Phys 340: FINAL EXAMINATION

Monday, 21st April, 2008; 12.00 -2.30 pm

This exam will last 2 hrs and 30 mins. You should answer 3 questions from section A (these should take roughly 20 mins each), and 2 questions from section B (these will take roughly 45 mins each).

No electronic equipment or devices of any kind are allowed into the exam (including calculators). You can use pens, pencils, and erasers, but no notes may be brought to the exam.

SECTION A

A(1): According to the uncertainty principle, an uncertainty in position of Δx implies an uncertainty in energy $\Delta E \sim \hbar^2/2m(\Delta x)^2$. Explain what this means for a particle of mass m.

Now suppose the energy uncertainty comes from thermal fluctuations (which cause a system to fluctuate between states of different energy). A hydrogen atom, whose mass $m_H \sim 2 \times 10^{-27}$ kg, will at room temperature have a quantum uncertainty in position, coming from its thermal energy, of 1 $\mathring{A}$, ie., $\Delta x \sim 10^{-10}$ m. Using this information, estimate the quantum position uncertainty of a bacterium at the same temperature (so that the energy uncertainty is the same), but whose mass is 10^{12} times greater than the Hydrogen atom.

A(2): Describe the famous 2-slit experiment for light. Explain what you would see in this experiment depending on whether one or two slits are open, and on whether the light intensity is weak or strong (this gives a total of 4 different possible cases). You will find diagrams very helpful here - make them clear.

A(3): Polarised light waves have their directions of magnetic and oscillations fixed. Show a diagram of how this works.

Light polarised vertically can pass through polarised glass whose polarisation plane is also vertical, but not if the plane is horizontal. Explain why this is.

Now imagine vertically polarised light passing through 2 sheets of polarised glass. What would you see if their polarisation planes were (a) both vertical, (b) the first was vertical and the second horizontal (c) the first was vertical and the second oriented at 45^o (halfway between horizontal and vertical).

Finally, suppose we pass vertically polarised light through a set of 3 glasses - the first has its polarization plane vertical, the third is horizontal, and now we insert a second plane between these, with its polarisation plane at 45^o. What will you then see? Explain your answer.

A(4): One of the most remarkable achievements of the ancient Greek astronomers was the measurement of the diameter of the earth by Eratosthenes, by comparing shadows of vertical sticks in 2 places. Explain how this was done, using pictures to illustrate your argument.

A(5): Suppose you wish to investigate the magnetic field around a circular wire which is carrying electric current.

Explain in detail how you would do this in practise, and show what pattern of magnetic field lines you would obtain if you did so.

A(6): The main reason that epicycles were originally invented to describe planetary motion was to explain the apparent retrograde motion of planets in the sky.

Taking the planet Mars as an example, show how the modern picture of planetary motion explains retrograde motion - you should use a picture to explain your argument.

SECTION B

B(1): Spacetime and the Universe

Before Einstein space and time were thought of as *receptacles* for material objects. After him we understood that the unified object known as 'spacetime' was itself a field, with its own dynamical properties.

(i) Explain what is meant by curved space, and how it is possible to determine whether a space is curved by making measurements of the geometry 'from within' (instead of going outside it). Explain in particular how one would measure the area enclosed by a circle, and the internal angles of a triangle, for the example of a closed 2-dimensional surface (like a balloon or a sphere); and say what results you would get for these quantities. Diagrams are always useful.

(ii) One prediction of Einstein's theory was that the universe would be expanding. Explain what evidence was used by Hubble to show that this was correct.

(iii) Another prediction of the theory was the gravitational bending of light. Describe how this was confirmed in measurements in 1919, and how it is now used to map out the distribution of mass in clusters of galaxies (and demonstrate the existence of dark matter).

(iv) Now give a description of the history of the universe from the time of the Big Bang, up until the crucial decoupling of radiation from matter some 280,000 years after the Big Bang. You should include a discussion of the initial Big Bang as now understood, and explain why radiation decoupled from matter, and how this then led to the microwave background.

B(2): Quantum Mechanics and Reality

(i) In the Stern-Gerlach experiment, a beam of atoms, all of which are polarized with spin $|\rightarrow\rangle$, is passed through a non-uniform magnetic field (between 2 magnets) in such a way that an outgoing beam of spin-up particles get separated from an outgoing beam of spin-down particles. However, one can then pass each beam through a very narrow slit, and let the particles be incident on a screen.

Draw a figure showing this set-up. Now, show where the particles will arrive on the screen if (a) only one slit is open, and (b) both slits are open.

Suppose now you measure the spin of a single one of these particles arriving on the screen. What will it be, if (a) only one slit is open, and (b) both slits are open?

(ii) Suppose you decide to measure the spin of the particle by allowing it to interact with another 'probe' particle. The probe spin goes into a spin-up state $|\Uparrow\rangle$ if the first spin is in state $|\uparrow\rangle$, and a state $|\Downarrow\rangle$ if the first spin is in state $|\downarrow\rangle$.

What will the state of the PAIR of particles be if (a) only one slit is open, and (b) if both slits are open, after they have interacted?

(iii) Suppose now we have 2 spins, each of which can exist in 2 states $|\uparrow\rangle$ and $|\downarrow\rangle$, and which are entangled in a state of form

$$|\Psi\rangle \quad = \quad (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Describe now the Einstein-Podolsky-Rosen (EPR) paradox, arising when we *measure* the state of one of these spins. To make the paradox clearer, you should note that we can also write the entangled state above as

$|\Psi\rangle \;\; = \;\; (|\rightarrow\rightarrow\rangle \;\; + \;\; |\leftarrow\leftarrow\rangle)$

where $|\rightarrow\rangle \;=\; (|\uparrow\rangle + |\downarrow\rangle)$, and $|\leftarrow\rangle \;=\; (|\uparrow\rangle - |\downarrow\rangle)$. To explain fully the EPR paradox, you should consider what happens when (a) you measure the spin of one of the spins along the vertical direction, or (b) along the horizontal direction.

Now, can we say what is the individual spin state of one of these spins in this entangled state? Is it objectively real? If not, what is objectively real in this system?

(iv) Now, describe what happens if we replace one of the 2 spins by a cat, assuming we still have an entangled state, such that $|\uparrow\rangle$ represents 'alive cat' and $|\downarrow\rangle$ represents 'dead cat'. What now would you argue about the physical reality of the state of the cat? You should refer back to your discussion of physical reality in the first part of this section.

(v) Finally, let us look at what this means for our understanding of physical reality. First, explain briefly what was Plato's argument for the existence of 'Forms', and why it was that these were considered to be more 'real' than the world of appearances. Then, whether you agree with Plato or not, explain what you think he would have said about the reality of the various quantum states in (iv) above.

B(3): Classical Physics

'Classical Physics' generally refers to all parts of physics independent of quantum mechanics - including all ideas based on particles and fields.

(i) Consider first the theory of particle dynamics. State what are Newton's 2nd and 3rd Laws of motion, and then explain carefully how it is that one can define operationally the mass of any object using these 2 laws.

(ii) Suppose an invisible black hole, with a mass 4 times that of the sun, were to move extremely rapidly through our solar system - so fast that its main effect would be to suddenly perturb the motion of the sun and all the planets. We observe that the sun is very rapidly accelerated, and ends up moving at a velocity of 1 km/sec away from its former position, in a direction we will call 'left'.

What would have happened to the motion of the black hole, after it has left the solar system?

Recall that Newton's law of gravitation says that the force between 2 bodies is proportional to the product of their masses, and also proportional to the 'inverse square' of the distance r between them (ie., proportional to $1/r^2$. Suppose the black hole passed just as close to the earth as it did to the sun - what would have happened to the earth's motion afterwards? And suppose it passed twice as close to Jupiter as it did to the earth and to the sun - what would have been the effect on Jupiter?

(iii) Newton thought that light could be understood as a particle. If you observe a light beam traveling across any interface (eg, between glass and air) you will note that some of it is reflected and some refracted. Show how this happens at an interface, in a diagram, and explain why this is a problem for a particle theory of light.

(iv) We now know that light is an electromagnetic wave distortion of an electromagnetic (EM) field. Consider first static distortions of the EM field. Show how the magnetic field varies around a wire carrying an electric current; and show how the electric field varies near an electric charge. Now, compare this with a gravitational field - show how the gravitational field lines vary near a mass.

Now explain how we would determine these fields lines experimentally, for one of these 3 examples (electric, magnetic, gravitational).

Finally, show how the electric and magnetic fields vary in space when an EM wave is present. Given that the velocity of light is 3×10^8 m/sec, what will be the wavelength of the light if its frequency is 6×10^{14} Hz? And what would be the frequency of the wave if its wavelength were 10 cm?

B(4): From Atoms to the Universe

(i) An electron localised around an atom in a region about 1 Angstrom (ie., 10^{-10}m) across has a kinetic energy which can be estimated from the uncertainty principle as about 9 eV (we recall that an uncertainty in position of Δx implies a typical kinetic energy $E \sim \hbar^2/2m(\Delta x)^2$). Now suppose the electron is able to spread out to a neighbouring atom, so that it spreads out over a length of about 3 Angstroms.

What is the new kinetic energy of the electron in this spread out state? What does this result have to do with chemical bonding? If we ignore any other energies in this example (like the repulsion between electrons) how much energy does this imply there would be in the covalent bond between the 2 atoms here (measured in eV, or 'electron Volts').

(ii) Give a brief description of the atomic theory of Democritus, explaining how the world was built from atoms, how the atoms were differentiated, and how change occurred in the world.

Now explain the modern understanding of atoms, composed of fermionic protons, neutrons, and electrons. You should explain the shapes they take, why only a few of these are possible and what they look like, and how atoms are able to combine to form compounds; and why we have a periodic table of the elements. Note that the Pauli principle is essential for several reasons in this explanation.

Note also that in both of these discussions, pictures will be very useful.

(iii) At the centre of an atom we find the nucleus. Show in a diagram what is the potential energy of a proton near another one, as a function of the distance between them. Then show in another diagram what is the potential energy of a neutron near a proton, as a function of the distance between them.

How do now understand nuclear fusion in terms of these diagrams? And how does thermonuclear fusion works in the stars? Explain the process leading to the formation of a star, and what then initiates fusion at the beginning of a star's life.

Finally, explain what happens to a very massive star at the end of its life, and why this is so important for the formation of life.

(iv) At the present time physicists are trying to go beyond the theory of bosonic and fermionic particles to a more fundamental 'string theory'. Give a brief description of what strings are supposed to be. Why are they beyond the reach of experiments?