Nov 19th, 2012

P340: Homework Assignment No. 4

DUE DATE: Monday, 3rd December 2012

Please note that late assignments will not be marked

(1) QUANTUM SUPERPOSITIONS:

1.1: Superpositions can be most easily discussed in terms of the 2-slit experiment, so we begin with this.

1.1(a): describe the 2-slit experiment for a set of quantum 'particles' like photons or electrons. Discuss in particular what will be seen when (i) one slit is open, (ii) when 2 slits ore open, and (iii) when one lowers the rate at which particles go through the slits so that only one is going through at any one time.

1.1(b): what would be seen under the same circumstances if the electrons or photons behaved classically?

1.2: Superpositions can also be discussed between states in potential wells. Let's consider a simple potential well, modeling the electron energy in a single atom of He, which is 10 eV deep in energy, and has a spatial extent of 1 $\mathring{A}$. Assume that an electron has a 'ground state' in this well with an energy of 1 eV above the bottom of the well (ie., 9 eV below the top of the well), and also assume that we can treat this potential well as a *square well.*

1.2 (a): Draw a diagram of this potential well, and show the 3 lowest energy levels for the electron in this well, along with their energies. Suppose we formed a superposition of the ground state and the first excited state - what is the energy difference between these states, in eV units?

1.2(b) Now imagine that we are able to join 10 He atoms in a row, so that the electron can travel now along a potential well which is 10 times as large in spatial extent (ie., it is 10 $\mathring{A}$ long). Assuming that you can also treat this as a square well, find now the energies of the 3 lowest levels in this potential well. How much have they decreased from the energies they had for the single He atom? Explain why this energy decrease takes place, and why it is important for chemical bonding.

1.2(c) Explain the structure of the benzene molecule, and how it is that a superposition of 2 different "localized bond" states produces a ground state of lower energy. Show what the wave functions for these states look like.

1.3: Nuclear Fission is a classic example of a quantum tunneling process, which can be used to explain radioactive decay, as well as many processes occurring in astrophysics.

1.3(a) Explain how nuclear fission works, taking the example of a heavy nucleus which decays while emitting a gamma ray and an alpha particle.

1.3(b) Suppose the gamma ray leaves the nucleus with an energy of 1 MeV (ie., of 10^6 eV). If we assume that the wavelength of a photon with energy 1 eV is 12,000 $\mathring{A}$ (ie., $\lambda = 1.2 \times 10^{-6}\ m$), then find (i) the wavelength of the gamma ray, and (ii) its frequency, noting that the velocity of light is $c = 3 \times 10^8$ m/sec.

(2) ENTANGLEMENT:

2.1: One of the most extraordinary features of quantum mechanics is what Einstein called its 'non-locality', exemplified in the EPR 'thought experiment' (now a real experiment). Suppose 2 spin-1/2 particles are emitted with their spins constrained to be opposite. One can write the state of the 2 emitted spins as

$$|\psi\rangle \quad \sim \quad (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \tag{0.1}$$

This is an entangled state, with the 2 spins constrained to be opposite. One spin ("spin A") heads for Paris, the other ("spin B") for Tokyo; and experimental teams are waiting for them at each of these 2 places.

2.1 (a) The Tokyo measuring apparatus is set up to measure the $\uparrow$ or $\downarrow$ polarization of spin A. What is the probability that it will be $\uparrow$? Independently the Paris apparatus measures the other spin- what is the probability that it will find $\uparrow$?

2.1(b) Now suppose we do a measurement on the first Tokyo spin, and at a time $t = 0$ it is found that it has $\uparrow$ spin. What is the probability that the second Paris spin, measured a nanosecond after the first Tokyo measurement, will be $\uparrow$?

2.1(c) Interestingly, a 'right' spin state $|\rightarrow\rangle$ is a superposition of up and down spin states. We can actually write

$$|\rightarrow\rangle \quad \sim \quad (|\uparrow\rangle + |\downarrow\rangle) \tag{0.2}$$

where the arrows indicate the direction of the spin.

Suppose we now measure the spins with an apparatus which looks at the vertical spin polarisation, and it finds that spin 1 has spin polarisation "up", ie., it is in a state $|\uparrow\rangle$. What is the probability now that we will find spin 2 in a state $|\rightarrow\rangle$? And what is the probability it will be found in a state $|\leftarrow\rangle$?

2.2: Let us now consider the philosophical implications of these results. In your opinion, is the spin of either of the 2 spins discussed above 'physically real', in any direction of the spin polarisation? If none of the individual spin states are 'physically real', then what is physically real for these photon pairs? Justify your answer by explaining very carefully what you mean by 'physically real', and showing how your definition applies to this physical situation.

2.3: Now explain what is meant by a "quantum state". Explain this first for a single quantum system; you should compare and contrast this with the notion of a "classical state" for a physical system. Then explain what extra features enter when one is discussing the state of a pair of entangled systems.