March 3rd, 2015

P340: Homework Assignment No. 3

DUE DATE: Wed, 18th March 2015

Please note that late assignments will not be marked

(1) The ELECTROMAGNETIC FIELD: Here we look at what EM fields are, a little bit at how they behave, and the electric charges and currents that they couple to.

(i) E and B fields, and EM induction: Explain how one can detect in an experiment (a) a static electric field $\mathbf{E}(\mathbf{r}, t)$ and (b) a static magnetic field $\mathbf{B}(\mathbf{r}, t)$, using "test probes". Describe what these probes are, and what is seen (diagrams are helpful). Then carefully describe an experiment which shows how electromagnetic induction works - how a time-varying **E**-field causes a **B**-field, and how a time-varying **B**-field causes an **E**-field. You should show where all the fields are in this experiment, and explain fully what is going on in each part of the experiment.

(ii) EM Field: Explain *in detail* the relationship between the electric charge, the underlying EM field $\mathbf{A}(\mathbf{r}, t)$, and the polarizations $\mathbf{B}(\mathbf{r}, t)$ and $\mathbf{E}(\mathbf{r}, t)$ caused by the electric charge (here $\mathbf{r}$ is the position in space, and t is the time). Explain how the charges affect the fields and also how the fields then influence the charges. You should find it helpful to draw diagrams showing how this works.

(iii) Ontological status of EM Fields: Given what you now know about EM fields, what sort of "physical reality" would you assign to the **E**- and **B**-fields, and to the **A**-field? Given that in classical electromagnetism we can never detect or experience in any way the **A** field, does this mean that it has a different ontological status from the other 2 fields? In what way can it be said to exist, and why should we be convinced that it does exist at all? Give your own analysis of this question; and then explain what you think what Plato might have said. To help in your own analysis of this question, I suggest that you consider some other example of an intangible entity that is claimed to exist, but which we are unable in principle to ever experience (pick your own examples - mine would include phlogiston, the 'silent majority', and a whole host of psychological conditions that have been described over the years by psychologists).

(iv) EM waves: Explain why EM waves exist, and show how the **E**- and **B**-fields vary in space in an EM wave, and why they take this form.

Then, explain how it is that the discovery of EM waves solved the problem that Huyghens had with his wave theory of light. What was the problem, and how did EM theory resolve it?

(2) GENERAL RELATIVITY: Here we look at Einstein's theory of General Relativity, and the idea of a spacetime field. To do this we begin with a simplified model, and then go on to look at the real world.

(i) Curved Space: Suppose you lived in a 2-dimensional world - meaning that you had no way whatsoever of perceiving anything outside the 2-dimensional space in which you lived. Describe how you would decide whether or not the geometry of this world was 'flat' or not, by measurements performed on (a) the paths traversed by 'straight line' beams of light, and (b) the angles in a triangle (eg., a right triangle, in which one of the angles was 90^o).

Explain what results you would see as you increased the size of the triangle, in the case where the geometry was closed to form a 2-dimensional 'sphere'.

Finally show what happens to a circle in this geometry, as you make the circle larger and larger. What would you see from inside the 2-d geometry?

(ii) Paths in curved space: Suppose we have a 2-d surface embedded in a 3-d world, in which it looks like a doughnut. Now suppose someone living in the this "surface world" starts drawing circles, beginning small but getting bigger and bigger. What would we see, watching the doughnut surface from inside our 3d world? Suppose now that the 2-d being starts with a circle which is 'wrapped around' the doughnut ring (as seen from outside). What will happen if the being tries to make this circle bigger? Or smaller? From observations like these, how could the 2-d being deduce the 'shape' of its universe?

(iii) The spacetime Field: Explain, in a way similar to what you did for EM fields, the relationship between (a) the 'source charge' for gravity, which we call 'mass', (b) the spacetime field (usually called $g_{\mu\nu}(x)$ by physicists, where x labels a point in spacetime), and (c) the distortion of the spacetime field caused by a mass, which we think of as being a gravitational field. Explain how the mass affects the spacetime field and also how the field then influences

the mass. You should draw diagrams showing how this works.

(iv) Evidence for GR: Finally, explain how one can verify the predictions of GR. You should (a) describe an earth-based experiment, and then (b) an astronomical observation, each of which confirms some aspect of Einstein's theory.