March 5th, 2011

P340: Homework Assignment No. 3

DUE DATE: Friday, 18th March. 2011

Please note that late assignments will not be marked

(1) FIELDS: We wish to examine the analogies between electric, magnetic and gravitational fields, viewed in a simple Newton in a framework of forces and 'charges' (sources of field).

(1.1) First show, using a Faraday picture of field lines: (a) the electric field generated by a stationary positive point electric charge; (b) the gravitational field generated by a stationary point 'gravitational charge' (ie., a point mass); and (c) the magnetic field generated by a stationary single current loop.

(1.2) Now, to clarify the forces that are exerted by these fields, we do the following:
(a) imagine that you surround the positive electric charge with a few 'test' electric charges, some of which are positive and some negative - show the forces on these. Add to these a few masses with no electric charge, and show the forces on these.
(b) Now surround the current ring with some test electric charges, positive and negative - what are the forces on these? And what will be the forces on a few test masses, with no electric charge?
(c) Now show the forces that will be generated on a test few current loops which are distributed around the central current loop. And finally, show the force on a single wire carrying electric current.

In all these questions it is important to show the direction of the forces correctly.

(1.3) From the fact that a moving electric charge - which is equivalent to an electric current - generates a magnetic field, we know that changing an electric field in time will generate a magnetic field.
(a) now briefly describe an experiment that shows that if we change a magnetic field, then an electric field will be generated. Thus the two fields (electric and magnetic) are intertwined, into a single electromagnetic (EM) field.
(b) An oscillatory disturbance of the electric and magnetic fields will propagate as a wave-like disturbance through the vacuum. If the velocity of light is $c = 300,000$ km/sec, then what is the frequency of these oscillations for an EM wave with wavelength $\lambda = 0.5\ \mu m$? (where $1\ \mu m = 10^{-6} m$ is a micron)? This is actually the wavelength of green light.
(c) What would be the wavelength of an EM wave with frequency 100 MHz (a typical radio station frequency)?

(1.4) The theories of both Newton and of Huygens had severe problems/paradoxical features. Explain what these were for each of them, and then explain how the discovery of electric charge and the EM field allowed these paradoxes to be resolved.

(2) RELATIVITY and COSMOLOGY:

(2.1) The relationship between the local *deformation* of spacetime and the local energy density was found by Einstein by starting from the equivalance principle.
(a) Explain the reasoning of Einstein that lay behind the 'equivalence principle' in a local frame of reference; you should use his own 'thought experiment' involving the person in an elevator, to do this, and say what the equivalence principle is.
(b) Now discuss how if one looks at the way gravitational forces *vary* as one moves around the vicinity of some local frame of reference, one can then distinguish this from simple local acceleration, and then relate the local gravitational field strength to the spacetime curvature. You can explain this either with a simple model of 2-dimensional spacetime, or in our own 4-d spacetime. There is no need to try and explain this mathematically - pictures will do.
(c) Now let's look at the *global* spacetime structure. Consider some closed -2-dimensional geometry (like a sphere). Explain how one can show, if one lives inside the sphere, that it is still possible to show that the geometry is curved, by doing measurements inside it.

(2.2) Various phenomena predicted by General Relativity have been demonstrated by astronomical observations. Describe, for each of the following three cases, one observation that confirms the theory:
(a) the expansion of the universe
(b) black holes
(c) gravitational lensing

(2.3) The philosopher Kant argued that space and time were *a priori* intuitions. His views were, in a certain sense, contradicted by Einstein's work. Nevertheless, Einstein remained sympathetic to certain parts of Kant's ideas.

(a) Explain what Kant's views on space and time were, in the context of his more general theory of epistemology.

(b) Now explain in what ways these views were contradicted by Einstein's theory if Relativity.

(c) Finally, explain briefly what it was about Kant's ideas that Einstein liked, and why.