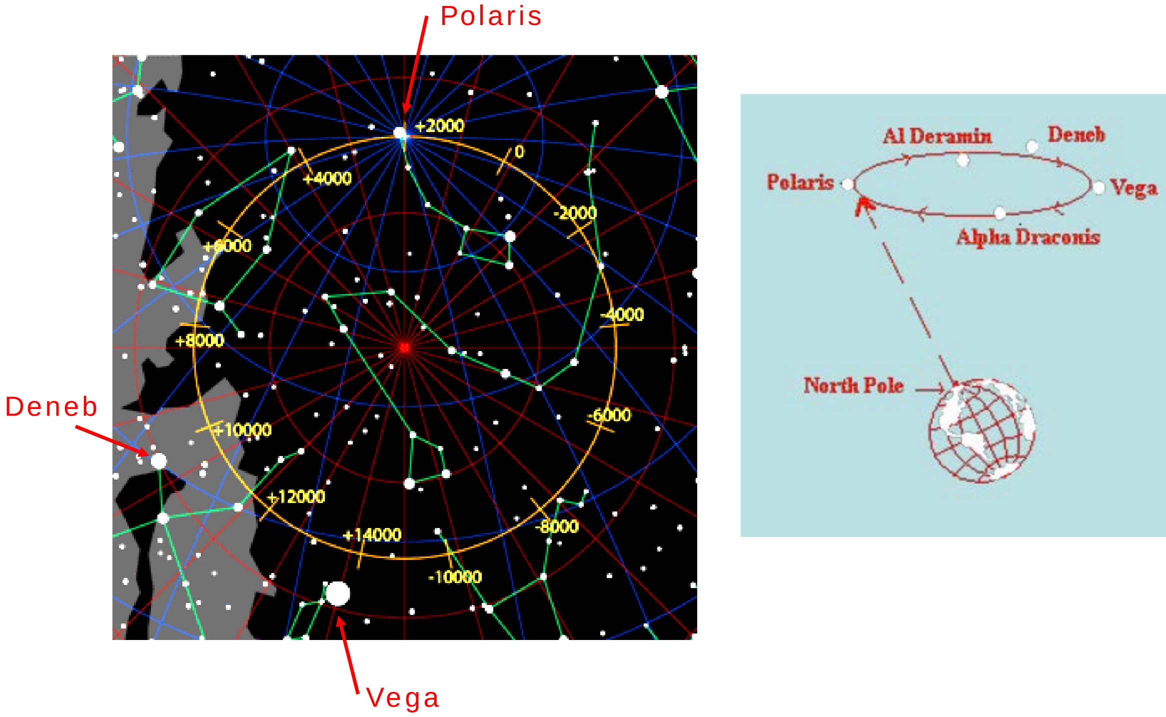
His contributions to mathematics were very extensive, although we know about them almost entirely from commentaries by later writers. Some of his results appeared in a work called "*Platonicus*", in which he apparently studied problems in number theory, geometry, and musical theory; most notable of the contributions herein were his discussion of the problem of duplicating the cube. Another book written by Eratosthenes, "*On means*" is also lost, but was mentioned by Pappus as one of the great books of geometry. A remarkable contribution (described in the "*Introduction to arithmetic*" by Nicomedes) is his prime number sieve, the 'Sieve of Eratosthenes'. In modified form, this is still an important tool in number theory.

However what Eratosthenes is best known for is his astronomical work. On the observational side, he is remembered for his invention of the armillary sphere, around 255 BC; this was a mechanical representation of the motion of the heavenly bodies, which was widely used until the invention of the orrery. He is also said to have compiled a star catalogue containing 675 stars

His most famous contribution to astronomy was his measurement of the diameter of the earth (see Fig. 2(c)). His initial idea was based on his observation that the sun is directly overhead at noon in Cyrene in southern Egypt (now Aswan, the site of the famous dam) on the first day of summer. While visiting Cyrene, Eratosthenes apparently noticed that the noon sun reflected directly back from the water in the bottom of a well. Knowing that the earth was a sphere, he correctly reasoned that if he could determine the altitude of the noon sun at some other location on the first day of summer, AND if he knew the distance between these two locations, he could compute the circumference of the earth. The other location was Alexandria - the distance between these two is equivalent to 7^o in angle, so tha tthe angle of the shadow cast at Alexandria was 7^o off the vertical. Eratosthenes reasoned that since a full circle subtends 360^o, the distance between Cyrene and Alexandria is a fraction $\frac{7}{360}$of the circumference of the earth. The distance form Alexandria to Cyrene was known very accurately - so the circumference could then be calculated. His answer was accurate to roughly 1 per cent - a remarkable achievement.

Eratosthenes also measured the distance to the sun, giving the result as 804,000,000 stadia, and the distance to the Moon as 780,000 stadia. Since 1 stadium is approximately 160 m, this means that he estimated the sun's distance as roughly 120,000,000 km, and the moon's distance as roughly 120,000 km (the correct values are 149,000,000 km and 400,000 km, respectively). He computed these distances using data obtained during lunar eclipses. Ptolemy tells us that Eratosthenes also measured the tilt of the Earth's axis with great accuracy, obtaining the value of 11/83 of 180^o, equivalent to 23^o $51'$ 15" (the correct answer is roughly 23^o $30'$).

The amazing accuracy of these results, obtained using naked eye observations, tells us that Eratosthenes must have been a superb observer with very accurate equipment at his disposal. Unfortunately we have little idea of what instruments he had.



(a) (b)

Figure 3: The precession of the equinoxes, discovered by Hipparchus. This can be understood as a precession of the earth's axis of rotation, which occurs over a 25,000 yr time period. The direction of the North pole in the sky with time is shown in (a). Currently this direction is close to Polaris, which we call the "pole star" for this reason. In 10,000 AD it will be much closer to Deneb, and in 14,000 AD much closer to the very bright star Vega. In (b) we show how the axis is precessing, as viewed from outside.

HIPPARCHUS of RHODES (190 - 120 BC: Little is known of Hipparchus's life, but he is was born in Nicaea in Bithynia (now this town is called Iznik, on the eastern shore of Lake Iznik in north-western Turkey). Hipparchus is thus often referred to as Hipparchus of Nicaea or Hipparchus of Bithynia and was listed among the famous men of Bithynia by Strabo, the Greek geographer and historian. There are coins from Nicaea which depict Hipparchus sitting looking at a globe (see course slides) and his image appears on coins minted under five different Roman emperors between 138 AD and 253 AD.

So little is known of the life of Hipparchus that much of our knowledge of his movements comes from the observations that are attributed to him. Apart from those made in Niceea, there were those made from the north of the island of Rhodes, plus several made in Alexandria. Thus apparently Hipparchus was in Alexandria in 146 BC and in Rhodes near the end of his career in 127 BC and 126 BC. Only one work by Hipparchus has survived, his "*Commentary on Aratus and Eudoxus*", which is certainly not one of his major works.

Most of the information which we have about the work of Hipparchus comes from Ptolemy's "*Almagest*". Where one might hope for more information about Hipparchus himself would be in the commentaries on the "*Almagest*". There are two in particular by Theon of Alexandria and by Pappus, but these add little information about Hipparchus. When Ptolemy refers to results of Hipparchus he usually assumes that the reader also has his original writings.

Nevertheless Hipparchus is considered by many to have been the greatest astronomical observer of pre-Renaissance times, if not the greatest overall astronomer of antiquity. He was the first Greek to give accurate quantitative models for the motion of the Sun and Moon, using records of observations going back many centuries, made by the Babylonians; he measured the lunar month to within an error of 1 second, and the year to an accuracy of 6 minutes. In the course of doing this he compiled trigonometric tables, and not only developed simple trigonometric ideas, but also solved some problems of spherical trigonometry. Using his solar and lunar theory and his knowledge of trigonometry, he may have been the first to develop a reliable method to predict solar eclipses; he also attempted to measure the distances of the sun and moon. His other best-known results were the discovery of the precession of the equinoxes (ie., of the earth's axis - see Fig. 3), and the compilation of the first comprehensive and accurate star atlas (containing the measured positions of 850 stars). His value of the annual precession of the equinoxes was 46" (the actual value is 50.26"), much better than the figure of 36" that Ptolemy derived nearly 300 years later! To do all this he developed or invented several new instruments, including the astrolabe, and possibly also the armillary sphere. On the theoretical side he also developed the epicycle as first formulated by Apollonius.

It is interesting to look in a little more detail at his best-known discovery, that of the precession of the equinoxes. This is seen as the slow drift of the position on the celestial sphere of the points at which the ecliptic crosses the celestial equator. His discovery came from his attempts to calculate an accurate value for the length of the year. One can define the 'year' in 2 ways, viz., (i) the time it takes the sun to return to the same place amongst the fixed stars (the sidereal year); or (ii) the length of time before the seasons repeat ie., to pass once between the equinoxes (the tropical year).

To calculate the length of these two years Hipparchus required many years of observations. He used Babylonian data, along with his own, to derive a value for the tropical year of 365 days, 5 hrs, and 50 minutes (the actual value is 365 days, 5 hrs, 48 minutes and 46 seconds, ie., 1 minute and 14 seconds less). He then checked this against observations of equinoxes and solstices, using both his own observations and those of Aristarchus in 280 BC and Meton in 432 BC. His calculation of the length of the sidereal year, again with the help of older Babylonian data, gave a result of 365 days, 6 hrs, and 10 minutes.

(2) LATER DEVELOPMENTS: The PTOLEMAIC THEORY

We now shift to Alexandria in very different times - by the 2nd century AD, Ancient Greece itself was a distant memory, and the Roman empire was at the height of its power. It had taken over most of the regions conquered by Alexander, and added enormously to them, particularly in what we now call Western Europe.

However intellectual attitudes were very different in Roman times - rather than adding to the Greek achievements, the Romans preferred to consolidate them. Innovative in engineering, and in warfare, the attitudes in the Roman empire bore more than a slight resemblance to those prevailing in the USA or in Ottawa today. There was a disregard for radical intellectual innovation prevailed amongst those in power, and amongst most of the populace, except in a few quarters where intellectuals and artists looked back to the Greeks for inspiration.

In the same way, astronomy was characterized by a real development in observational capabilities, but almost no change in the underlying theoretical ideas used to understand them. Many key ideas were forgotten, and in the end, those that survived were not the best of those that had been developed in earlier times.

(a) Life of CLAUDIUS PTOLEMY (85 - 165 AD: We know very little of Ptolemy's life. His name, Claudius Ptolemy, is a mixture of the Greek Egyptian 'Ptolemy' (the name taken by the rulers of Egypt starting from the annexation of Egypt by Alexander the Great - the name was quite common among the

Macedonian upper class at the time of Alexander), and the Roman 'Claudius'. This suggests that he was descended from a Greek family living in Egypt and that he was a citizen of Rome.

He clearly spent a large fraction of his life as an astronomer. He was apparently a student of the astronomer and mathematician Theon of Smyrna, who had written without much depth of understanding on conjunctions, eclipses, occultations and transits. Ptolemy is known to have made observations from Alexandria in Egypt at least during the years 127-141 AD; He appears to have lived all his life in Alexandria.

All of Ptolemy's most important works have survived. This includes the 13-volume "*Almagest*", his 8-volume work on Geography, and his 5-volume work on Optics; and he also wrote a popular account of his results in 2 books under the title "*Planetary Hypothesis*", and a book on astrology. He was one of the most influential astronomers and geographers of all time; in particular, his version of the geocentric theory of the cosmos prevailed for 1400 years, until a century after the work of Copernicus in 1543. However his reputation is rather controversial. The work of some historians show that Ptolemy was a remarkable mathematician, whereas that of others show that he was no more than a very good writer. Some have even argued that he plagiarised the work of his predecessors, notably that of Hipparchus.

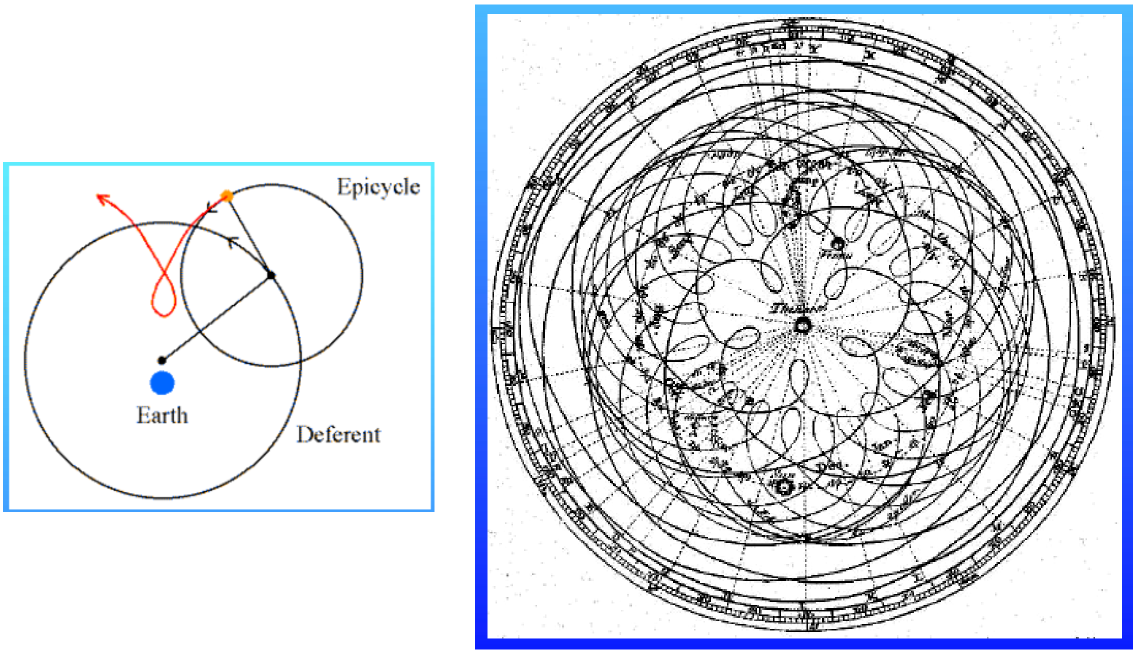
The "*Almagest*", originally written in Greek, was first entitled "*The Mathematical Compilation*" , which later became "*The Greatest Compilation*". This was translated directly into Arabic as "*al-Majisti*" and from this the title "*Almagest*" was given to the work when it was finally translated from Arabic to Latin in the Middle Ages. It is the earliest of Ptolemy's works, and gives in great detail his theory of the motions of the Sun, Moon, and planets. The "*Almagest*" shares with Euclid's "*Elements*" the distinction of being the scientific text with the greatest longevity; for 1400 years it defined astronomy as a subject for most of Western Europe and much of the Islamic world.

In the opening books of the "*Almagest*" Ptolemy explains his description of a universe based on the earth-centred system of Aristotle. Ptolemy used geometric models to predict the positions of the sun, moon, and planets, using combinations of circular motion known as epicycles (see course slides). With this model Ptolemy then describes the trigonometric methods which he uses in the rest of the work.

Ptolemy then proceeds in vol. 3 to the motion of the sun, comparing his own observations of equinoxes with those of Hipparchus and Meton in 432 BC. Based on observations of solstices and equinoxes, Ptolemy found the lengths of the seasons, and proposed a simple model for the sun which was a circular motion of uniform angular velocity, but with the earth displaced from the centre by a distance called the eccentricity. In Books 4 and 5 Ptolemy gives his theory of lunar motion, studying the 4 different periods associated with this motion (the time taken to return to the same longitude; the time taken to return to the same velocity - the lunar anomaly; the time taken for to return to the same latitude; and the synodic month, ie., the time between successive oppositions of the sun and moon). He also outlines Hipparchus's epicycle model for the motion of the moon, but then improved on it. In book 6 he used these results to give a theory of eclipses, and in Book 7 he used his own observations along with those of Hipparchus to show that the fixed stars always maintain the same positions relative to each other. Much of Books 7 and 8 are devoted to Ptolemy's star catalogue, which contained over 1000 stars.

The final five books of the Almagest outline his planetary theory. There had been no satisfactory theoretical model to explain the rather complicated motions of the five planets before the Almagest. Ptolemy combined the epicycle and eccentric methods to give his model for the motions of the planets. In this model, the path of a planet consisted of circular motion on an epicycle, the centre of the epicycle moving round a circle whose centre was offset from the earth. Ptolemy's most important innovation was to make the motion of the epicycle centre uniform not about the centre of the circle around which it moves, but around a point called the equant, which is symmetrically placed on the opposite side of the centre from the earth (see course slides, and the more detailed discussion below)

(b) Ptolemaic Theory: The Epicycle/Deferent Construction: The planetary theory which Ptolemy developed was a remarkable achievement. It was able to fit existing and previous observational data, and although complicated, it predicted the motions of the planets fairly well. As time went by the predictions became less and less reliable, but for 1300 years no better model was available, and his theory



(a) (b)

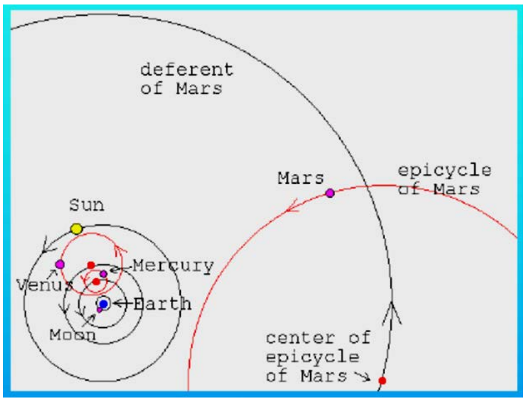
Figure 4: The basic idea of deferents and epicycles. In (a) this is shown in simple form for a single planet - the planet orbits around the epicycle, which is centred at a point which is going around the deferent. The path of the planet is then shown in red. In (b) we see the level to which this picture was taken by later astronomers, working in the early renaissance.

was the point of departure for astronomers during this time, both in Europe and in the Islamic world. By insisting on a foundation of accurate observations for any study of the sky, Ptolemy's work decisively influenced the work which in the early Renaissance eventually led to the overthrow of the Ptolemaic system itself. Thus Ptolemy's methods, which he inherited from his forbears - notably Hipparchus - led ultimately to the destruction of the ideas on which he tried to understand his observations.

Ptolemy's theory was based on the use of deferents and epicycles - he brought it to the form in which it was passed down to later generations. This theory was a fairly natural continuation of the earlier ideas on celestial spheres, discussed above. It was first discussed by the remarkable Greek mathematician Apollonius of Perga (c. 262-190 BC), who was best known for his pioneering work on conic sections. The Apollonian construction was also used by Hipparchus, and by the time of Ptolemy it was a widely used construction for the planetary and solar orbits around the earth. In what follows I first explain the Apollonian construction, and then the modifications made by Ptolemy. The explanation is by no means comprehensive. To understand the following text it is very helpful to look also at the course slides (particularly slide 1.39), and Fig. 4 .

The basic idea of Apollonius was fairly simple - the sun moved around the earth, as did the moon. The 'inferior planets' Mercury and Venus each orbited around an epicycle, which was a circle centred on a point P, and this point P was itself was moving around another circle called a 'deferent'. The deferents were centred on the earth - the deferent circles of Mercury and Venus were larger than the orbit of the moon around the earth, but smaller than the orbit of the sun (see Fig. 5(a)). The period of the sun around its orbit was one year, and likewise for the points P_{Merc} and P_{Ven} on their deferents; all three of these, as well as the moon, moved at uniform speed around their deferents and orbits. The points P_{Merc} and P_{Ven}

(a)



(b)

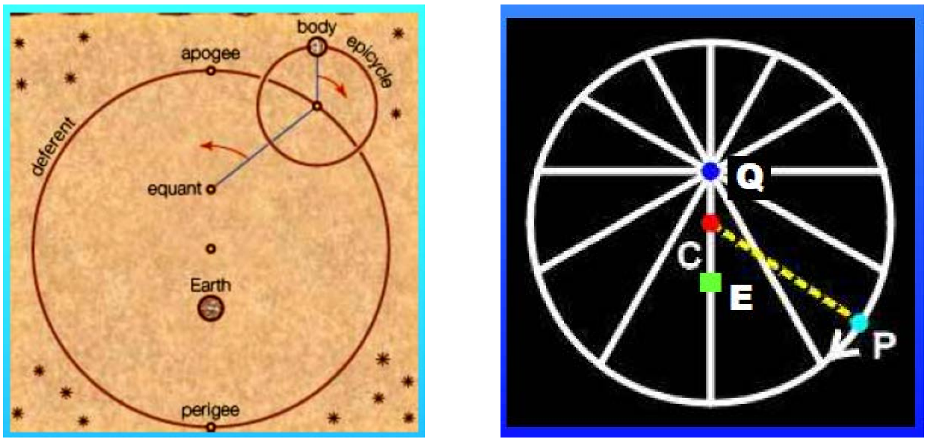


Figure 5: A closer look at the deferent/epicycle construction. In (a) we see how this was supposed to work for the planets Mercury, Venus, and Mars, as well as for the sun - notice the sun has no epicycle. In (b) we see the 'equant' construction in more detail; and in (c) we see how Ptolemy arranged for the motion of the planet around the deferent so that equal angles in equal times were swept out around the equant.

were locked to the sun's motion around its orbit - each moved around their deferents in such a way as to be always in the same direction as the sun, as viewed from the earth, as it orbited around its orbit. The motion of Mercury and Venus around their epicycles then made it seem, as viewed from the earth, as though they were simply moving back and forth on either side of the sun. Note that the planets Mercury and Venus each moved rather rapidly around their epicycles (88 days for Mercury, 224.7 days for Venus).

The 'superior planets', Mars, Jupiter, and Saturn, also orbited in epicycles around points P on their respective deferents, and these points also moved around their deferents. The deferents were again centred on the earth. However these deferents were larger than the orbit of the sun. Moreover, the motions of the epicycle centre points P were no longer connected with the motion of the sun around its orbit. Each of the superior planets took one year to orbit around its epicycle, but the epicycle centres P_{Mars}, P_{Jup}, and P_{Sat} took longer than a year to orbit around the earth (in the case of Jupiter and Saturn, much longer). The resulting motion looked from the earth as though the planets were moving in a rather curious fashion across the sky, with both 'prograde' motion in the same sense as the sun, and 'retrograde' motion in the opposite direction.

The main innovation introduced by Ptolemy was the idea of the 'equant' (see Fig. 5(b)). This displaced the earth from the centre of the deferents. We imagine 2 points on opposite sides of a deferent centre, equidistant from it. The earth occupies one of these points, while the other point, called the equant, is empty. The motion of the epicycle centres around the deferents is then the same as before, *except* that the motion of the points P was no longer uniform. Instead of moving at constant speed, the epicycle centres P moved around their deferents in such a way as to seem to be moving at constant angular speed around the equants (rather than at constant angular speed around the deferent centres); see Fig. 5(c). This meant that the epicycle centres were not only moving more slowly around their deferents when at 'apogee' (ie., the part of the deferent nearest the equant), but that that because they were then at their maximum distance form earth, the deferent centres seemed to move even more slowly from earth.

The net result of this construction was that the motion of the planets was no longer uniform in the heavens, and appeared even less so from the earth. A construction of this kind was forced on Ptolemy by observations - the departure of planets like Mars from the simple picture of circular motions was so large as to be completely incompatible with the very long-lived set of astronomical data at his disposal. If he was to keep a construction involving circles, some solution like this was necessary.

From a modern point of view these constructions seem hopelessly complicated and also "*ad hoc*" - they were not based on any clearly formulated theoretical idea, and the various elements of the theory were introduced in a way which seems completely arbitrary if divorced from the data they were trying to explain. Viewed from a more modern standpoint, we would that the theory, insofar as it was a theory at all, was in no way predictive or even falsifiable. Indeed, it was continually being modified to fit the data.

That this seems ridiculous now is a measure of how both science and 'common sense' have evolved since then; and of how much more sophisticated is our contemporary understanding of what a theory should be. But in judging it, we have to imagine what it was like before the modern notion of a theory, and how it was supposed to work, had even been imagined. And we also have to imagine trying to work while emprisoned in the straightjacket of old ideas - that everything had to be made from perfect circles, that the earth was central, and that explanations had to be of the kind described by Aristotle.