P340: Homework Assignment No. 4

DUE DATE: Monday, March 29th. 2004

Please note that late assignments will not be marked

(1) QM: WAVES and PARTICLES

(a) Explain what is the 'wave-function' of a quantum particle? In what way is it related to the physical properties of the particle, and how is one supposed to calculate these? If the wave-function is zero in some region, what does this imply about the behaviour of the particle in that region?

(b) Explain what is meant by the "wave-particle duality". You should discuss this question by referring to some realistic set-up in physics- it is probably easiest to do this by looking at the 2-slit interference experiment, and discussing it for something like photons or electrons. It will be helpful to show what happens in such experiments under different conditions, with the aid of diagrams. Discuss under what circumstances the system behaves like a particle, and when it behaves like a wave, by showing, eg., what would be seen on the screen in a 2-slit experiment, for both photons and electrons.

(2) ATOMS

(a) Describe the Hydrogen atom as it is now understood by quantum physics. You should begin at the scale of the atom itself (ie., a scale of roughly 1 Angstrom), and discuss how we now understand the behaviour of the single electron in this atom- describing the electronic states, their sizes, shapes, etc., and their energies, and what is responsible for these. You should also briefly note the role of the Hydrogen nucleus, and say what you know about this.

(b) We can estimate the typical velocity of an electron in a Hydrogen atom, using the uncertainty principle. Assume that in the Hydrogen atom the electron is confined in a region of size $\Delta r \sim 10^{-10}$ m around the nucleus. From this estimate the uncertainty in momentum of the electron. Assuming that the typical momentum is the same as this uncertainty, now give the typical velocity of the electron. You can use the following rough numbers: an electron mass is approximately 10^{-30} kg, and Planck's constant is $h \sim 7 \times 10^{-34}$ J secs. Note that no calculator should be necessary to do this problem- I am not asking for exact answers, only approximate ones.

(c) The electronic energy levels of the Hydrogen atom are usually considered to have a negative energy $\epsilon_n = -R/n^2$, where $R \sim 13.6\ eV$, and 1 eV is an "electron Volt" (the energy acquired by an electron if it is accelerated in a field across a potential difference of 1 Volt). The energies are negative because these are "bound states", with energy less than free states- they have been trapped in the potential well around the Hydrogen nucleus. Photons can be emitted if an electron jumps from one of these states to another- the energy of the photon will be equal to the *difference* in energy between the 2 electron states.

To kick an electron out of its lowest energy state (the 'ground state') so that it escapes completely from a Hydrogen atom needs an energy of roughly 13.6 eV. This corresponds to a photon of wavelength roughly 1000 Angstroms (ie., about 10^{-7} m).

Suppose now that an electron drops from the $n = 3$ to the $n = 2$ state. Using the above result, say what will be the energy of the emitted photon, and also what will its wavelength. No calcul‘¡ator is needed here- I only want approximate answers.