Phys 340: HOMEWORK ASSIGNMENT No (5)

Monday, March 24th

This assignment is to be handed in during or before the class on Friday April 4th. Please note that late assignments will not receive a mark.

(1) **Wave-packets**

The electrons moving on the surface of a metal (which can be seen using a scanning tunneling microscope) might have a typical wavelength of about 10 Angstroms (ie., 1 nanometre, or 10^{-9} m).

(i) How fast are these electrons moving?

Suppose now we confine the motion of these electrons, by having a beam of them incident on a "gate" in a solid barrier, whose width is 50 Angstroms. Assume that they arrive "head-on" at the gate (ie., the barrier is perpendicular to the direction of propagation of the electrons). This is a microscopic version of a pinhole camera. When the electrons emerge on the other side of the gate, there will be some diffraction on each side, by an angle of roughly α. Find out:

(ii) what is the uncertainty in momentum of the electrons in the direction of motion perpendicular to the beam (use the uncertainty principle); and

(iii) what is the angle α?

(2) **Energy levels**

The electronic energy levels of the Hydrogen atom are usually considered to have a negative energy $\epsilon_n = -R/n^2$, where $R \sim 13.6\ eV$, and 1 eV is an "electron Volt" (the energy acquired by an electron if it is accelerated in a field across a potential difference of 1 Volt). The energies are negative because these are "bound states", with energy less than free states- they have been trapped in the potential well around the Hydrogen nucleus.

Photons can be emitted if an electron jumps from one of these states to another- the energy of the photon will be equal to the *difference* in energy between the 2 electron states.

(i) Give the energy of 2 such possible photon states, in electron Volts.

(ii) Now take the 2 photons you have chosen, and find out their wavelength- this can be done by using the fact that a photon with energy E has a momentum given by $E = cp$, where $c = 3 \times 10^8\ m$ is the velocity of light; and the photon wavelength λ is given by the usual relation with the photon momentum p. NOTE: to get the right answer one has to convert the energies for the photons from electron Volt units to Joules. To do this you can use the fact that 1 eV = 1.6×10^{-19} Joules, ie., multiply your energies in eV by 1.6×10^{-19}, to get them in Joules.

(3) **Entanglement**

Imagine an experiment in which 2 photons are emitted in such a way that their helicities (these are like spins) have to be opposite (eg., decay of positronium). Classically this actually corresponds to "circularly polarized light", in which the transverse electric and magnetic fields in the EM wave are rotating clockwise or counterclockwise. Quantum-mechanically you can think of this rotation as being the "spin" of the photon. If we use a notation for 2-spin states in which, eg., $|+-\rangle$ means that the first spin is in a "+" helicity state, and the second spin in a "$-$" state, one can actually write the spin state of the photons coming out as

$$\psi \sim (|+-\rangle + |-+\rangle) \tag{1}$$

which is an entangled state with the spins constrained to be opposite.

(i) Suppose you set up an apparatus to measure the helicity of one of the photons. What is the probability that it will be +? Suppose instead you look at the other photon- what is the probability that it will be +?

(ii) Suppose now you set up a pair of apparatus, one to look at the first photon, and the other the second. They can be very far apart. Now suppose you find that the first photon has + helicity. What is the probability that the second one will be +?

(iii) Interestingly, a state $|+\rangle$ of a photon is a superposition of states in which the photon is "linearly polarised"- ie., the transverse electric field is no longer rotating but is fixed to be in one plane. We can actually write

$$|+\rangle \sim (|\uparrow\rangle + |\rightarrow\rangle) \qquad (2)$$

where the arrows indicate the direction of the polarisation of the electric field.

Suppose we now measure the photons with an apparatus which looks at the vertical polarisation- it finds that photon 1 has polarisation "up", ie., it is in a state $|\uparrow\rangle$. What is the probability now that we will find the 2nd photon in a state $|+\rangle$?

(4) **Miscellaneous**

(i) Explain briefly how it is that the atoms of which we ourselves are made are created. Nuclear reactions of a certain kind are involved- explain what is the quantum-mechanical process by which these work.

(ii) Atoms are surrounded by electrons, and so it is very hard to see classically how they could do anything but repel each other. Yet they actually do the opposite- they like to make a huge variety of molecules. What, roughly, is the quantum-mechanical reason for this?

(iii) Explain what is the superfluid fountain effect.

NB: I am not asking for long essays here- just a short explanation, with the main points, will do.