PHYS 340 ("From Atoms to the Universe"): FINAL EXAM

(Friday, April 20th, 2012)

3.30 - 6.00 pm

This exam will last 2 hrs and 30 mins. The only material allowed into the exam will be pens, pencils, and erasers. No notes of any kind are permitted, nor any calculators.

There are 2 sections. Students should answer THREE QUESTIONS ONLY from section A, and TWO QUESTIONS ONLY from section B. No extra marks will be given for extra questions answered. The questions in section A should take roughly 15-20 minutes to answer, and the questions in section B roughly 45-50 minutes to answer.

SECTION A

A1: A geostationary satellite is 40,000 km above the earth. Suppose you telephone somebody on the other side of the world, so that the microwave signal travels from you to the satellite, and then back down to the other person. Draw a picture of the configuration, and estimate *roughly*, to the nearest round number, the total length the microwave signal travels (NB: the radius of the earth is 6,000 km).

Suppose now that part of the signal from you is simply reflected back to you, because the circuit is faulty - so you hear your own voice as an echo, shortly after you have spoken. Given that the velocity of light is 300,000 km/sec, how long will it be after you speak that you hear the echo?

Finally - suppose that the wavelength of the microwaves is 10 cm. How many wavelengths will fit along the total path between you and the other person? And what is the frequency of these microwaves?

A2: Imagine you have a gas or liquid of particles passing through one or more channels or holes between 2 containers. By analogy with what happens when a crowd of people tries to run in all directions through a door or doors between 2 very large rooms, explain what is the source of the viscosity here (i.e, the resistance to flow).

Now give a description of the experiment which demonstrates the superfluid fountain effect. What happens here, and why is it different from what you just explained?

A3: Explain briefly what were the 4 'Causes' of Aristotle. Give an example of each. What role did these causes play in explaining how the Aristotelian universe was constructed?

A4: Imagine that a dog, running at 10 m/sec, is chasing a chicken running at 1 m/sec. However the dog stars 100 m away from the chicken coop, and chicken 10 m away.

How long will it take for the dog to reach the chicken coop, and how long for the chicken?

Explain how Zeno of Elea treated this is a paradox, and explain what you think was the fault in his reasoning.

A5: Describe the famous 2-slit experiment for light. Explain what you would see in this experiment, depending on whether one or two slits are open, and on whether the light intensity is weak or strong (this gives a total of 4 different possible cases). You will find diagrams very helpful here - make them clear.

A6: What is a 'supermassive black hole'? Where are these typically to be found, and how do we know they are there? If two supermassive black holes were orbiting around each other, what do you think would happen, and why? And how do you think we might know about it?

SECTION B

B1: FIELDS:

(i) Conventionally we distinguish between the EM (Electromagnetic) field, the magnetic field **B**, and the electric field **E**. Explain

- how each of **B** and **E** is detected;

-the effects that each has on electric charges and currents.

- two different ways in which an electric field can be generated, and two ways in which a magnetic field can be generated.

Please be precise: you will find it helpful to draw diagrams to illustrate some of your answers.

Now, describe the analogy between the EM field, the magnetic field **B**, and the electric field **E**, on the one hand, and an elastic medium and its deformations, on the other. What would be the distortions, in the elastic medium, that correspond to the fields caused by (i) an electric dipole (ie., 2 opposite electric charges placed near to each other); and (ii) a current ring? You should draw pictures here to illustrate your answers.

Suppose you had an electric charge moving through a vacuum at a constant velocity. Would there be any resistance to its motion? Justify your answer.

(ii) Consider now the quite different case of a spacetime field. In pre-relativistic physics, space and time were considered to be quite separate, and measured quite differently. Explain how Galileo measured both spatial and temporal intervals.

- In modern physics, spacetime can be curved. There is no way by examining a metre rule that one can tell if space is curved, since everything around it will be distorted in the same way as the space itself. But we can, for example, measure the distortion of a shape. Explain how for a 2-dimensional geometry this could be done, and use the example of the internal angles of a triangle.

- Suppose we had a 2-dimensional geometry that was cigar-shaped as seen in 3 dimensions. Now consider a succession of circles drawn with each of their centres at one tip of the cigar, but with larger and larger radii as measured from the centre. Draw this, and then draw a graph of how you think that the area of the circles will increase as we increase the radius of the circle (all the way up to the maximum possible radius).

(iii) Describe, in as much detail as you can, an astronomical observation which shows that space is curved near massive objects (you can pick the sun, or a cluster of galaxies). You should show in each case how the path of a light beam is affected by the curvature, and what is observed by us on earth.

B2: The UNIVERSE:

(i) We wish to compare and contrast the view of the universe given by Ptolemy and by Copernicus. Explain first, using diagrams, how the planets were supposed to move around, in each of their models. In the case of the Ptolemaic picture explain what the 'equant' was, and show this in the diagram.

To see how these two models were different, explain how in each of them, we are supposed to understand the apparent motion of Jupiter in the sky. First show how Jupiter is seen to move in the sky from the earth, and then show how this motion is explained in each theory.

(ii) The modern universe is much more complicated than this. We begin with the universe as seen up to the largest scale. Give a short description of what we see as we travel to ever greater distances from the earth: out from the solar system to the nearest star (4 light years away), to the centre of the Milky Way (25,000 light years away), to galaxies of the Virgo cluster (50 million light years away), and to the furthest distances we can see, 10 billion light years away. What is it that all of this made of? What else is there? And by what process is the universe supposed to have formed in the first place? Try to give as much precise detail as possible.

(iii) We wish to give the life history of a Hydrogen atom, which we can find in any organic molecule on earth (and in water). First, describe the Hydrogen atom as it is viewed in quantum mechanics - discussing the form of the electron states around the nucleus, and the basic facts about the particles making it up. Now - when and where would this atom have been formed? And how did it get to where it is now, on Earth? Give as much detail as you can.

(iv) Now we consider a Carbon atom which we can also find in any organic molecule on earth - and in ourselves. This atom has an atomic weight of 12, and has 6 electrons around the nucleus. First, describe the structure of this atom, beginning with the nucleus, and then going on to the electrons. Now - when and where would this molecule have been formed? And how did it get from there to where it is now, in you? Again, give as much detail as you can.

B3: MATTER and LIGHT

(i) In the Newtonian picture, all matter, and also light, was made from tiny corpuscles. The one thing they all had in common was mass. Explain how Newton, using his 2nd and 3rd laws, was able to give a definition of mass which also allowed it to be measured.

Then explain how Newton thought about the motion of light. What caused refraction and reflection, what happened to the light corpuscles during this process, and why was it that some light corpuscles were refracted and others reflected? What was it that kept light corpuscles moving, and how fast did they move; and what did they interact with?

Now explain the picture that Huyghens gave for light. How did his picture explain refraction and reflection, and what happened to the light during these processes? And how is that one obtained simultaneous refraction and reflection in this picture? And how did light interact with matter?

Finally, give one way in which you think one can distinguish between the predictions of Newton and those of Huyghens for the properties of light.

(ii) In the Quantum theory, all particles have to be either fermions or bosons. By considering what happens to probabilities when we swap 2 identical particles that can either both be in the same quantum state $|\alpha\rangle$, or be in 2 different states $|\alpha\rangle$ and $|\beta\rangle$, show how it is that we get the distinction between bosons and fermions. Now give 2 examples of fermions in Nature, apart from electrons.

Suppose now I take a Hydrogen atom, made from an electron and a proton, which are bound together by an attractive force, in a state $|\Psi(A,\alpha)\rangle$, meaning that the proton is in state A and the electron in state α. Now I swap these with another identical hydrogen atom, which is in exactly the same quantum state. Show that if I do this, I find that the hydrogen atoms behave like a pair of bosons.

(iii) In EM theory, light is supposed to be a wavelike oscillation of the EM field. Describe the effect of varying an electric field in time, and of varying a magnetic field in time. From this show how an EM wave arises.

In a vacuum, light travels at roughly 300,000 km/sec. What is the wavelength of FM radio waves (with frequency 100 MHz, ie., 10^8 Hz); and what is the frequency of an X-ray with wavelength 1 $\mathring{A}$, ie., 10^{-10} metres?

Suppose we tried to pass a microwave of wavelength roughly 5 cm through a set of metallic Venetian blinds, with parallel slats roughly 3 cm apart. What would we expect to see if we rotated the blinds from horizontal to vertical orientation? Explain why.

B4: QUANTUM MECHANICS

(i) In quantum mechanics, particles are often confined by attractive forces to 'potential wells'. Show a graph of the potential well which describes the energy of an electron at some distance r from a proton. How does this energy $E(r)$ depend on r?

Now show what happens if we have 2 protons at some distance a_o apart from each other - show how the potential for the electron looks like in this case.

Now consider a chain of 6 atoms, which are close enough so that tunneling allows the electron states to spread out over the whole chain. Given that the energy of a particle confined to a length L is roughly $E(L) = \hbar^2/2mL^2$, suppose that (a) the ground-state energy of an electron confined to a single atom, of size 1 $\mathring{A}$, is 5 eV (where eV means "electron Volt"), and that (b) the length of the 6-atom chain is 10 $\mathring{A}$. What is the new ground state energy of the electrons once they have spread over the whole chain?

Explain how results like this allow us to have chemical bonding.

(ii) The nucleus of a Nitrogen atom, with atomic weight 14, contains 7 protons. Draw a graph which shows how the energy $E(r)$ of another positively charged proton will vary as one varies r (assuming that the nucleus is at $r = 0$), and show where the energy levels of the proton will be in this graph. Explain why your graph of $E(r)$ has this shape. If the proton is absorbed by the nucleus we then have 8 protons - ie., an Oxygen nucleus; what is its atomic weight?

This new nucleus is actually unstable to fission. Explain, using the graph you have already drawn, what happens during a fission process.

(iii) A single electron has a spin. Part of the quantum description of spin involves having states like $|\uparrow\rangle$ and $\downarrow\rangle$ (pointing 'up' or 'down'). However we can also have superpositions like $|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$, pointing 'right',and $|\leftarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$, pointing 'left'.

First, find the expressions for the states $|\uparrow\rangle$ and $\downarrow\rangle$ in terms of $|\rightarrow\rangle$ and $|\leftarrow\rangle$.

Now, consider the entangled state $|\Psi_{EPR}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$. Using what you have been told above, show that this state can also be written as $\Psi_{EPR}\rangle = \frac{1}{\sqrt{2}}(|\leftarrow\leftarrow\rangle + |\rightarrow\rightarrow\rangle)$.

(iv) The remarkable thing about the EPR state above is that we cannot say which direction either of the two spins is pointing in - all we can say is that they are parallel.

Describe now what will happen if we set up the standard EPR apparatus with two widely-separated spin measuring systems M_I and M_{II}. Suppose first of all that both M_I and M_{II} are set up to measure the vertical component of spin ('up' or 'down'). What is then the probability that we will see 'up' in M_I? What is the probability we will see 'up' in M_{II}? And if we do see 'up' in M_I, what will we see in M_{II}?

Now suppose that we set M_I to measure 'up', but set M_{II} to measure the horizontal spin component, ie., 'left' or 'right'. If we now see 'up' in M_I, what is the probability we will see 'right' in M_{II}?

Now the EPR paradox imagines that we do this experiment, but that at the very last minute we switch the orientation of one of the measuring systems. Describe what happens, and why this is paradoxical.

END of EXAM