Wednesday, March 15th, 2006

Phys 340: ASSIGNMENT No. (4): QUANTUM MECHANICS

Due Date: MONDAY, MARCH 27th, 2006

. To be handed on MONDAY, MARCH 27th. Late assignments will not receive a mark

(1) Waves and Particles

(a) Explain what is the 'wave-function' of a quantum particle? What is the precise relation between the wave function and the physical properties of the particle, and how is one supposed to calculate these? If the wave-function is zero in some region, what does this imply about the behaviour of the particle in that region?

(b) Explain what is meant by the "wave-particle duality". You should discuss this question by referring to some realistic set-up in physics- it is probably easiest to do this by looking at the 2-slit interference experiment, and discussing it for something like photons or electrons. It will be helpful to show what happens in such experiments under different conditions, with the aid of diagrams. Discuss under what circumstances the system behaves like a particle, and when it behaves like a wave, by showing, eg., what would be seen on the screen in a 2-slit experiment, for both photons and electrons.

(2) Energy levels

The electronic energy levels of the Hydrogen atom have a negative energy $\epsilon_n = -R/n^2$, where $R \sim 13.6$ eV, and 1 eV is an "electron Volt". The energies are negative because these are "bound states", with energy less than free states- they have been trapped in the potential well around the Hydrogen nucleus.

(a) Draw a figure showing these energy levels for a few different values of n. Give the energies of the levels having $n = 1, 2, 3$ using the formula above, in units of eV.

(ii) Photons can be emitted if an electron jumps from one of these states to another- the energy of the photon will be equal to the *difference* in energy between the 2 electron states. Give the energy of 2 such possible photon states, in electron Volts, and show in an energy level diagram which 2 photons you have chosen.

(iii) The energy E of a photon is proportional to its momentum p (actually for a photon, $E = cp$, where c is the velocity of light). Since the photon momentum is related to its wavelength λ by $p = h/\lambda$, we can find the wavelength of a photon if we know its energy. In fact a photon of energy 1 eV has wavelength of roughly $1.24 \times 10^{-6} m$, or 12,400 Angstroms. Using this information, find:

(a) the wavelengths of the 2 photons you have chosen in (ii) above

(b) the energies of blue light (wavelength about 4,500 Angstroms) and red light (wavelength about 7,000 Angstroms).

(c) the wavelengths of an X-ray (energy 100 keV = 100,000 eV), of a fairly high-energy gamma ray (energy 10^{10} eV, and the energies of a microwave (wavelength $1cm$ and a typical radio wave as used in local radio stations (wavelength about $3m$).

(iv) Suppose a particle is attracted very strongly to a particular region in space, but that if it it is outside the region, it is repelled away from it. Draw a diagram showing how the potential energy of the particle varies as one moves away from the centre of the attractive region. Now suppose that the attractive region is roughly $10^{-15} m$ across. If the mass of the particle is $M = 10^{-30} kg$, use the uncertainty principle to find out (a) what the typical momentum of a particle confined to this space in a 'quasi-bound state' would be, and (b) how deep the potential well of the attractive region must be if the particle is to have a 'quasi-bound' state inside it.

Why will this state only be 'quasi-bound' , even if the depth of the potential well exceeds this depth?

(3) **Entanglement**

Imagine an experiment in which 2 photons are emitted in such a way that their helicities have to be opposite (eg., decay of positronium). Quantum-mechanically you can think of this helicity as being the "spin" of the photon. If we use a notation for 2-photon states in which, eg., $|+-\rangle$ means that the first photon is in a "+" helicity state, and the second photon in a "$-$" state, one can actually write the state of the 2 emitted photons as

$$\psi \sim (|+-\rangle + |-+\rangle) \tag{1}$$

This is an entangled state, with the 2 helicities constrained to be opposite.

(i) Suppose you set up a measuring apparatus to measure the helicity of one of the photons. What is the probability that it will be +? Suppose instead you look at the other photon- what is the probability that it will be +?

(ii) Suppose now you set up a pair of measuring systems, one to look at the first photon, and the other at the second. They can be very far apart. Now suppose you do a measurement on the first photon, and find it has + helicity. What is the probability that the second one, measured elsewhere, will be +?

(iii) Interestingly, a state $|+\rangle$ of a photon is a superposition of states in which the photon is "linearly polarised"- ie., the transverse electric field is no longer rotating but is fixed to be in one plane. We can actually write

$$|+\rangle \sim (|\uparrow\rangle + |\rightarrow\rangle) \tag{2}$$

where the arrows indicate the direction of the polarisation of the electric field.

Suppose we now measure the photons with an apparatus which looks at the vertical polarisation- it finds that photon 1 has polarisation "up", ie., it is in a state $|\uparrow\rangle$. What is the probability now that we will find the 2nd photon in a state $|+\rangle$?

(4) **Entanglement and 'Physical Reality'**

Compare the answers you get in question for the helicities of the photons (and/or their linear polarisations). How are we supposed to understand what is physically 'real' here? In particular

(i) Is the helicity of either of the 2 photons physically real? Or is their linear polarisation? Note that both the circularly polarised states (having a definite helicity) and the linearly polarised states (with a definite direction of polarisation) are legitimate quantum states.

(ii) If neither of these are 'physically real', then what is real for these photon pairs?

The most important thing to do here is to justify your answer by explaining very carefully what you mean by 'physically real', and showing how your definition applies to this physical situation.

(5) **Measurements and 'Schrodinger's Cat'**

Consider an experiment in which a measuring apparatus has 2 states $|\Uparrow\rangle$ and $|\Downarrow\rangle$, which can correlate with the states $|\uparrow\rangle$ and $|\downarrow\rangle$ of an electronic spin (so that, eg., if the spin is in state $|\uparrow\rangle$

before a measurement, then after the measurement the combined spin-apparatus system is in state $|\Uparrow\uparrow\rangle$).

(i) Explain what is the measurement paradox, by assuming that the spin comes in to be measured while in a some 'quantum superposition' of spin up and spin down states.

(ii) Now suppose that we set up an apparatus to measure the spin state in which the measuring system is connected to a Cat, whose final state depends on the initial state of the spin. Describe, referring if necessary to your answer to (i), how you might set up this experiment. Then explain what is Schrodinger's 'Cat paradox'.

(iii) It is usually claimed that the solution to the Cat paradox is that 'decoherence' destroys the live/dead superpositions. Explain in simple terms what this decoherence argument is, and how it is supposed to solve the Cat paradox.