March 10th, 2012

P340: Homework Assignment No. 3

DUE DATE: Monday, 26th March 2012

Please note that late assignments will not be marked

(1) NATURE of LIGHT: We have now seen how our understanding of light has evolved over the last 300 or more years. A key feature of this evolution has been the debate over the 'corpuscular' and wave pictures.

1(a) Newton vs. Huyghens: The debate between Newton and Huyghens (which was done at arm's length, and continued long after their deaths) can be seen in retrospect to have been one of the most important discussions in the history of science. To get to grips with it, we look at the following questions:

(i) Discuss Newton's arguments for his corpuscular theory - what arguments led him to the theory, and what arguments did he use to support it? Then explain the theory in detail, noting in particular what role the 'aether' played in the theory (for this you will have to explain a little how Newton saw the aether), and what determined the velocity of light in different media (including the vacuum). Explain how reflection and refraction at an interface between two media work in this theory; and explain how light was supposed to interact with matter in his theory, as well as with the aether. Finally, explain what you think are the strengths and weaknesses of this theory (but do not assume knowledge of things that were unknown at the time of Newton).

(ii) Now discuss the arguments that Huyghens used to get to his wave theory of light, and the arguments he used to support it. Explain in detail how light was supposed to propagate in this theory, the role played by the aether (and how Huyghens viewed the aether)and what determined the velocity of light in different media. Then explain in detail how Huyghens's theory explained reflection and refraction, and how light was supposed to interact with matter. Finally, explain what you think are the strengths and weaknesses of this theory (but do not assume knowledge of things that were unknown at the time of Huyghens).

(**1(b) Quantum Mechanics and Light:** The discovery of Quantum mechanics transformed our understanding of matter, including light.

(i) Explain in detail the 2-slit experiment with light, which is an adaptation of the 1803 experiment of Thomas Young. Discuss what results you would see in this experiment when the light intensity is very low, both for an arrangement in which both slits are open, and one in which only one of them is open.

(ii) Now explain why these results are so strange. Explain why it is that the results are incompatible with both a purely corpuscular picture of light, and with a purely wave picture.

(2) BASIC FEATURES of QUANTUM MECHANICS: The following questions are intended to help you review the basic physical features of quantum mechanics (we will discuss some of the more philosophical aspects, as well as the application of quantum mechanics to the real world, in the next and last assignment).

(a) Interference and Superposition: Consider again the 2-slit experiment. Describe how quantum mechanics explains the results of this experiment, in terms of a 'probability amplitude' which 'superposes' (ie., sums over) the 2 possible alternatives in this experiment (when both slits are open), and then gives the probability of seeing a photon hit the screen at a given point. Show how the results given in the last question (for the pattern seen on the screen) are explained when we have either one or 2 slits open.

Then consider the case when the photons will travel through a whole sequence of barriers, each containing some number of slits (1,2,3 or more). Describe how the results of this experiment are summing over different alternative 'amplitudes', and hence over all possible paths.

(b) Discrete Quantum States: Many quantum systems only have a finite discrete set of states available to them. The simplest is the spin-1/2 system, which is a 2-state system (another example is the photon, which can exist in 2 different 'polarization states').

Suppose we set up an experiment in which a beam of atoms with spin 1/2 is sent through a 'Stern-Gerlach' system which measures their spin along some axis. Describe what you would see

(i) When the spins are oriented along the x-axis, and the apparatus is measuring spin along the z-axis.

(ii) When the spins are oriented along the x-axis, and the apparatus has been rotated to measure spin along the x-axis.

How do you explain this result?

(c) Wave aspects of Probability Amplitudes: Any system in quantum mechanics propagates between points according to Schrodinger's 'wave equation'. In this wave equation, a shorter wavelength means higher velocity, according to the de Broglie relation $p = mv = \hbar/\lambda$, where m is the particle mass, v is its velocity, λ is the wavelength, and $\hbar = h/2\pi \sim 10^{-34}\ Js$ is Planck's constant.

(i) Consider first an electron, which has a mass $m_e \sim 10^{-30}\ kg$, and imagine it is moving at a velocity $v = 5\ m/s$. What is the wavelength of the probability amplitude?

(ii) Now consider a bacterium, of mass $m \sim 4 \times 10^{15} m_e$. If it is moving at $v = 5\ m/s$, what is its wavelength?

(iii) Finally consider a person of mass $m = 100\ kg$, moving at $v = 5\ m/s$. what is its wavelength?

(d) Entanglement: Suppose we have a *pair* of 2-state spin-1/2 systems, which are set up in such a way that (i) when they are produced, they have to have the same spin, but (ii) the direction of this spin is not fixed. Now let us suppose that they propagate so that after some time, they are very widely separated in space, at 2 different points A and B.

(i) Suppose we now measure, at point A (ie., on spin A) the spin along the z-axis. What will we find? We then measure along the x-axis. What do we find? Let us instead measure the spin of the other spin, at point B, in each of the 2 directions. What do we find? And how do we interpret these results in quantum mechanics?

(ii) Now suppose we measure the spin of spin A at point A, and find some specific result (say, that it is spin $\uparrow$, meaning its quantum state is $|\uparrow\rangle$). Then, immediately afterwards, we measure the spin of spin B at point B. What will we find? And how do we interpret this result in quantum mechanics?

(3) CURVED SPACETIME: We will revisit this topic in the context of modern astrophysics and cosmology, but now we confine ourselves to some basic features of Einstein's theory of General Relativity. To do this we look at a simplified spacetime which is only 2-dimensional.

(a) Curved Space: Suppose you lived in a 2-dimensional world - meaning that you had no way whatsoever of perceiving anything outside the 2-dimensional space in which you lived. Describe how you would determmine whether or not the geometry of this world was 'flat' or not, by measurements performed on (i) the area of a circle, and (ii) the angles in a triangle (eg., a right triangle, in which one of the angles was 90^o).

Explain what results you would see as you increased the size of these objects, in the case where the geometry was closed.

(b) Dynamics of Matter: Now we consider the mutual influence of mass and spacetime - the key to the whole theory.

(i) In Einstein's theory, the effect of a mass on the spacetime is similar to that of a charge on the electromagnetic field - it 'polarizes' it. We think of this as a *curvature* of the spacetime metric. Roughly what this means can be imagined by drawing lines on the spacetime when it is flat - and then seeing how these distort in the presence of a mass. Show how this works by drawing a 2-dimensional spacetime (a) when there are no masses around, and (b) when there is a single very large point mass in one very confined region of this same spacetime (assume however that this mass is not sufficiently concentrated to create a black hole).

(ii) The second key part of the theory is the effect of spacetime on mass. In the theory, a 'test mass' (which could be a light ray/photon) moves between two points in such a way as to minimize the spacetime distance between them (called in relativity the 'interval'). In flat spacetime this is simply a straight line. Show how the test mass will move in the presence of a large mass - as happens, eg., when light rays pass near the sun (or even more dramatically, near a large very concentrated mass, like a neutron star).

NB: Note that we have missed out here one key aspect of General Relativity, which is very different from a field theory like electromagnetism. This is that spacetime curvature has energy, and by the famous relation $E = mc^2$, this energy can be thought of as equivalent to mass. Thus the energy associated with spacetime curvature can generate further spacetime curvature - the 'spacetime field' can act on itself (which does not happen with an EM field). A runaway process of this kind can produce a black hole.