Monday, March 17th, 2008

Phys 340: ASSIGNMENT No. (4): QUANTUM MECHANICS and MODERN PHYSICS Due Date: FRIDAY, APRIL 11th, 2008

To be handed on Friday, April 11th. Late assignments will not receive a mark

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(1) Waves and Particles

(a) Explain what is the 'wave-function' of a quantum particle. What is the precise relation between the value of the wave function at some point in space, and the physical properties of the particle at that point, and how is one supposed to calculate these? If the wave-function is oscillating in space with a wavelength λ in some region, what does this imply about the behaviour of the particle in that region? If it is oscillating in time with a frequency ω, what does this imply?

(b) Explain what is meant by the "wave-particle duality". You should discuss this question by referring to some realistic set-up in physics- it is probably easiest to do this by looking at the 2-slit interference experiment, and discussing it for particles like photons or electrons. You should show what happens in such experiments under different conditions, with the aid of diagrams. Discuss under what circumstances the system behaves like a particle, and when it behaves like a wave, by showing, eg., what would be seen on the screen in a 2-slit experiment, for both photons and electrons.

Finally, explain why these results are impossible to understand if the particles are considered to be classical; and why it is also impossible to understand if they are considered to be waves.

(2) Entanglement, Superposition, and 'Physical Reality'

Imagine an experiment in which 2 spin-1/2 particles are emitted with their spins constrained to be opposite. One can write the state of the 2 emitted spins as

$$\psi \sim (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \tag{1}$$

This is an entangled state, with the 2 spins constrained to be opposite. One spin ("spin 1") heads for Newfoundland, the other ("spin 2") for Tokyo; and experimental teams are waiting for them at each of these 2 places.

(i) The Tokyo measuring apparatus is set up to measure the $\uparrow$ or $\downarrow$ helicity of spin 1. What is the probability that it will be $\uparrow$? Independently the Newfoundland apparatus measures the other spin- what is the probability that it will find $\uparrow$?

(ii) Now suppose we do a measurement on the first Tokyo spin, and at a time $t = 0$ it is found that it has $\uparrow$ spin. What is the probability that the second Newfoundland spin, measured a nanosecond after the first Tokyo measurement, will be $\uparrow$?

(iii) Interestingly, a 'right' spin state $|\rightarrow\rangle$ is a superposition of up and down spin states. We can actually write

$$|\rightarrow\rangle \sim (|\uparrow\rangle + |\downarrow\rangle) \tag{2}$$

where the arrows indicate the direction of the spin.

Suppose we now measure the spins with an apparatus which looks at the vertical spin polarisation, and it finds that spin 1 has spin polarisation "up", ie., it is in a state $|\uparrow\rangle$. What is the probability now that we will find spin 2 in a state $|\rightarrow\rangle$? And what is the probability it will be found in a state $|\leftarrow\rangle$?

(iv) In your opinion, is the spin of either of the 2 spins physically real, in any direction of the spin polarisation?

(v) If none of the individual spin states are 'physically real', then what is physically real for these photon pairs? Justify your answer by explaining very carefully what you mean by 'physically real', and showing how your definition applies to this physical situation.

(vi) Now consider an experiment in which a measuring apparatus has 2 states $|\Uparrow\rangle$ and $|\Downarrow\rangle$, which interact with the states $|\uparrow\rangle$ and $|\downarrow\rangle$ of an electronic spin, such that if the electron spin is in state $|\uparrow\rangle$ before a measurement, then after the measurement the combined spin-apparatus system is in state $|\Uparrow\uparrow\rangle$). Suppose now that the spin comes in to be measured while in a 'quantum superposition' state $\psi = (|\uparrow\rangle + |\downarrow\rangle)$. What then is the final state of the system plus apparatus?

(vii) Now suppose that we set up an apparatus to measure the spin state in which the measuring system in (vi) is connected to a Cat, whose final state depends uniquely on the final state of the apparatus. What then is the final state of the system (electron spin + apparatus + Cat), if the electron spin is initially in the superposition given in (vi)? Finally, on the basis of this answer, explain what then is Schrodinger's 'Cat paradox'.

(3) Quantum Mechanics on the Small Scale

(i) The energy E of a photon has an energy $E = cp$, where c is the velocity of light, and p is the photon momentum. Since $p = h/\lambda$, where λ is the photon wavelength, we can find the wavelength of a photon if we know its energy. In fact a photon of energy 1 eV has wavelength of roughly $1.24 \times 10^{-6} m$, or 12,400 Angstroms. Using this information, find:

(a) the wavelength of orange light (energy 2 eV) and very blue light (energy 3 eV).

(c) the wavelengths of an X-ray (energy 100 keV = 100,000 eV), of a fairly high-energy gamma ray (energy 10^{10} eV), of a microwave (energy 10^{-4} eV) and a typical radio wave as used in local radio stations (energy about 3×10^{-7} eV).

NOTE: You do not need to know Planck's constant to do the above questions.

(ii) A proton is attracted very strongly to the central region of a nucleus by the strong nuclear force, but once outside the nucleus, it is repelled away from it. Draw a diagram showing how the potential energy of the particle varies as one moves away from the centre of the attractive region.

Now we use the uncertainty principle, written in the form $p \sim h/x$ (and the relation that kinetic energy $E = p^2/2m$) to make estimates of nuclear energies. Suppose that the attractive nuclear region is roughly $10^{-15} m$ across. If the mass of the proton is roughly $m = 1.5 \times 10^{-27} kg$, use the uncertainty principle to find out what is the typical momentum of a particle confined to the nucleus must be, and what is the typical kinetic energy of the proton inside the nucleus.

Finally, suppose we have 1 ton of Hydrogen (ie., 10^3 kg). What would be the energy produced if 1% of this Hydrogen was converted into kinetic energy (eg., by nuclear fusion)? This is what happens in a hydrogen fusion bomb (and on a far bigger scale, in the sun and other stars).

Note: if you do these computations in units of metres, kg, and seconds, the answer for the momentum will be in kg.m/sec, and the answer for the energy will be in Joules.

(iii) Explain how it is that we end up with 2 different kinds of particle in Nature, bosons and fermions. You should do this by considering what happens to the quantum state of 2 particles when they are *exchanged*. ie., when 2 identical particles are swapped. Explain also why it is that matter (which is made up from fermions) does not collapse, even if the fermions attract each other (as in an atomic nucleus).

Now briefly compare and contrast the 4 fundamental interactions in Nature, each of which is mediated by a particular species of bosonic field.

(iv) At first glance it seems strange that different atoms should want to combine and share electrons to make 'chemical bonds'. Why is that making a chemical bond lowers the energy of a pair of atoms (it may help to choose the example of a pair of H atoms which pair to make a H_2 molecule)?

(4) Quantum Mechanics on the Large Scale

Write a short essay (600 words) in which you explain, to a non-scientific lay-person, where we are now in unraveling the fundamental building blocks of Nature. You can if you like stress string theory, or modern cosmology, or experiments in accelerators, and the theory behind these - all of these themes are part of the whole picture. In such a short essay you can only highlight one or two of these aspects, and I leave it to you to choose what you want to emphasize - but you should try to be as clear as possible. Pictures are of course a help.