March 28th, 2011

P340: Homework Assignment No. 4

DUE DATE: Friday, 8th April. 2011

Please note that late assignments will not be marked

(1) TUNNELING PHENOMENA: Quantum Tunneling is one of the most important processes involved in Nature - much of what we know in the natural world would be impossible without it. Here we look at some examples.

(1.1) Before looking at tunneling, note that this occurs out of, or between, potential wells. So let's first look at a simple potential well, that existing around the Hydrogen nucleus, for any electron that happens to be around.
(a) Draw a diagram of this potential well, and show the energy levels for the electron in this well.
(b) Suppose that the energy of the 'ground state' $|0\rangle$ of the electron in this well is 160,000 K below the lowest energy of 'free' or 'continuum' states. Then what are the energies of the first 2 excited states $|1\rangle$ and $|2\rangle$ above this ground state?
(c) Show in a picture how one can get transitions between the ground state and the states $|1\rangle$ or $|2\rangle$, and also just between $|1\rangle$ and $|2\rangle$. What are the energies of these 3 different possible transitions?

(1.2) One of the most important tunneling processes in Nature is that responsible for the chemical bond.
(a) Explain how this works by using the simple hydrogen molecule as an example. Show the potential in which the electrons move, and the electronic energy levels, both for widely separated atoms and for ones that are close together.
(b) Then consider a pair of Carbon atoms. Suppose the electrons in the outer shell are confined to a diameter of 2 $\AA$ in the atom, but are allowed to spread over a length of 4 $\AA$ when the 2 atoms bond. From the uncertainty principle, how much is the energy of one electron lowered by doing this?

(1.3) Nuclear fission is also a tunneling process. Explain how nuclear fission works for a heavy nucleus, using diagrams; you will need to explain the roles of the strong and electrostatic force here.

(1.4) Nuclear fusion powers the stars. Explain how this works in determining the life cycles of the stars, from the moment they are born to when they die, as follows:
(a) for a star like the sun.
(b) for a star like Deneb or Rigel.

(1.5) The currently most popular theory of the formation of the universe (the inflationary universe theory) has the universe itself formed by a quantum tunneling event. Explain how this theory works, and explain one astronomical observation that apparently supports it.

(2) ENTANGLEMENT and SUPERPOSITION: At the very heart of quantum mechanics is the existence of state superpositions, and the possibility of 'entangled' quantum states. We want to bring out the essential ideas in what follows:

(2.1) Superpositions can be most easily discussed in terms of the 2-slit experiment, so we begin with this.
(a) describe the 2-slit experiment for a set of quantum 'particles' like photons or electrons. Discuss in particular what will be seen when one slit is open, when 2 slits ore open, and when one lowers the rate at which particles go through the slits so that only one is going through at any one time.
(b) what would be seen under the same circumstances if the electrons or photons behaved classically?

(2.2) Imagine an experiment in which 2 spin-1/2 particles are emitted with their spins constrained to be opposite. One can write the state of the 2 emitted spins as

$$|\psi\rangle \sim (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \quad (0.1)$$

This is an entangled state, with the 2 spins constrained to be opposite. One spin ("spin A") heads for Newfoundland, the other ("spin B") for Tokyo; and experimental teams are waiting for them at each of these 2 places.
(a) The Tokyo measuring apparatus is set up to measure the $\uparrow$ or $\downarrow$ polarization of spin A. What is the probability that it will be $\uparrow$? Independently the Newfoundland apparatus measures the other spin- what is the probability that it will find $\uparrow$?

(b) Now suppose we do a measurement on the first Tokyo spin, and at a time $t = 0$ it is found that it has $\uparrow$ spin. What is the probability that the second Newfoundland spin, measured a nanosecond after the first Tokyo measurement, will be $\uparrow$?

(c) Interestingly, a 'right' spin state $| \rightarrow\rangle$ is a superposition of up and down spin states. We can actually write

$$| \rightarrow\rangle \quad \sim \quad (| \uparrow\rangle + | \downarrow\rangle) \tag{0.2}$$

where the arrows indicate the direction of the spin.

Suppose we now measure the spins with an apparatus which looks at the vertical spin polarisation, and it finds that spin 1 has spin polarisation "up", ie., it is in a state $| \uparrow\rangle$. What is the probability now that we will find spin 2 in a state $| \rightarrow\rangle$? And what is the probability it will be found in a state $| \leftarrow\rangle$?

(2.3) Now we look at some of the consequences of the existence of superposition and entanglement. The most striking of these concerns our idea of 'physical reality':

(a) In your opinion, is the spin of either of the 2 spins discussed above 'physically real', in any direction of the spin polarisation? If none of the individual spin states are 'physically real', then what is physically real for these photon pairs? Justify your answer by explaining very carefully what you mean by 'physically real', and showing how your definition applies to this physical situation.

(b) Now consider an experiment in which a measuring apparatus has 2 states $| \Uparrow\rangle$ and $| \Downarrow\rangle$, which interact with the states $| \uparrow\rangle$ and $| \downarrow\rangle$ of an electronic spin, such that if the electron spin is in state $| \uparrow\rangle$ before a measurement, then after the measurement the combined spin-apparatus system is in state $| \Uparrow\uparrow\rangle$). Suppose now that the spin comes in to be measured while in a 'quantum superposition' state $|\psi\rangle \ = \ (| \uparrow\rangle + | \downarrow\rangle)$. What then is the final state of the system plus apparatus?

(c) Now suppose that we set up an apparatus to measure the spin state in which the measuring system is connected to a Cat, whose final state depends uniquely on the final state of the apparatus. What then is the final state of the system (electron spin + apparatus + Cat), if the electron spin is initially in the superposition given in (vi)? Finally, on the basis of this answer, explain what then is Schrodinger's 'Cat paradox'. What do you consider to be 'physically real' in this final state?