Phys 340: FINAL EXAMINATION

Tuesday, 18th April, 2006; 2.00 pm-4.30 pm

This exam will last 2 hrs and 30 mins. You should answer 3 questions from section A (these should take roughly 20 mins each), and 2 questions from section B (these will take roughly 45 mins each).

No electronic equipment or devices of any kind are allowed into the exam (including calculators). You can use pens, pencils, and erasers, but no notes may be brought to the exam.

SECTION A

A(1): Give a brief history of the universe, beginning from the Big Bang 15 billion years ago up to our times. You have to explain the most interesting and relevant things we now know, and briefly explain how our modern knowledge of them depends on theory and observation. All in 20 minutes! Marks will be given for an understanding of the topic, and also for an interesting essay.

A(2): Suppose we bring a single proton in towards an atomic nucleus- we will assume a nucleus of Neon, containing 10 protons and 8 neutrons. Now show in a graph what are the contributions to the energy of the incoming proton, as a function of the distance out from the centre of the nucleus.

Now, suppose we increase the size of the nucleus to very large values (eg., to a nucleus of Uranium, with 92 protons and 146 neutrons). Why does it start to become unstable to fission? And in what way does the fission occur?

A(3): Suppose we have 2 identical particles, in states "x" and "y" (for example, state "x" could mean that a particle is a position "x"). Now let the quantum 'wave-function' $\Psi(x, y)$ describe a state where the first of the two particles is in state x and the second is in state y. The state $\Psi(y, x)$ describes the situation where they are swapped, ie., the first particle is in state y, and the second in state x. The probability that we find a particle at state x and another at state y is $P(x, y) = |\Psi(x, y)|^2$.

Now, explain what indistinguishability means for the relation between (i) $P(x, y)$ and $P(y, x)$; and (ii) between $\Psi(x, y)$ and $\Psi(y, x)$. Then (iii) show from these results that there must be 2 kinds of particles, fermions and bosons, and show also that no 2 fermions can be simultaneously in, say, state x.

A(4): Give a brief discussion of the way in which Galileo's work upset the established Aristotelian view of the universe at he time. You should explain what the Aristotelian view was and how it had arisen, and then explain what observations and ideas of Galileo were so crucial to its overthrow.

Also explain what later work of Galileo's had for the subsequent rise of classical Newtonian physics.

A(5): An electron localised around an atom in a region about 1 Angstrom (ie., 10^{-10}m) across has a kinetic energy which can be estimated from the uncertainty principle as about 9 eV. Now suppose the electron is able to spread out to a neighbouring atom, so that it spreads out over a length of about 3 Angstroms. The uncertainty principle relates the typical momentum $\tilde{p}$ to the typical spread in position $\tilde{x}$ by $\tilde{p} = h/\tilde{x}$ (approximately).

(i) Now since the typical energy of the electron is $\tilde{p}^2/2m$, where m is the electron mass, estimate the new kinetic energy of the electron in this spread-out state.

(ii) What does this result have to do with chemical bonding? If we ignore any other energies in this example (like the repulsion between electrons) how much energy do you estimate there would be in the covalent bond between the 2 atoms here?

(iii) Suppose we let a proton delocalise over a length of 3 Angstroms. The proton has a mass 2000 times larger than an electron, so what would be its typical energy in this case?

A(6): Explain what is nuclear fusion. Why can it only happen between light nuclei? Show the relevant potential energy involved in the problem.

Now explain how thermonuclear fusion works in the stars. How is it that fusion continues for so long in the stars? Explain the process leading to the formation of a star, what then initiates fusion at the beginning of a star's life, and what eventually causes it to stop.

SECTION B

B(1): Fields

(i) Explain what is usually meant by a 'scalar field', and give an example of this. Then explain what is meant by a 'vector field', and give an example of this. Use diagrams if necessary.

(ii) Show how the magnetic field varies around a wire carrying an electric current; and show how the electric field varies near an electric charge. Show how the gravitational field lines vary near a mass.

Finally, draw how you think the electric field varies in space if I very rapidly move a charge a distance to the right, and then move it back again to where it was just as fast. Suppose it takes me 1 microsecond (10^{-6} seconds) to accomplish this entire manoeuvre. Assuming the velocity of light is $c = 3 \times 10^8$ metres/second, over what distance will this "EMP" distortion of the electric field stretch in space? And during this time, how far will an Olympic sprinter move (running at 10 metres/second)?

(iii) Suppose I have 2 electric charges. One has magnitude $q_1 = +2$ Coulombs and is attached to a mass of 3 kg, while the other has magnitude $q_2 = -8$ Coulombs and is attached to a mass of 1kg. If I put one close to the other, and what will be the ratio of their initial accelerations? Now suppose we introduce a small 'test charge' q_3; how would you use this to measure the ratio q_1/q_2 of the first 2 charges?

(iv) Francis Bacon talked in his philosophy about the concept of 'immaculate perception'. Amongst other things this involves the idea that one can imagine observing, at least in some cases, 'objective' physical phenomena (independent of the observer). Discuss first how one can approximately realise this idea when one measures either an electric or a gravitational field (specifying how you would measure the field). Now consider the field $\psi(\mathbf{r})$ which is the wave function for a quantum particle. Is this objectively real? If not, what is objectively real for this particle?

(v) Pythagoras emphasized that for him, it was not matter that was fundamental, but rather the mathematical forms in which it appeared. Explain briefly what he meant by this. Consider now the possible wave functions of a 'particle in a box' (in which one can only fit certain waves into the box). Assuming the width of the box is L, and recalling that the momentum of the particle is $p = h/\lambda$, where h is Planck's constant and λ is the wavelength of the wave-function, write down the wavelength and corresponding momentum of the lowest 3 states of the particle. Then, since the energy of the particle is given by $E = p^2/2m$, write down the energies of these 3 states (assuming the particle has mass m). In what way would these results have realised Pythagoras's ideas?

B(2): Light

Light of intensity I and frequency f illuminates a metal plate. Noting that (a) it takes a finite energy E_o to kick an electron out of a metal, and (b) that in quantum mechanics a photon of frequency f has an energy $E = hf$, where h is Planck's constant, explain

(i) what you would see in this experiment if light could be understood as classical waves - you should show here graphs of how the number of electrons kicked out of the metal would vary with (a) intensity of the light waves (ie., the number of photons per second), at some fixed frequency f, and (b) with the frequency f of the light, at some fixed intensity.

(ii) Now show what is actually seen in this experiment. You should show the same 2 graphs as before. Explain why this proves that light comes in photon packets with energy $E = hf$.

(iii) Now suppose we allow the light to pass through 2 slits. Explain, and show in a diagram, what you would see on a screen on the other side of these slits as you pass light of very low intensity through the slits. Show what you would see if only one slit is open, and if the 2 slits are open. Explain, in terms of the quantum-mechanical wave propagation, why you see this.

(iv) Now show what happens if you pass the light through a lens. Why is a light beam traveling from a distant object focussed to a point in the wave theory of light? How would you explain it in a particle theory of light?

(v) If you look carefully at the light traveling though a lens (or indeed across any interface) you will see that some of it is reflected and some refracted. Show how this happens at an interface, in a diagram, and explain why this is a problem for a particle theory of light.

(vi) 2 photons are emitted with opposite spin (also called 'helicity'). If the 2 helicity states are called $|+\rangle$ and $|-\rangle$, then what sort of quantum state do we expect to see the pair of photons in (you should write it down)? In this state you have written down, what is the probability that you will see one of the photons with + polarisation? Suppose you do a measurement and find that the other photons has + polarisation- what now is the probability that the first one will have + polarisation? Finally- the vertical polarised state $|V\rangle \sim (|+\rangle + |-\rangle$. Suppose you measure one photon to have vertical polarisation; what then is the probability the other will have + polarisation? You do not have to give detailed reasons for your answers here, but an incorrect answer will get no marks if it is unsupported by arguments.

B(3): Astronomical Objects

(i) Explain with a diagram the apparent path traced out in the sky by Jupiter (which is roughly 5 times farther from the sun than the earth is). Then, show how you think Venus must move in the sky as seen from earth (the distance of Venus from the sun is roughly 3/4 that of the earth). Finally, suppose we observe a binary star moving across the sky with a very powerful telescope - what path do you think will be traced out in the sky by the 2 stars as they orbit around each other?

(ii) Show how the motion of Jupiter would have been explained in the Ptolemaic epicycle system of motion.

(iii) State what are Newton's 2nd and 3rd Laws of motion, and then explain carefully how it is that one can define operationally the mass of any object using these 2 laws (giving an example if necessary)

(iii) Suppose I have 2 masses, one 10^4 times as large as the other, orbiting round their common centre of mass. From Newton's 3rd law, the force of the large mass on the small is equal and opposite to the force of the small mass on the large. Give an argument to show, using Newton's 2nd Law, that the orbit of the small mass must be 10^4 times as large as that of the large one. If the distance of the sun from the earth is 150,000,000 km, and the mass of the sun is 300,000 times that of the earth, how big is the orbit of the sun around the centre of mass of the earth-sun system? Given that the earth is moving at roughly 30 km/sec in its orbit, how fast is the sun orbiting in its smaller orbit?

(iv) Two ways that have been found to verify predictions of the General Theory of Relativity are (i) observations of the binary pulsar (2 neutron stars orbiting each other, and (ii) the bending of light by massive objects. Explain one of these two, saying what observations have been made and how they confirm the theory. If ordinary classical mechanics was obeyed (ie., not the General Theory of Relativity) what would be seen instead?

B(4): Physical Reality

(i) Plato considered that there were 5 'perfect solid' shapes. Draw any two of these. What made them perfect, according to his definition? Did such shapes exist in the real world, in Plato's theory? What was his reason for this conclusion? You should start here from the theory of Forms, outlining what this is, and explaining carefully what are the relevant aspects of this theory for the question at hand.

(ii) Consider now a molecule having atoms at each of the vertices of one of Plato's solids (or, alternatively, a crystal having the same shape as one of his solids, because the atoms assemble microscopically in the right arrangement). Explain the quantum-mechanical reason why it is that such shapes exist in Nature. What precisely is it, in quantum mechanics, that is adopting these shapes?

Do you consider that the existence of real molecular or crystalline shapes that are *exactly* the same as those discussed by Plato has any bearing on the correctness of his theory? Base your arguments carefully on Plato's formulation of his ideas, and contrast these with the quantum-mechanical picture.

(iii) Explain carefully the relationship in EM theory between the underlying EM Field, the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$, and the electric charges. To illustrate this you could choose the example of a moving or stationary charge.

Define carefully a criterion you would use to say whether a physical entity is objectively real. According to your definition, which of the above objects (the EM field, the 2 fields, and the charges) are objectively real? Which of them can be measured?

(iv) Suppose now we have 2 quantum objects, each of which can exist in 2 states $|\uparrow\rangle$ and $|\downarrow\rangle$, and which are entangled in a state of form

$|\Psi\rangle \;=\; (|\uparrow\uparrow\rangle \;+\; |\downarrow\downarrow\rangle)$

These 2 states could be atomic spins, so that "$\uparrow$" represents an 'up' spin. Describe now the EPR paradox, arising when we *measure* the state of one of these spins. To make the paradox clearer, you should note that we can also write the entangled state above as

$|\Psi\rangle \;=\; (|++\rangle \;+\; |--\rangle)$

where $|+\rangle \sim (|\uparrow\rangle + |\downarrow\rangle)$, and $|-\rangle \sim (|\uparrow\rangle - |\downarrow\rangle)$; these states represent spins pointing to the right or left along the horizontal direction. To explain fully the EPR paradox, you should consider what happens when (a) you measure the spin of one of the spins along the vertical direction, or (b) along the horizontal direction.

Now, can we say what is the individual spin state of one of these spins in this entangled state? Is it objectively real? If not, what can we say about the system that is objectively real (use exactly the same definition here as you did in part (iii) of this question).

(v) Finally, describe what happens if we replace one of the 2 spins by a coin, assuming we still have an entangled state, such that $|\uparrow\rangle$ represents 'heads' and $|\downarrow\rangle$ represents 'tails'. What now would you argue about the physical reality of the coin state?