March 29th, 2012

P340: Homework Assignment No. 4

DUE DATE: Thursday, 5th April 2012

Please note that late assignments will not be marked

(1) BOUND STATES: We can quickly estimate energies in quantum mechanics using the simple 'zero point energy' formula, according to which the zero point energy of a particle of mass m in a bound state confined to a potential well of size L is $E_o \sim \hbar^2/2mL^2$, where $\hbar \sim 10^{-34}\ Js$. For an electron with mass m_e (where in fact $m_e \sim 10^{-30}\ kg$), bound inside an atom of size roughly 1 $\mathring{A}$ (ie., $10^{-10}\ m$), this zero point energy is very roughly 10 eV (where eV signifies 'electron volts); and we shall think in terms of this convenient energy unit from now on.

1(a) Atomic and molecular bound states: Suppose we let the electron cloud extend out from a single atom into a molecule, so that it is now spread out over a distance of roughly 2 $\mathring{A}$. What then is its zero point energy? Show what the energy levels of the electron will be in both the original atom, and then in the new molecular configuration of the electron; and explain the significance of this result for chemical bonding.

(**1(b) Nuclear states:** In the nucleus we can use the same arguments to find the energies of nucleons bound inside the nucleus. However now the masses and the length-scales are different. Consider a deuterium nucleus, with one proton and one neutron, each having a mass $m_N \sim 2000 m_e$. They are confined by the strong force to be bound together - the size of the nucleus is about 1 Fermi, ie., $L_N \sim 10^{-15}\ m$. What now is the zero point energy of the 2 nucleons inside their mutual potential well, in eV? This is the characteristic energy involved in nuclear physics.

(2) TUNNELING: One simple phenomenon that explains many different physical phenomena is that of quantum tunneling, whereby a particle can 'tunnel' through some potential barrier.

(a) Nuclear fission: Explain how it is that a nucleus containing some number A of protons and also N neutrons (so that the total number of nucleons in the nucleus is $A + N$), can undergo fission. Explain what are the forces between the nucleons, and show a diagram of how this then results in a particular shape of the potential well for a proton in a nucleus, and how one can then get fission. Finally - what do you think the potential looks like for a neutron?

(b) Nuclear fusion: Now explain, starting from the same arguments that were developed above, how it is that under certain circumstances (and you should explain these circumstances) one can also get fusion of nuclei.

Using this result, explain briefly the life cycle of a typical star like the sun, starting from when it begins as a very large gas cloud.

(c) Beginning of the universe: Imagine you are a journalist working for the 'Vancouver Metro' newspaper (heaven forbid!). You are asked by your editor to write a short piece on how the universe began, and you decide to try and do a good job of this, even up to explaining that it began with a tunneling event. You have a maximum of 200 words - see what you can do (making it understandable, informative and interesting, but not misleading, and not leaving anything important out).

(3) FERMIONS and BOSONS:

(a) Symmetry under interchange: Explain how it is that 2 indistinguishable particles in quantum mechanics can be either fermions or bosons.

(b) Atomic shells: Electrons are fermions. Explain how it is that this then leads to the atomic shell structure of atoms - you might find it useful to draw pictures here.

(c) Bose Condensation: A set of atoms like He atoms are bosons; and so are photons. Explain how it is that a set of bosons can then form a 'Bose-Eintein Condensate' (a 'BEC'); and what we have physically when we Bose condense He atoms and photons.