LECTURES – COLLEGE de FRANCE, MAY 2016

4th LECTURE

PCE STAMP (UBC)

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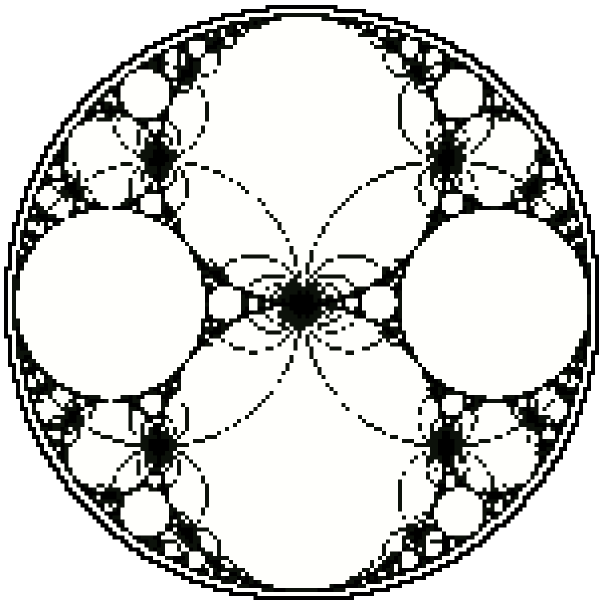
LECTURE 4

FROM OPTOMECHANICS

to

QUANTUM GRAVITY

and back



A: LARGE-SCALE Q PHENOMENA & DECOHERENCE MECHANISMS: the PROBLEMS

- Non-traditional decoherence mechanisms (3rd party, etc.)
- Definition of large-scale coherence

B. GRAVITY & QUANTUM MECHANICS – INTRODUCTORY REMARKS

- Experimental/Observational confirmations of classical GR
- Relativistic Astrophysics
- Quantum Gravity – key features & Problems
- Formal Remarks on Quantum gravity
- Low-Energy conflict between Quantum Mechanics & General Relativity

C. CORRELATED WORLDLINE THEORY

- Basic idea of CWL theory
- How a particle propagates in CWL theory
- Remarks on Formal structure

D. An OPTOMECHANICAL EXPERIMENT

- Optomechanical tests of Large-scale quantum phenomena
- CWL theory for an N-particle system
- CWL theory for a driven oscillator

APPENDIX: TECHNICAL DETAILS of CWL THEORY

LARGE-SCALE QUANTUM PHENOMENA

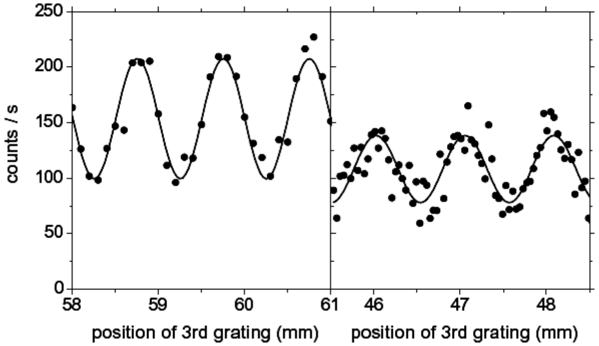
and

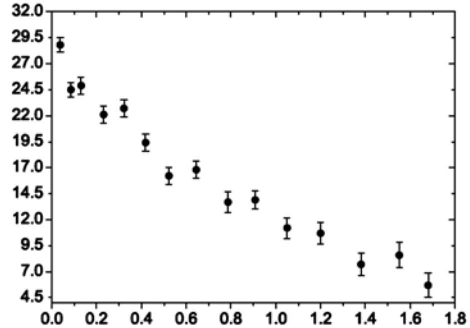
DECOHERENCE MECHANISMS

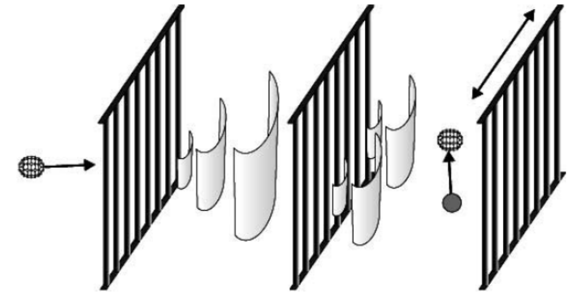
The PROBLEMS

2-SLIT EXPERIMENTS with BIG MOLECULES

The "Talbot-Lau" modified 2-slit experiment tests the superposition principle – it involves masses up to 10^4.



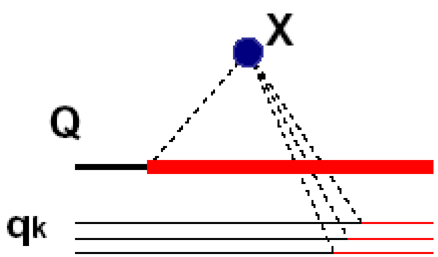




In the experiment, decoherence is assumed to come from photons & gas molecules

B Brezger et al., PRL 88, 100404 (2002)
K Hornberger et al, PRL 90, 160401 (2003)

3rd PARTY DECOHERENCE: This is decoherence in the dynamics of a system A, with coordinate Q caused by *indirect* entanglement with an environment E - the entanglement is achieved via a 3rd party B (coordinate X).



The 2-slit experiment with heavy molecule is an interesting example of this. The COM coordinate Q of the molecule does not couple to the "environmental" vibrational or rotational modes of the molecule while it is moving in free space.

However – these modes do couple to the slit system, in a way which distinguishes which path the molecule moves when passing through the slits.

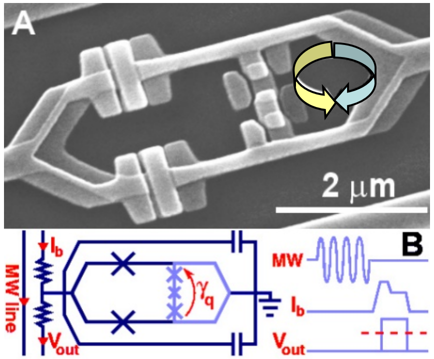
NB: The slit system is essentially a PASSIVE 3rd PARTY here – it is basically acting as an infinitely massive scattering potential, and its state does not change appreciably. One can also have ACTIVE 3rd PARTIES.

PCE Stamp, Stud. Hist Phil Mod Phys 37, 467 (2006)

This is one example of a 'hidden' decoherence mechanism

LARGE-SCALE COHERENCE – HOW LARGE?

Various claims have been made for the observation of "macroscopic" quantum phenomena – notably "macroscopic quantum coherence" and/or "Schrodinger's Cat" states – most dramatically in the case of superconducting SQUID systems.



Actually the 'size' of these superpositions is by no means clear. Calculations of a well-defined measure for the "macroscopicity" of these superpositions (taking account of the very small change in the states of individual modes between different branches of the superposition) actually give rather small values (this number essentially representing a "number of particles").

		L	ΔI_p	$\Delta\mu$	ΔN_{tot}
SUNY	Nb	560 μm	2–3 μA	$5.5 - 8.3 \times 10^9 \mu_B$	3800–5750
Delft	Al	20 μm	900 nA	$2.4 \times 10^6 \mu_B$	42
Berkeley	Al	183 μm	292 nA	$4.23 \times 10^7 \mu_B$	124

W Durr, C Simon, J Cirac, PRL 89, 210402 (2002)
J Korsbakken et al., Phys Rev A75, 042106 (2007)
" " , Europhys Lett 89, 30003 (2010)
M Arndt, K Hornberger, Nature 10, 271 (2014)

SUMMARY of PROBLEMS to do with ENVIRONMENTAL DECOHERENCE

(1) Many decoherence sources are not visible in dissipation/energy relaxation (eg., spin bath mechanisms). Use of "noise models" or 'quantum noise' analyses (including the use of fluctuation-dissipation theorems) is just wrong.

(2) Some of them are not understandable in terms of direct coupling to the bath at all (eg., 3rd party decoherence)

(3) The degree to which a system is displaying macroscopic quantum behaviour is often not obvious

ALL OF THESE POINTS COME INTO PLAY IN OUR NEXT TOPIC

GRAVITY

and

QUANTUM MECHANICS

INTRODUCTORY REMARKS

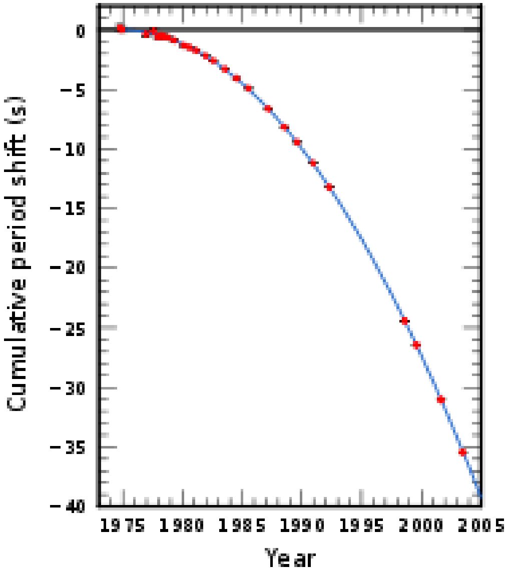
The STUNNING SUCCESS of CLASSICAL GENERAL RELATIVITY

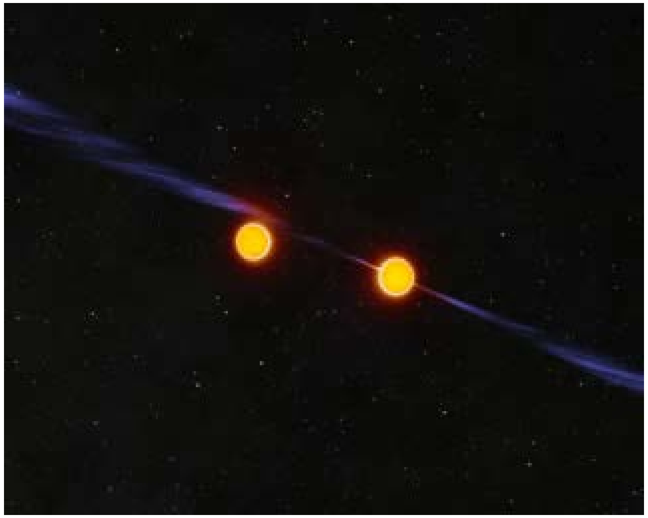
The revolutionary changes brought to physics by General Relativity – and its incredible success in explaining and predicting diverse phenomena - are not always appreciated by scientists in other fields. So we begin by recalling some of these.

EXPERIMENTAL TESTS of GENERAL RELATIVITY

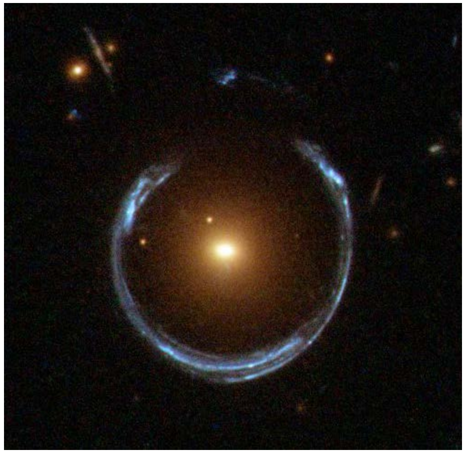
Although these tests have only checked GR in the v. restricted regime of weak fields, some of them are very accurate.

There are actually now many such tests – here I only look at two of them, which are quite striking.





The famous binary pulsar (video at right); timing of the orbital decay (left) by Hulse & Taylor confirmed GR predictions of grav. wave emission



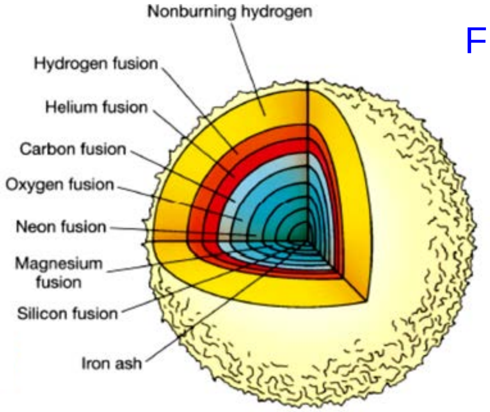
GRAVITATIONAL LENSES: At left, the "smiling galaxy cluster" SDSS-J1038+4849; at right, The "Horseshoe" Einstein ring

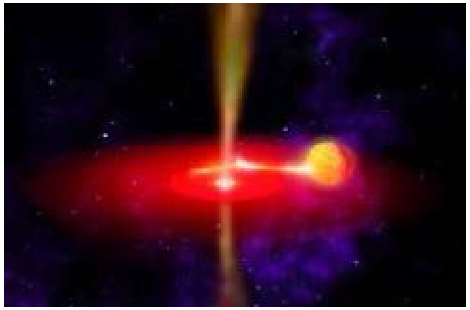
The 1st binary pulsar to be discovered has now very accurately confirmed the theory of gravitational wave generation.

Gravitational lensing, which started as a curiousity, has now become an extremely powerful tool.

Many of these are indirect tests of the Einstein equivalence principle.

From SUPERGANT STARS to SUPERMASSIVE BLACK HOLES



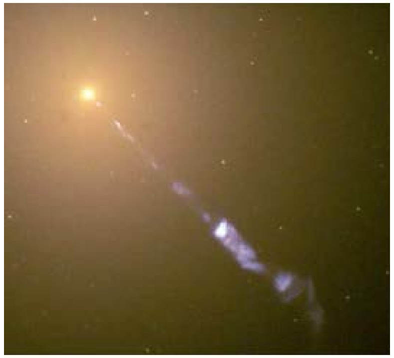


TOP: structure of a supergiant
MIDDLE: M1, the crab nebula
BOTTOM: artist's rendition of Cygnus X1, a stellar blackhole

Far stronger fields are involved in massive stars & of course in black holes.

So far there have been no direct observations of any black hole (ie., of any phenomena around the event horizon).

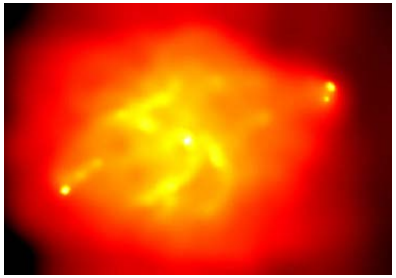
This is not really relevant to the question of whether they exist – this is hardly possible to doubt, given the wealth of different astrophysical processes which depend on their existence, & the agreement of theoretical predictions with observation.

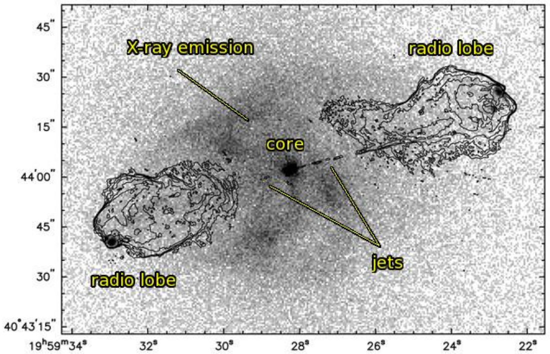


The ultra-relativistic jet emitted by the central black hole in M87 – its total length is roughly 100,000 light yrs. The black hole has mass 6.7 billion suns

M87 is a monster elliptical galaxy, of mass 3.5 trillion suns. It has been sterilized to 2 million K by the jet from the central black hole

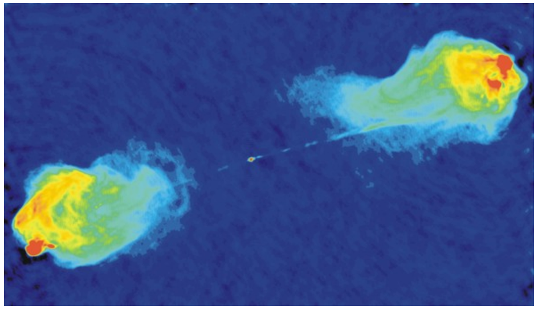
RIGHT: Cyg A optical



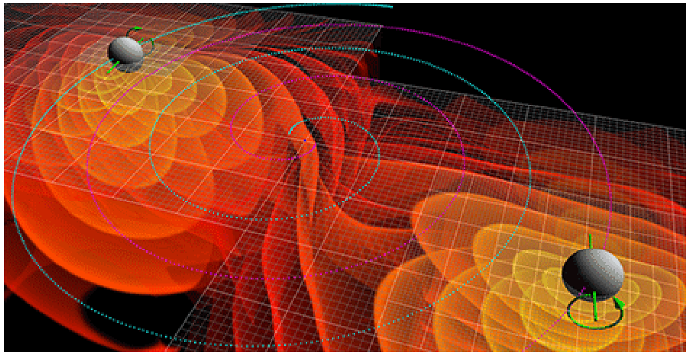


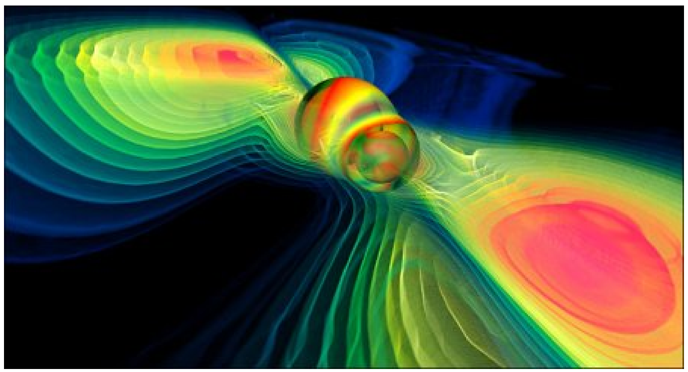
ABOVE & BELOW: radio
LEFT: X-ray

The radio source Cyg A, distant 750 million lyrs. The radio and X-ray lobes extend 2.2 million lyrs out from the central AGN (originally thought to be a pair of colliding galaxies)



VIOLENT EVENTS on COSMIC SCALES

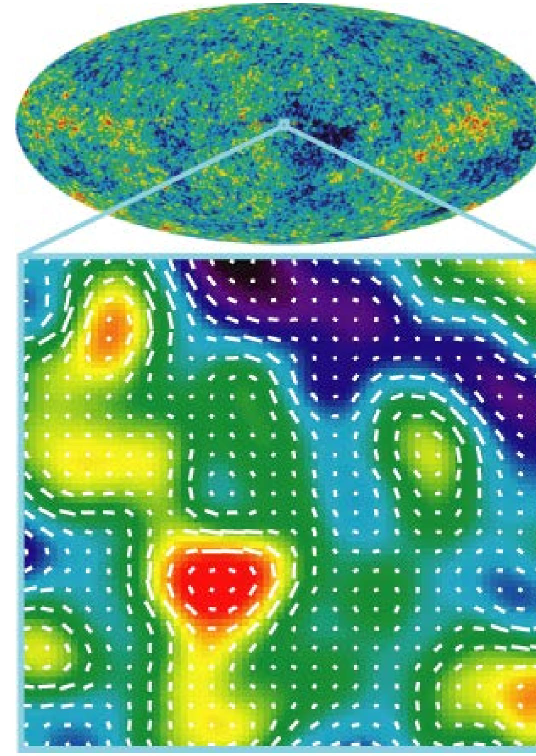




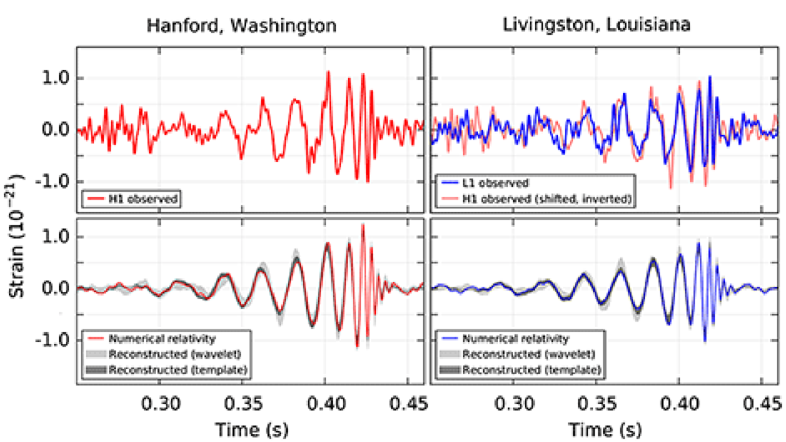
Distortions of spacetime occurring when 2 black holes collide and coalesce

The recent observation of gravitational waves from a pair of coalescing black holes serves to reinforce a basic point - that black holes have played a key role in the evolution of the universe for a long time (the significance of the result was actually in the demonstration of an entirely new observational tool).

A key part of the analysis involved comparison of the signal with calculations done prior to the observations.



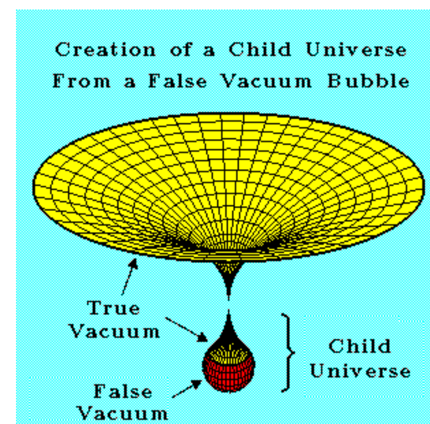
WMAP observation of the microwave background – the blow-up shows the polarisation



Signals from the 2 LIGO detectors, one in Louisiana & the other in Hanford WA.

Of course one of the earliest predictions of GR was the Big Bang – another strong field phenomenon.

The predictions of the 'inflationary universe' model involve QM, in the form of a massive tunneling event at the origin of the Big Bang.



the 'decay of the false vacuum' scenario (which led to the inflationary universe theory)

GRAVITY & INHOMOGENEITY on the LARGEST SCALES

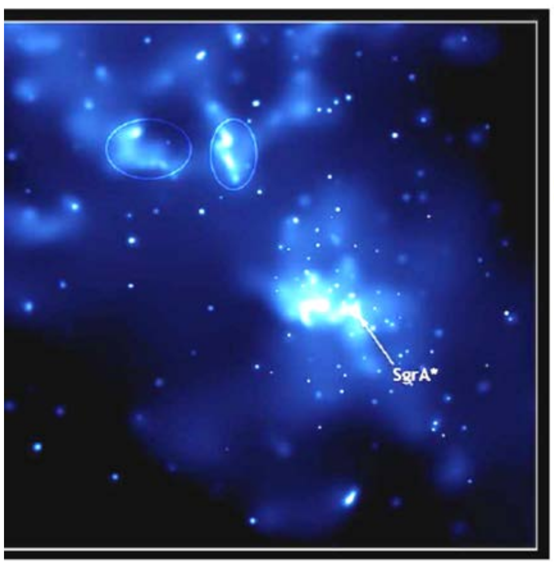
Energy in the universe is concentrated in stars and stellar heated dust – that contained in CMB photons, neutrinos, and black holes is down on these by 2 orders of magnitude. The number below do not include the contributions from dark matter.

But the entropy is almost entirely in supermassive black holes, outweighing that in CMB photons & relic neutrinos by 14 orders of magnitude! Stars contain a tiny fraction (~ 10^{-23}) of the total entropy in the universe.

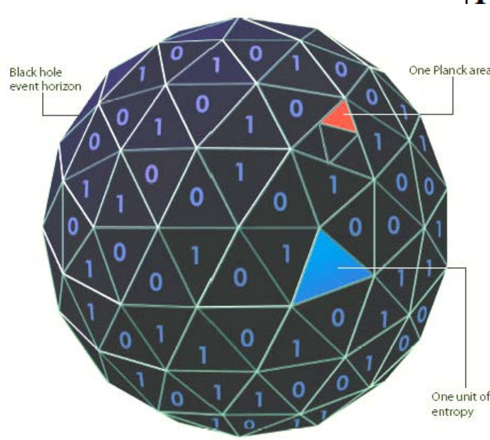
This result is quantum-mechanical – it comes from the Hawking relation between black hole area and its information content.

There are all sorts of questions here (not least of which: what are the mechanisms by which the entropy increases during stellar collapse to a black hole)? And – what happens to the info in the black hole? Our understanding of black holes has hardly begun.

Objects	Entropy	Energy
10^{22} stars	10^{79}	$\Omega_{stars} \sim 10^{-3}$
Relic neutrinos	10^{88}	$\Omega_{\nu} \sim 10^{-5}$
Stellar heated dust	10^{86}	$\Omega_{dust} \sim 10^{-3}$
CMB photons	10^{88}	$\Omega_{CMB} \sim 10^{-5}$
Relic gravitons	10^{86}	$\Omega_{grav} \sim 10^{-6}$
Stellar BHs	10^{97}	$\Omega_{SBH} \sim 10^{-5}$
Single supermassive BH	10^{91}	$10^{7} M_{\odot}$
$10^{11} \times 10^{7} M_{\odot}$ SMBH	10^{102}	$\Omega_{SMBH} \sim 10^{-5}$
Holographic upper bound	10^{123}	$\Omega = 1$



TOP: Radio map of Sgr A complex – distance 26,000 light-yrs



Q Info distributed over the Black hole horizon

MASSES of some BLACK HOLES

SDSS J102325.31+514251	33 x 10^{9}
H1841+643	30 x 10^{9}
....	.....
NGC 1600	17 x 10^{9}
.....	....
M87	6.3 x 10^{9}
Cygnus A	10^{9}
.....	....
M31	230 x 10^{6}
Milky Way	4.3 x 10^{6}

QUANTUM GRAVITY – CLUES & PROBLEMS

1) SPACETIME MUST BE QUANTIZED: Attempts to couple quantum fields to a curved classical spacetime lead to disaster – acausal propagation of information and violation of energy conservation, etc. More on this below.

2) QUANTUM GRAVITY is UV DIVERGENT: All attempts so far to create a UV-finite theory of quantum gravity have failed. The reason is that gravity is non-linear – high-energy quantum fluctuations of spacetime attract each other, and this gets worse ($\sim E^2$) as the energy gets higher. In classical GR this creates black holes; the quantum version is non-renormalizable.

3) LOW-ENERGY QUANTUM GRAVITY is WELL-DEFINED: In low-energy or ‘semiclassical’ quantum gravity, one has quantum fluctuations on a ‘background spacetime field’ which is treated according to classical GR, and varies slowly. In this well-defined framework, one calculates, eg., Hawking radiation, Unruh radiation, or Q fluctuations in the early universe. One sees clear departures from classical GR (deflection of light, time flow effects, Hawking radiation, etc.). The biggest single problem is that the quantum field vacuum varies in curved spacetime, so it is hard to define QFT properly.

4) SUPERPOSITIONS ARE NOT WELL-DEFINED: As will be discussed in more detail below, superpositions of matter states in Q Gravity are not well-defined – this is because they then create superpositions of background spacetime. The quantum fields living in communion with the spacetime are then not well-defined.

Perhaps most important of all – Gravity is extremely non-linear, & this means that Q Gravity must be too

FORMAL REMARKS on QUANTUM GRAVITY

Recall again that we are interested in a path integral formulation, which for a single particle is just the usual

$$K(x,x') = \int_{x'}^{x} \mathcal{D}q \, e^{\frac{i}{\hbar}S[q]}$$

We need to generalize this to gauge fields like the EM field or the spacetime field.

(1) QUANTUM ELECTRODYNAMICS: We first recall the simpler case of QED. The following is a summary of 50 yrs of work. In QED one starts with a Lagrangian:

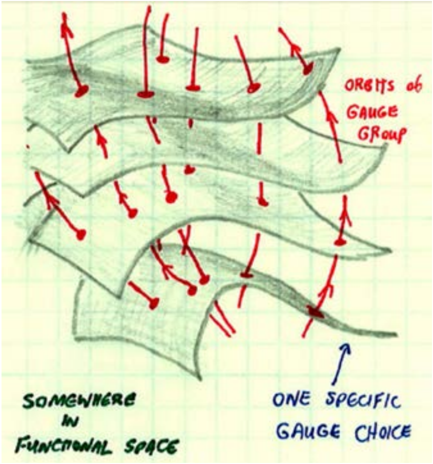
$$\mathcal{L} = \bar{\psi}_\alpha(i\gamma^\mu D_\mu - m)\psi_\alpha - \frac{1}{2\mu_0}F_{\mu\nu}F^{\mu\nu}$$

with covariant derivative $D_\mu = \partial_\mu + iqA_\mu$

Now suppose we write a functional integration of form $\int \mathcal{D}A^\mu \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \, e^{\frac{i}{\hbar}S[\bar{\psi},\psi,\mathbf{A}_\mu]}$ between different field configurations.

A key question then arises, viz., what do these functional integrals mean? There are 2 key points to note here. First we have to define a measure in the space of all possible configurations of each field. Thus, eg., the EM field has the 'internal metric' defined by

$$\langle \delta A, \delta A \rangle = \int d^4x \; \delta A_\mu(x) \, \eta^{\mu\nu} \, \delta A_\nu(x)$$



Integrating over different gauge-equivalent EM fields configurations in function space.

ie., a Gaussian measure appropriate to free photons.

2nd, we must not over-count configurations of the EM field – those related by gauge transformations are physically equivalent. We must therefore 'divide out' gauge-equivalent configurations in the integral over A_μ. It turns out that in QED, we can deal with this by adding an extra term to the Lagrangian, which then becomes

$$\mathcal{L} \rightarrow \mathcal{L} - \frac{1}{2\alpha\mu_o}(\partial_\mu A^\mu)^2$$

(Faddeev-Popov / 't Hooft)

(ii) PATH INTEGRAL for GRAVITY: The case of gravity has some similarities to QED, but also some key differences. Suppose we have some relativistic particle, travelling through a curved spacetime with some metric $g^{\mu\nu}(x)$; then the particle propagates according to

$$K(x,x') = \int_{x'}^{x} \mathcal{D}q \, e^{\frac{i}{\hbar}S_M[q,g^{\mu\nu}]} \rightarrow \int_{x'}^{x} \mathcal{D}q(s) e^{-\frac{i}{\hbar}\int ds[\frac{m}{2}g_{\mu\nu}(q)\dot{q}^\mu \dot{q}^\nu]}$$

and we can describe any quantum phenomenon in this metric using this result (eg., any interferometric phenomenon).

However, the metric is in reality a dynamic quantum field. So what we should really be looking at is

$$\mathcal{K}(x,x') = \int \mathcal{D}g^{\mu\nu} \Delta(g) \, e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \int_{x'}^{x} \mathcal{D}q \, e^{\frac{i}{\hbar} S_M[q,g^{\mu\nu}]}$$

where we also integrate over the metric. We might think we could do this in analogy with QED, but several key problems arise:

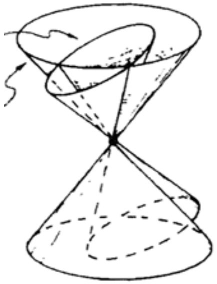
- It is much more problematic to define a measure for the integration over metrics – the natural inner product depends on the metric itself, and we get:

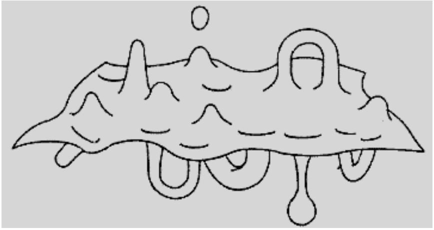
$$\langle \delta g, \delta g \rangle = \frac{1}{2}\int d^4x \sqrt{g(x)} \; \delta g_{\mu\nu}(x) \; [g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} + Cg^{\mu\nu}g^{\alpha\beta}] \; \delta g_{\alpha\beta}(x)$$

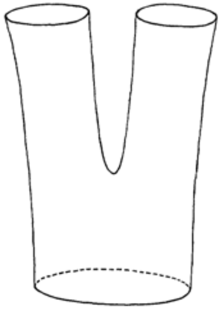
  Thus the result is not even diffeomorphism invariant, and is strongly non-linear.

- We find ourselves integrating over very strange configurations: by changing the metric we change the causal structure of spacetime. This changes the light-cone structure, so that when we sum over metrics, we are summing over processes in which time-ordering is switched. Attempts to fix this by, eg., rotating to a Euclidean base metric can make the problems even worse.

  For this reason many people feel that these path integrals make no sense.



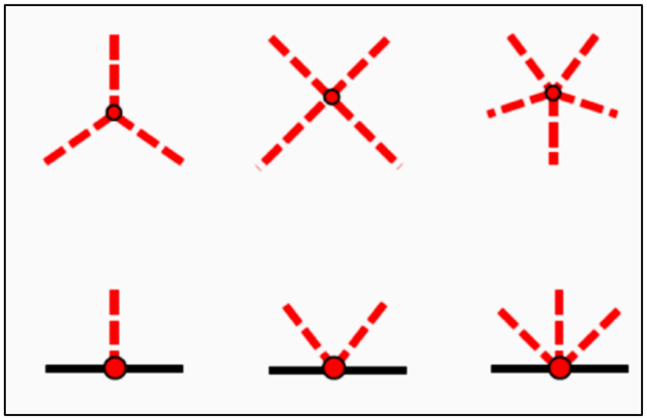




LOW-ENERGY GRAVITY

When we do experiments in the lab we are not interested in problems at very high energies. So we use a low-energy effective theory (analogous to, eg., low-energy QED, or low-energy QCD → nuclear physics).

Effective Quantum Gravity for terrestrial experiments writes $\tilde{g}^{\mu\nu}(x) = \eta^{\mu\nu} + \lambda h^{\mu\nu}(x)$ where $h^{\mu\nu}$ parametrizes the deviations from the flat spacetime metric, and $\lambda << 1$ is the dimensionless gravitational coupling. We then write the spacetime Lagrangian as a sum of a free field part (ie., non-interacting gravitons) plus the non-linear interacting part, viz.



GRAVITON VERTICES. Top: Self-interactions of spacetime Field. Bottom: graviton interactions with a matter field

$$L_G = L_o - \int d^4x \, U(h^{\mu\nu})$$

where everything is expanded in powers of λ (note that we can just as easily expand about a curved background metric – this is what is done in, eg., the derivation of Hawking radiation). If we then let the graviton field $h^{\mu\nu}$ interact with some matter field, we get graphs like those shown at left.

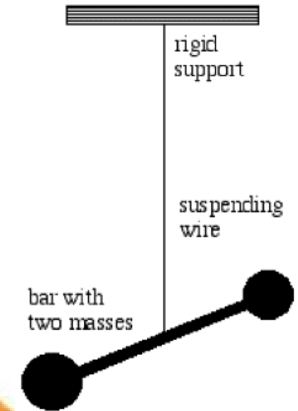
Now in practise this just means that the expression

$$\mathcal{K}(x,x') = \int \mathcal{D}g^{\mu\nu} \Delta(g) \, e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \int_{x'}^{x} \mathcal{D}q \, e^{\frac{i}{\hbar} S_M[q, g^{\mu\nu}]}$$

has a restriction on the allowed paths and/or field configurations – we are only allowed to include slow, long-wavelength configurations (RG philosophy).

Among the more dramatic objects described in this framework is a quantum radiating black hole

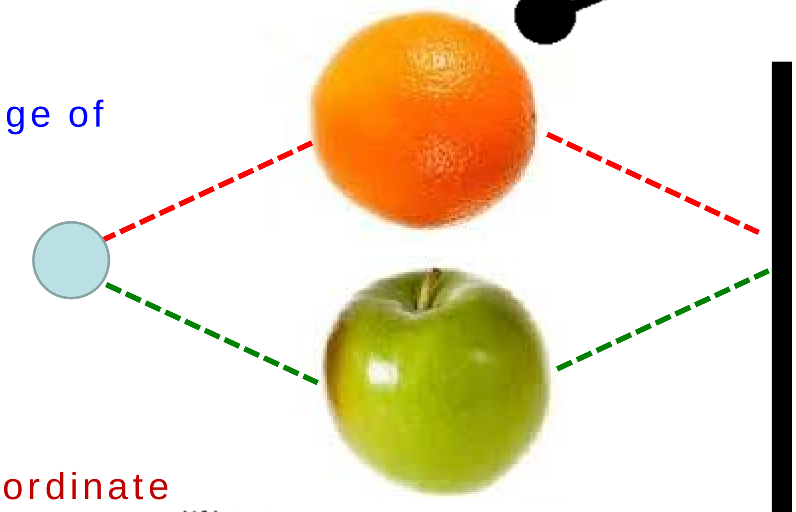
The low-E INCOMPATIBILITY of QM & GR



Feynman 1957, Karolhazy 1966, Eppley-Hannah 1977, Kibble 1978-82, Page 1981, Unruh 1984, Penrose 1996, all argued that there is a basic conflict between the superposition principle & GR at ordinary ‘table-top’ energies.

Consider a 2-slit experiment with a mass M. Assume a wave-fn in which the metric is quantized (& entangled with the mass):

$$|\Psi\rangle \;=\; a_1|\Phi_1;\tilde{g}^{\mu\nu}_{(1)}(x)\rangle + a_1|\Phi_2;\tilde{g}^{\mu\nu}_{(2)}(x)\rangle$$



If we ignored the change of metric with change of mass trajectory, we would have

$$\Phi(\mathbf{r},t) \equiv \langle\mathbf{r}|\Phi(t)\rangle \;=\; a_1\Phi_1(\mathbf{r},t) + a_2\Phi(\mathbf{r},t)$$

and then an interference term:

$$\langle\Phi_1(t)|\Phi_2(t)\rangle \;=\; \int d^3r\;\langle\Phi_1^*(\mathbf{r},t)|\Phi_2(\mathbf{r},t)\rangle$$

But now we have two problems.

(i) FORMAL PROBLEM: There are 2 different coordinate systems, $(\mathbf{r}_j, t_j)$, defined by the 2 different metrics: $\tilde{g}^{\mu\nu}_{(j)}(x)$, & in now we cant relate these – there is no formal meaning to the inner product $\langle\Phi_1|\Phi_2\rangle$, since we can’t relate $\langle\Phi_1|\mathbf{r}_1,t_1\rangle$ and $\langle\mathbf{r}_2,t_2|\Phi_2\rangle$. Indeed, in QFT, the 2 states don’t even have the same vacuum.

(ii) PHYSICAL PROBLEM: If we keep the spacetime metric in quantum form, we can never describe definite results. However a “wave-function collapse” for this superposition, with widely separated masses, causes non-local changes in the metric, which are drastically unphysical (non-conservation of energy, etc.). Clearly these problems are worse for large masses, where the causal structure of the 2 superposed metrics may be appreciably different.

As we'll see, this conflict between QM & GR is, in principle, capable of being probed in EARTH-BASED EXPERIMENTS

So, what do we do? We must weigh our options here....

We can't just drop one or the other theory – they both work incredibly well at low E.

Neither QM nor GR has ever failed an experimental test; and both have shown a shocking ability to predict & explain an amazing variety of new (very counter-intuitive) physical phenomena.

EACH is JUST as INCREDIBLY SUCCESSFUL as the OTHER.

Obviously we need a new theory that combines the virtues of each one.....

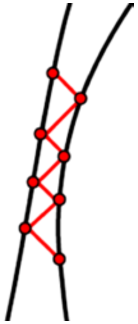
This is very hard; they are both very difficult to modify

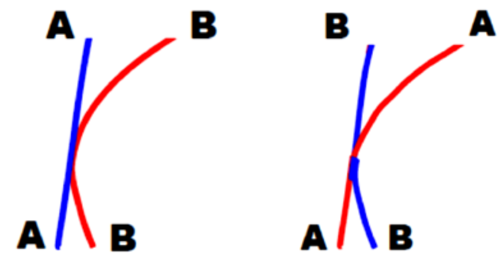
The

CORRELATED WORLDLINE THEORY

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
“ , New J. Phys. 17, 06517 (2015)

BASIC IDEA BEHIND CWL THEORY





This theory begins from several observations:

(i) In GR, the phase accumulated along a worldline is a LOCAL quantity (can be determined by mmts of proper time along the worldline). The metric is on the other hand determined by mmts between pairs of worldliness.

(ii) The equivalence principle – according to which gravity can't distinguish different forms of mass/energy – then implies that gravity can't distinguish, in a path integral, worldline pairs for a single particle from those for 2 particles.

One is then led to postulate the following generalization of QM/QFT:

(1) We replace path integrals, with their sums over paths, by sums over correlated paths (this amounts to a breakdown of the superposition principle). Thus, eg., for a single particle we have:

$$G_o(2,1) = \int_1^2 \mathcal{D}q(\tau) e^{\frac{i}{\hbar}S(2,1)} \longrightarrow \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_1^2 \mathcal{D}q_k(\tau)\, \kappa_n[\{q_k\}]\, e^{\frac{i}{\hbar}S[q_k;2,1]}$$

where the κ_n $[q_1, q_2, \ldots q_n]$ are the correlators between the paths.

(2) The correlations are mediated by the spacetime field itself (or more precisely by its curvature, which is responsible for the difference in vacua between different paths). We then have

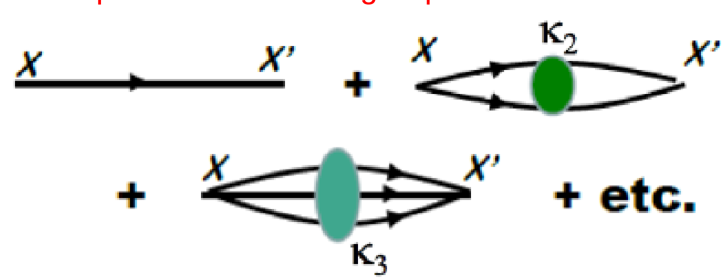
$$\kappa_n = \int \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x)\, e^{\frac{i}{\hbar}S_G} \Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)]$$

metric density

gravitational action

Faddeev-Popov determinant

Diagramatically we can represent the single particle propagator as



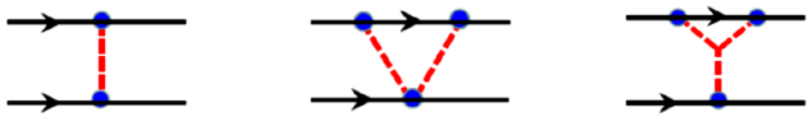
HOW A PARTICLE PROPAGATES: Rather than go into the gory details of the formal theory, let's show what happens to the propagator of a single point particle. According to what was said above, this takes the form:

$$\mathcal{K}(x,x') = \int \mathcal{D}g^{\mu\nu}\,\Delta(g)\, e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \left[\oint \mathcal{D}q\, e^{\frac{i}{\hbar}S[q,g^{\mu\nu}]} + \frac{1}{2}\oint \mathcal{D}q \oint \mathcal{D}q'\, e^{\frac{i}{\hbar}(S[q,g^{\mu\nu}]+S[q',g^{\mu\nu}])} + \ldots \right]$$

Lowest correction

where for the moment we drop higher corrections.

The lowest order irreducible diagrams for this first correction are at right. In de Donder gauge the graviton propagator is



$\kappa_2[q_1(\tau), q_2(\tau)]$

$$\mathcal{D}^o_{\mu\nu\lambda\rho}(q) = \frac{1}{2q^2}[\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\rho}]$$

and we get: $$\kappa_2[x,x'] = \exp\left[\frac{i\lambda^2}{2\hbar}\int d^4x \int d^4x'\, T^{\mu\nu}(x)\mathcal{D}^o_{\mu\nu\lambda\rho}(x-x')T^{\lambda\rho}(x')\right] - 1 \qquad + \; etc.$$

Let's write this as $\kappa_2[q,q'] = e^{i\chi_2[q,q']} - 1$ and take the 'slow-moving' limit where v << c.

Then $q \to (\mathbf{q}, t)$; define the relative coordinate $\mathbf{r} = \mathbf{q} - \mathbf{q}'$

$L_p = (\hbar G/c^3)^{1/2}$

$R_s = 2Gm/c^2$

and we find $$\chi_2[q,q'] = \int^t \frac{d\tau}{\hbar}\frac{m^2\lambda^2}{r(\tau)}\left[1 - \frac{R_s}{2r(\tau)} - \frac{17}{20}\frac{L_p^2}{r^2(\tau)} + \ldots\right]$$

so that $$\kappa_2[\mathbf{q},\mathbf{q}'] = \exp\frac{i}{\hbar}\int^t d\tau\, \frac{4\pi G m^2}{|\mathbf{q}(\tau) - \mathbf{q}'(\tau)|} - 1$$

for velocities << c

This attractive correlation leads to a 'path bunching' effect

SINGLE PARTICLE - SLOW DYNAMICS:

For an intuitive understanding of all this we look at the key scales in the problem. These are as follows:

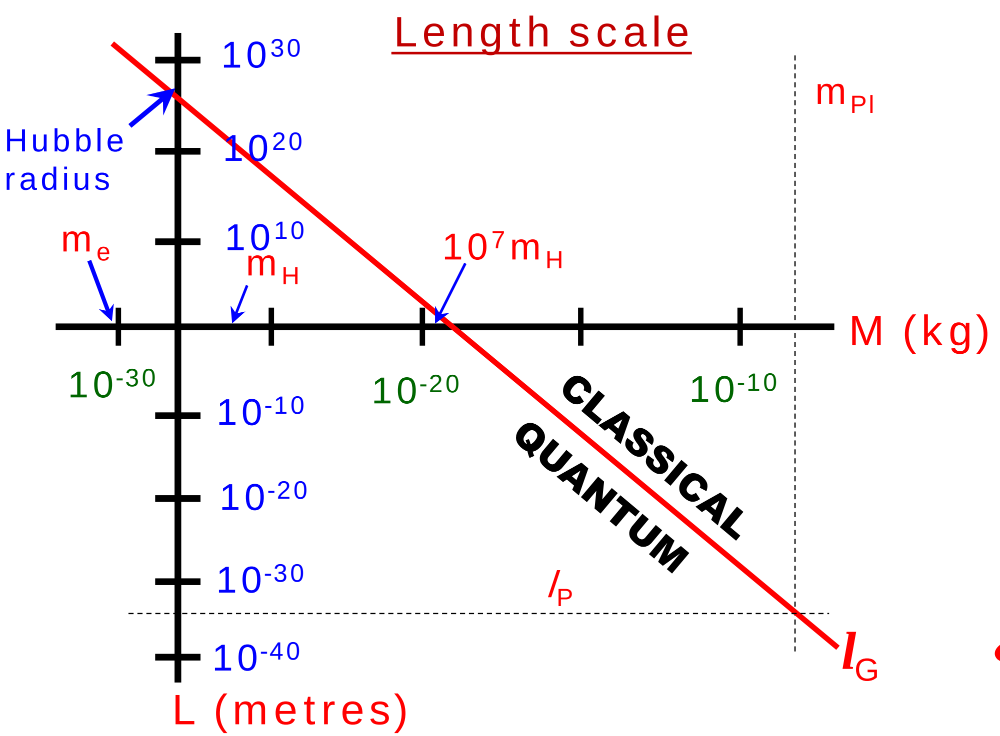
$l_G(m) = \left(\frac{M_p}{m}\right)^3 L_p$ Newton radius (gravitational analogue of the Bohr radius)

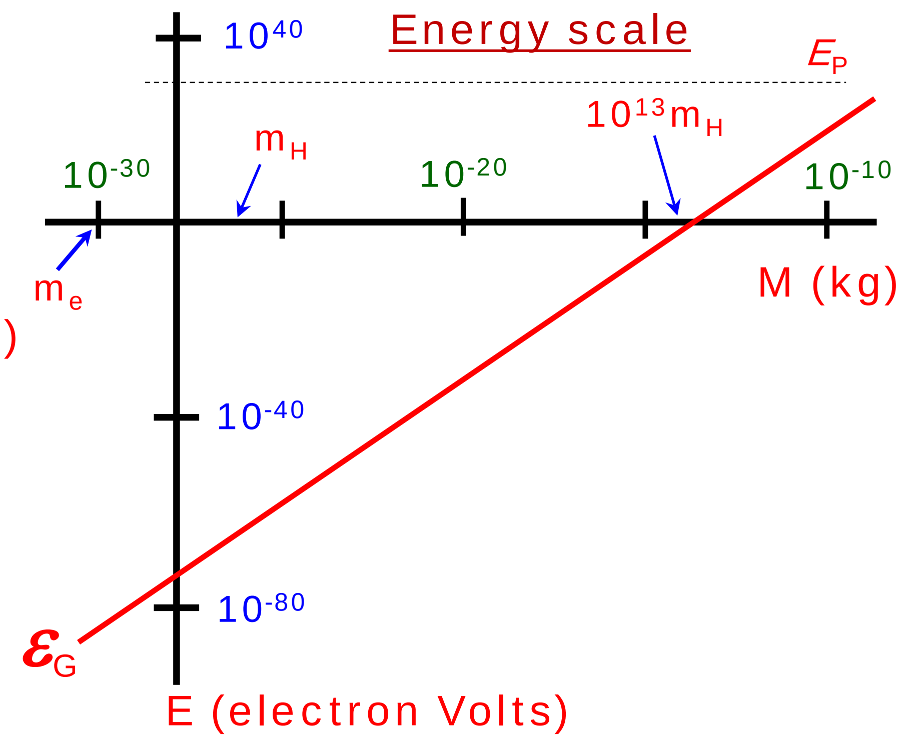
$\epsilon_G(m) = G^2 m^2 / l_G(m) \equiv E_p (m/M_p)^5$ Mutual binding energy for paths

} (QM)

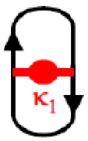
$R_s = 2Gm/c^2$ Schwarzchild radius for the particle (Classical)

The potential well giving this 'Coulomb-Newton' attraction causes a 'path bunching'. 2 paths will bind if $\epsilon_G > E_Q$ where E_Q is the energy scale associated with any other perturbations from impurities, phonons, photons, imperfections in any controlling potentials in the systems, and, worst of all, dynamical localized modes like defects, dislocations, paramagnetic or nuclear spins, etc.

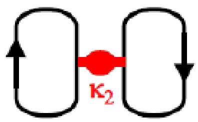




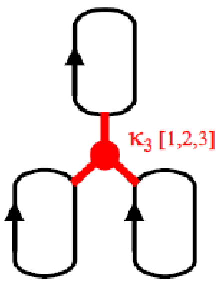
BRIEF REMARKS on FORMAL STRUCTURE of CWL THEORY



Q_1



Q_2



Q_3

To set up the theory we actually define via a generating functional (ie., a partition function) of form

$$\mathcal{Q}[j] \;=\; \frac{1}{\mathcal{N}} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}q_k \; \kappa_n[\{q_k\}] e^{\frac{i}{n\hbar}\sum_k (S[q_k] + \int j q_k)}$$

(for a single particle) which has the diagrammatic representation shown at left. For quantum fields we have a similar result:

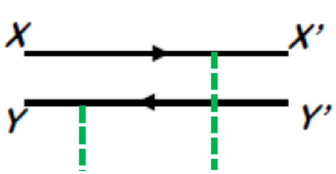
$$\mathcal{Q}[J] \;=\; \frac{1}{\mathcal{N}} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}\Phi_k \; \kappa_n[\{\Phi_k\}] \; e^{\frac{i}{n\hbar}(S[\Phi_k] + \int d^4x J(x)\Phi_k(x))}$$

(here given for a scalar field). As we saw previously, the correlators are given by functional integration over the quantized spacetime metric

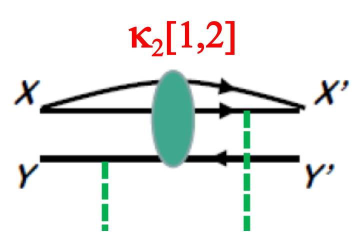
We define correlation functions & propagators by functional differentiation of these objects. We can easily generalize to an many-body condensed matter system; thus, eg., the N-particle propagator for such a system becomes

$$\mathcal{K}_N(x_1,...x_N; x'_1,...x'_N) \;=\; \prod_{j=1}^{N} \int \mathcal{D}g^{\mu\nu}\Delta(g)\; e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \sum_{n_j=1}^{\infty} \prod_{k_j=1}^{n_j} \int_{x'_j}^{x_j} \mathcal{D}q^{(j)}_{k_j} \; e^{\frac{i}{n_j\hbar}\sum_{k_j=1}^{n_j} S_M[q^{(j)}_{k_j}, g^{\mu\nu}]}$$

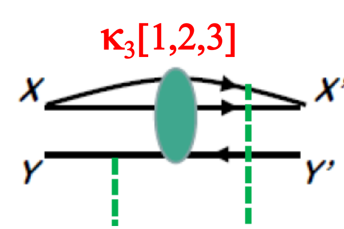
One can represent all of this in diagrammatic form – as example, we show below diagrams for a 2nd-order correlation function for a 2-particle propagator



2 free particles



CWL lowest correction

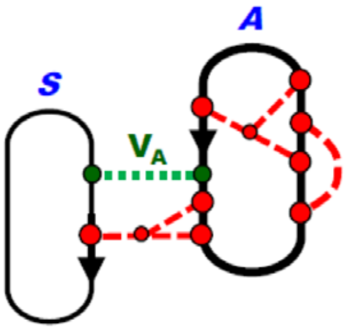


CWL next correction

For much more on the formal details, see appendix @ the end of this lecture

MEASUREMENTS & PROBABILITY in CWL THEORY

The difference between standard QM and the CWL theory comes down to the path bunching effect. Let's see how this works.



The system and apparatus now couple via some measurement coupling – one useful example is

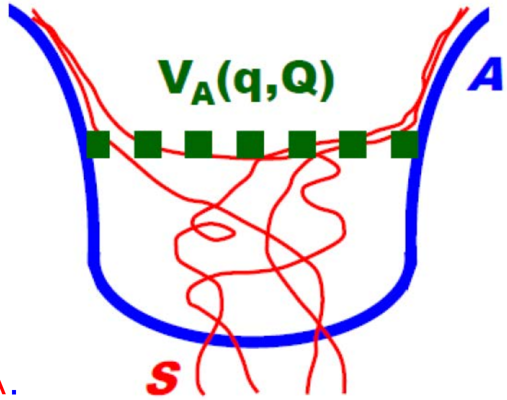
$$V_A(q,Q) = -i\hbar V(q, \partial/\partial q; t)\, \frac{\partial}{\partial Q}$$

where the coordinates Q & q refer to the apparatus *A* and the system *S* respectively. This coupling has the effect of slowly moving the apparatus coordinate Q into synchronization with the coordinate q of the system.

However although gravitational correlations between S and A are negligible, the CWL correlations in the dynamics of A are not; we assume its mass is sufficiently large so that path bunching eventually occurs in the dynamics of A (over some path bunching timescale).

Then, as the paths of A start to diverge into 2 classes (associated here with 2 different possible paths of S, because of the coupling between S and A), we see that the path bunching stops any interference between the 2 sets of corresponding paths for A – the paths of A separate into 2 bunches.

But this also means that the paths for S will do the same; they get 'dragged' into separate bunches because of the S-A interaction . Thus interference is suppressed for both S and A.

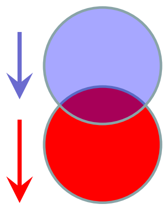


REAL WORLD PROBLEMs #4

QUANTUM DYNAMICS
of
OPTOMECHANICAL SYSTEM
(quantum gravity test)

OPTOMECHANICAL RESONATOR EXPERIMENTS

(1) The key idea is to look at interference between 2 separate states of a moving object – the 2 paths here, corresponding to the 2 different positions of the mass, will interact gravitationally in the CWL theory according to what we have seen. One can imagine lots of different ways to do this – eg., with a freely falling mass, or with a resonator put into a superposition of 2 different oscillating states.

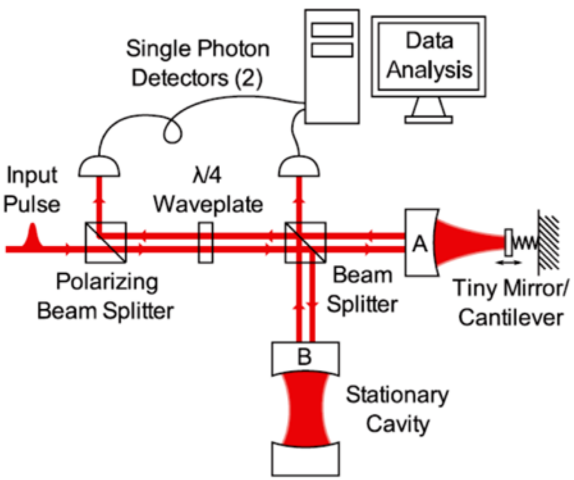


(2) Let's look 1st at interference between the 2 paths of an oscillating heavy mass. One way to do this is to entangle a photon with a heavy mirror, and then look for gravitational effects. Starting from a state

$$|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$$

we get

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B \; |\kappa(1 - e^{-i\omega_m t})\rangle_m],$$



and one looks at interference between the 2 branches. Another alternative is to look at interference between a 0-phonon and a 1-phonon state

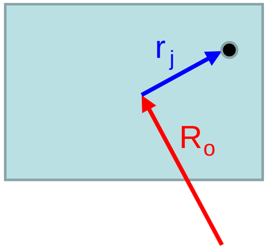
D Kleckner et al., N J Phys 10, 095020 (2008)

I Pikowski et al., Nat Phys 8, 393 (2012)

(3) The difficulty here is to reduce environmental decoherence effects – coming from the interaction with photons, or between, eg., charged defects in the system (or spin defects/nuclear spins) and EM fields.

A KEY RESULT: Gravitational effects depend in a completely different way on system parameters than do decoherence effects.

CWL ANALYSIS of OSCILLATOR DYNAMICS



We now need to develop the CWL theory for N-particle dynamics – we have a macroscopic oscillator

Define the centre of mass $\mathbf{R_o}(t) = \frac{1}{N}\sum_{j=1}^{N} \mathbf{q}_j(t)$ so that $\mathbf{q}_j = \mathbf{R_o} + \mathbf{r}_j$

The effective action is then $S_o[\mathbf{R}_o, \{\mathbf{r}_j\}] \;=\; \int d\tau \left[\frac{M_o}{2}\dot{\mathbf{R}}_o^2 + \sum_{j=1}^{N} \frac{m_j}{2}\dot{\mathbf{r}}_j^2 - \sum_{i<j}^{N} V(\mathbf{r}_i - \mathbf{r}_j) \right]$

Define sum & difference coordinates:

$$\mathbf{r_j} + \mathbf{r_j'} = \mathbf{x}_j \qquad \mathbf{R_o} + \mathbf{R_o'} = \mathbf{X_o}$$
$$\mathbf{r_j} - \mathbf{r_j'} = \xi_j \qquad \mathbf{R_o} - \mathbf{R_o'} = \mathbf{\Xi_o}$$

The extra contribution to the propagator is:

$$\Delta\mathcal{G}(2,1) \;=\; \int \mathcal{D}\mathbf{X}_o \int \mathcal{D}\mathbf{\Xi_o} \prod_j \int \mathcal{D}\mathbf{x}_j \int \mathcal{D}\xi_j$$
$$\times \;\; \kappa_2^N[\mathbf{\Xi}_o; \{\xi_j\}] \int d\mathbf{P} d\mathbf{K}\; e^{\frac{i}{\hbar N}(\mathbf{P}\cdot\mathbf{x}_j + \mathbf{K}\cdot\xi_j)}\; e^{i\Psi_2[\mathbf{\Xi}_o, \{\xi_j\}; \mathbf{X_o}, \{\mathbf{x}_j\}]}$$

where the C.o.m. correlates gravitationally with the individual particles according to

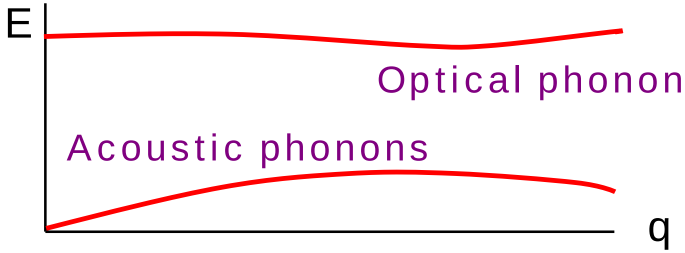
$$\kappa_2^N[\mathbf{\Xi}_o, \{\xi_j\}] \;=\; \left(\exp\left[\frac{i\lambda^2}{4\pi\hbar} \int d\tau \sum_{j=1}^{N} \frac{m_j^2}{|\mathbf{\Xi}_o + \xi_j|} \right] \;-\; \delta_{\mathbf{\Xi}_o}\delta_{\xi_\mathbf{j}} \right)$$

PHONON EFFECTS We parametrize this by looking at the displacement correlator

$$\langle u_i^\alpha(t_1) u_j^\beta(t_1)\rangle \;=\; \frac{1}{N}\sum_{\mathbf{Q}\mu} \frac{\hat{e}^\alpha_{\mathbf{Q}\mu}\hat{e}^\beta_{\mathbf{Q}\mu}}{2m\omega_{\mathbf{Q}\mu}}\; e^{i[\mathbf{Q}\cdot\mathbf{r}_{\mathbf{ij}}^{(\mathbf{o})} - \omega_{\mathbf{Q}\mu}(t_1 - t_2)]}$$

with phonon frequencies

$$\omega^2_{\mathbf{Q}\mu} \;=\; \frac{1}{m}\sum_{i\neq j} V_{ij} e^{i\mathbf{Q}\cdot\mathbf{r}_{ij}^{(o)}}$$



To calculate the oscillator dynamics we need to add a driving force to the Lagrangian for the oscillator, coupling to the centre of mass. Thus we now consider a total action:

$$S[\mathbf{R_o}, \{\mathbf{r_j}\}; \mathbf{F_o}] \;=\; S_o[\mathbf{R_o}, \{\mathbf{r_j}\}] \;+\; S_U[\mathbf{R_o}; \mathbf{F_o}]$$

with a driving term: $S_U[\mathbf{R_o}; \mathbf{F_o}] \;=\; -\int d\tau \left[\frac{1}{2} U_o \mathbf{R_o^2}(\tau) \;+\; \mathbf{F_o}(\tau) \cdot \mathbf{R_o}(\tau) \right]$

Rather complex calculations show that the main effects come from an integral of form

$$\int_0^0 \mathcal{D}\mathbf{\Xi_o}\; e^{-\frac{i}{\hbar} \int d\tau \frac{1}{2} [\mathbf{\Xi_o} \mathcal{D}_o \mathbf{\Xi_o}]} \left[e^{-i \frac{\lambda^2}{4\pi\hbar} \int d\tau \sum_j \frac{m_j^2}{|\mathbf{\Xi_o} + \xi_j|}} \;-\; \delta_{\mathbf{\Xi_o}} \right]$$

where $\hat{\mathcal{D}}_o \;=\; \frac{M_o}{2} \left[\frac{d^2}{d\tau^2} + \Omega_o^2 \right]$ is the differential operator for the harmonic motion of the oscillator

All the path bunching effects of interest are contained in this integral – to evaluate them we need to know the range of paths for the ions in the solid, which can be calculated completely from knowledge of the phonon spectrum for a given system.

Typically in a solid we have $\Delta \mathbf{u}_j \;\sim\; a_o (U_\Phi / U_C)$ where U_Φ / U_C is the ratio of elastic to electronic energies, and a_o is the inter-ion spacing. One then finds roughly

$$\Delta \mathbf{u}_j \sim 2 \times 10^{-12} \text{ m}$$

for typical path fluctuations (although this varies a lot between solids).

We now want to know what happens in an experiment – when do centre of mass superpositions break down?

TABLE-TOP EXPERIMENTS

Only wimps specialize in the general case. Real scientists pursue examples. MV Berry (1995)

I. MECHANICAL OSCILLATOR

Now we add a term to the action: $S_U[\mathbf{R_o};\mathbf{F_o}] \;=\; -\int d\tau \;\left[\frac{1}{2}U_o\mathbf{R_o^2}(\tau) \;+\; \mathbf{F_o}(\tau)\cdot\mathbf{R_o}(\tau)\right]$

In the absence of any coupling between the phonons and the centre of mass, we get

$$\begin{aligned}\mathcal{G}(2,1) \;&=\; G_{osc}(2,1)\;G_c(2,1)\\ &\equiv\; G_{osc}(\mathbf{X_o}^{(2)},\mathbf{X_o}^{(1)}|\mathbf{F_o}(t))\;\;G_c(\mathbf{\Xi_o}^{(2)},\mathbf{\Xi_o}^{(1)};\{\xi_\mathbf{j}^{(2)},\xi_\mathbf{j}^{(1)}\})\end{aligned}$$

where the latter term incorporates the reduction of the path-bunching coming from individual ion dynamics.

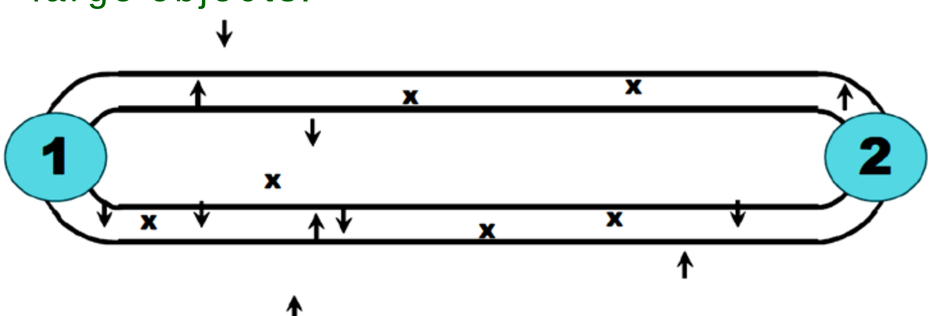
The final result depends strongly on both the phonon dynamics and on the coupling of phonons to defects and spin impurities. The onset of path bunching is now at mass scales $M \sim 10^{18}\ m_H$ with an effective path-bunching length $\sim 10^{-16}$ m.

Such an experiment has many attractive features.

II. 2-SLIT EXPERIMENT

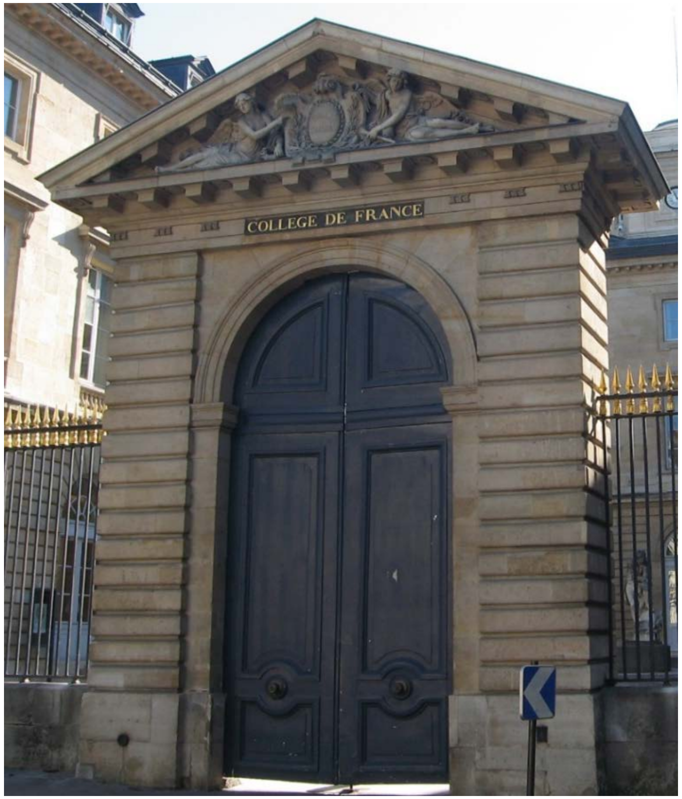
This is at first glance a very attractive experiment to analyse – but to realize it will be very difficult. For an extended mass the numbers come out similarly to those for the oscillator – but the influence of defects and impurities is much greater.

Such an experiment is likely impossible – even if one could do interference for such large objects.



CRUCIAL RESULT: The CWL CORRELATIONS & PATH BUNCHING MECHANISM DO NOT INVOLVE DECOHERENCE !!

UBC



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Mona Berciu
Dan Carney
Tim Cox
Alvaro Gomez
Lara Thompson
Igor Tupitsyn
Zhen Zhu

APPENDIX

TECHNICAL DETAILS

of

CWL THEORY

CWL THEORY: FORMAL STRUCTURE

I. GENERATING FUNCTIONAL

Start with the generating functional

$$\mathcal{Q}[j] \;=\; \frac{1}{\mathcal{N}} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}q_k \; \kappa_n[\{q_k\}] e^{\frac{i}{n\hbar} \sum_k (S[q_k] + \int j q_k)}$$

with normalization

$$\mathcal{N} = \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}q_k \; \kappa_n[\{q_k\}] e^{\frac{i}{n\hbar} \sum_k S[q_k]}$$

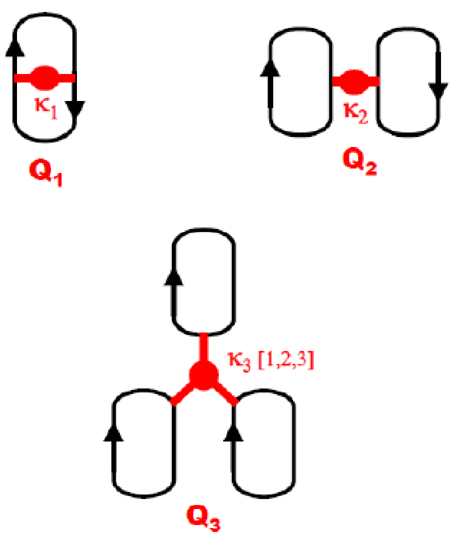
The inter-path correlator is given by

$$\kappa_n[q] \;=\; \frac{1}{n!} \int'' \mathcal{D}g^{\mu\nu} e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \Delta[g^{\mu\nu}] \qquad (\forall\, n > 0)$$

Thus, for the generating functional of a single particle we have

$$\mathcal{Q}[j] \;=\; \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int'' \mathcal{D}\tilde{g}^{\mu\nu} \int \mathcal{D}q_k$$

$$\times \; \frac{1}{n!} e^{iS_G/\hbar} \; \Delta[\tilde{g}^{\mu\nu}] \; e^{\frac{i}{n\hbar} \sum_k (S[g, q_k] + \int ds\; j(s) q_k(s))}$$



where the particle action is

$$S_M \;=\; \int d^4x \, \frac{m}{2} g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu \; \delta(x^\lambda - q^\lambda(s))$$

For a scalar field we have the simple generalization

$$\mathcal{Q}[J] \;=\; \frac{1}{\mathcal{N}} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}\Phi_k \; \kappa_n[\{\Phi_k\}]$$

$$\times \; e^{\frac{i}{n\hbar} (S[\Phi_k] + \int d^4x J(x) \Phi_k(x))}$$

II. CORRELATION FUNCTIONS

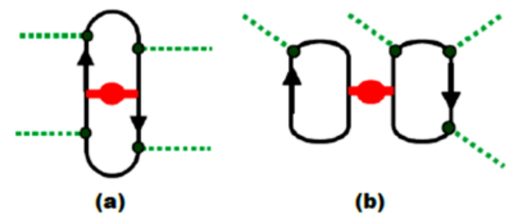
For a single particle we define the CWL correlator

$$\mathcal{G}_n^{\sigma_1,..\sigma_n}(s_1,..s_n) \;=\; \left(\frac{-i}{\hbar}\right)^n \lim_{j(s)\to 0}\left[\frac{\delta^n \mathcal{Q}[j]}{\delta j(s_1\sigma_1)..\delta j(s_n\sigma_n)}\right]$$

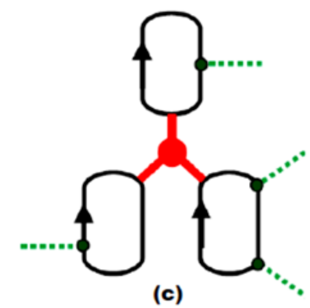
$$= \sum_{r=1}^{\infty}\prod_{\alpha=1}^{r}\int \mathcal{D}q^\alpha(\tau)\; e^{\frac{i}{r\hbar}\sum_\alpha S_o[q^\alpha]}\;\kappa_r(\{q^\alpha\})\;\prod_{j=1}^{n}\left(\sum_{\alpha=1}^{r} q^\alpha(s_j,\sigma_j)\right)$$

$$= \sum_{r=1}^{\infty}\frac{1}{r!}\int'' \mathcal{D}g^{\mu\nu} e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]}\Delta[g^{\mu\nu}]\prod_{\alpha=1}^{r}\int \mathcal{D}q^\alpha(\tau)\; e^{\frac{i}{r\hbar}\sum_\alpha S_o[q^\alpha,g^{\mu\nu}]}\;\prod_{j=1}^{n}\left(\sum_{\alpha=1}^{r} q^\alpha(s_j,\sigma_j)\right)$$

We can represent this messy formula diagrammatically by the sum shown at right, where the green hashed lines represent current insertions – we sum over all combinatoric possibilities.



The same structure exists for a set of fields. Thus, eg., for a single scalar field we have the explicit expansion, for the 4-point correlator, given by



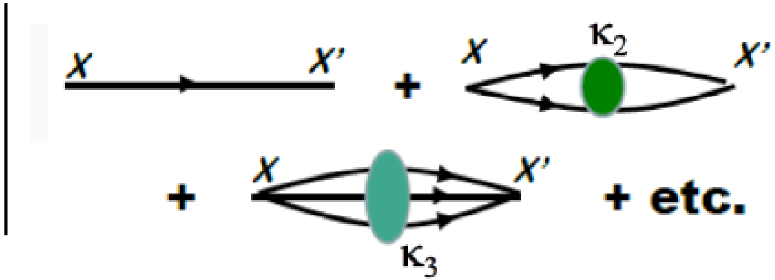
$$\mathbb{G}_4^{\sigma_1,..\sigma_4}(x_1,..x_4) \;=\; \oint \mathcal{D}\phi(x)\; e^{\frac{i}{\hbar}S[\phi]}\prod_{j=1}^{4}\phi(x_j,\sigma_j)$$

$$+\;\oint \mathcal{D}\phi(x)\oint \mathcal{D}\phi'(x)\; e^{\frac{i}{\hbar}(S[\phi]+S[\phi'])}\;\kappa_2[\phi,\phi']\prod_{j=1}^{4}\left[\phi(x_j,\sigma_j)+\phi'(x_j,\sigma_j)\right]$$

III. STRUCTURE of PROPAGATORS

Recall that in ordinary QM we have the 1-particle propagator:

$$K(x,x') = \int_{x'}^{x} \mathcal{D}q \, e^{\frac{i}{\hbar} S_M[q]}$$



In CWL theory we have the generalization:

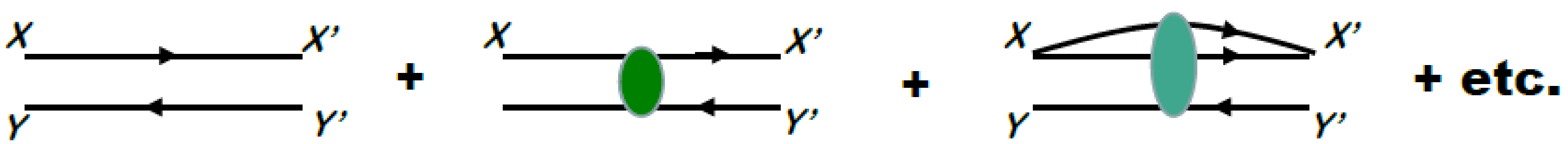
$$\mathcal{K}(x,x') = \int \mathcal{D}g^{\mu\nu} \Delta(g) \, e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_{x'}^{x} \mathcal{D}q_k \, e^{\frac{i}{n\hbar} \sum_k \; S_M[q_k, g^{\mu\nu}]}$$

which is shown diagrammatically at top right.

For a many-body system we can define the N-particle propagator

$$\mathcal{K}_N(x_1, ...x_N; x'_1, ...x'_N) = \prod_{j=1}^{N} \int \mathcal{D}g^{\mu\nu} \Delta(g) \, e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \sum_{n_j=1}^{\infty} \prod_{k_j=1}^{n_j} \int_{x'_j}^{x_j} \mathcal{D}q_{k_j}^{(j)} \, e^{\frac{i}{n_j\hbar} \sum_{k_j=1}^{n_j} S_M[q_{k_j}^{(j)}, g^{\mu\nu}]}$$

Diagrammatically we have:



All of this has an obvious generalization to fields – for propagation between initial and final field configurations

IV. CONDITIONAL / COMPOSITE PROPAGATORS

Let's first recall that in conventional QFT we can define the composite propagator/correlator:

$$\chi_1^{(p)}(x,x'|\{q(t_\alpha)\}) \;=\; \langle x|\hat{T}\{q(t_1),...q(t_p)\}|x'\rangle$$

which has the path integral representation:

$$\chi_1^{(p)}(x,x'|\{q(t_\alpha)\}) \;=\; \int_{x'}^{x}\mathcal{D}q\; e^{\frac{i}{\hbar}S[q]}\prod_{\alpha=1}^{p}q(t_\alpha)$$

$$= \;(-i\hbar)^p\frac{\delta^p}{\delta j(t_1)....\delta j(t_p)}\,K_1(x,x'|j(t))\Big|_{j=0}$$

where we have defined the external current-dependent propagator:

$$K_1(x,x'|j) \;=\; \int_{x'}^{x}\mathcal{D}q\; e^{\frac{i}{\hbar}(S[q]+\int dt j(t)q(t))}$$

Now in CWL theory we have

$$\chi_1^{(p)}(x,x'|\{q(t_\alpha)\}) \;=\; (-i\hbar)^p\frac{\delta^p}{\delta j(t_1)....\delta j(t_p)}\,\mathcal{K}_1(x,x'|j(t))\Big|_{j=0}$$

where now the propagator involves the CWL sum:

$$\mathcal{K}_1(x,x'|j) \;=\; \int\mathcal{D}g^{\mu\nu}\Delta(g)\;e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]}\sum_{n=1}^{\infty}\prod_{k=1}^{n}\int_{x'}^{x}\mathcal{D}q_k\;e^{\frac{i}{n\hbar}\sum_k\,(S_M[q_k,g^{\mu\nu}]+\int dt j(t)q_k(t))}$$

Working this out we get:

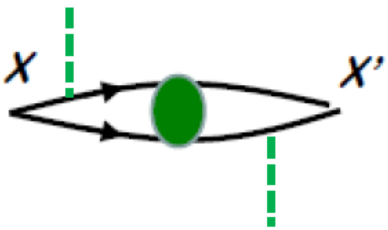
$$\chi_1^{(p)}(x,x'|\{q(t_\alpha)\}) \;=\; \int\mathcal{D}g^{\mu\nu}\Delta(g)\;e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]}\sum_{n=1}^{\infty}\prod_{k=1}^{n}\int_{x'}^{x}\mathcal{D}q_k\;e^{\frac{i}{n\hbar}\sum_k\;S_M[q_k,g^{\mu\nu}]}\prod_{\alpha=1}^{p}\left(\sum_{k'=1}^{n}q'_k(t_\alpha)\right)$$

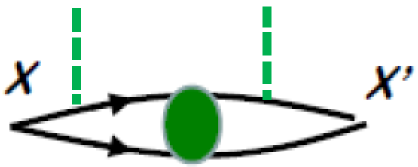
which has the diagrammatic interpretation shown on the next page

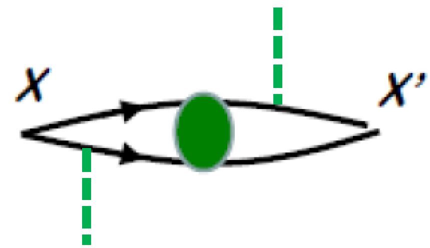
DIAGRAMMATIC INTERPRETATION

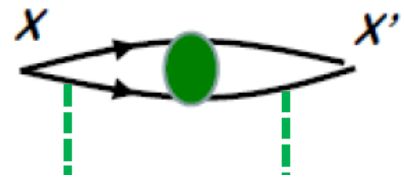
Consider for example a 1-particle propagator with 2 current insertions. Then the conventional QFT result is

The first set of CWL corrections looks like:

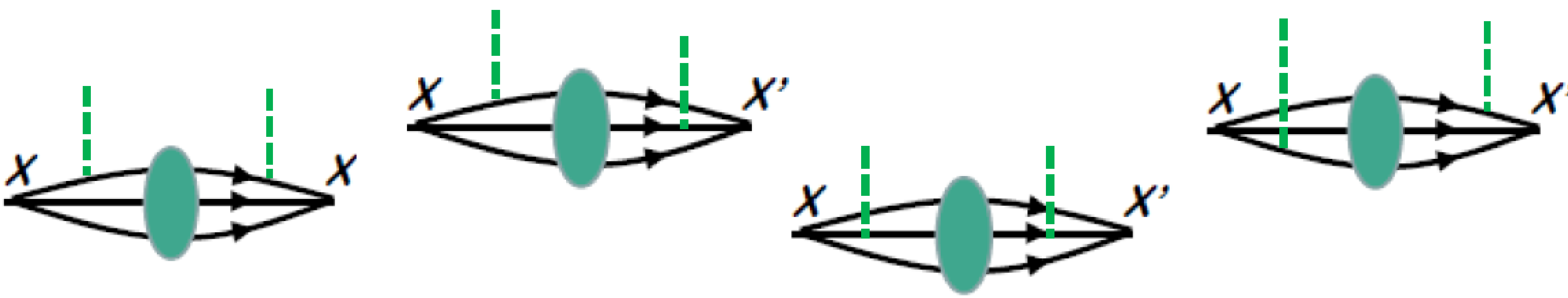








The next set of CWL corrections looks like:



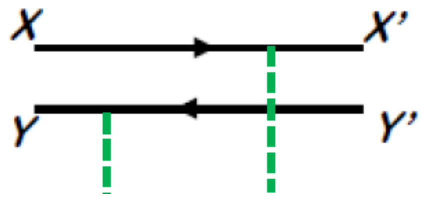
and so on....

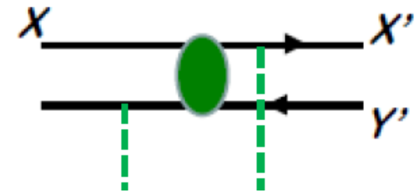
HIGHER CONDITIONAL / COMPOSITE PROPAGATORS

Consider, eg., the 2-particle propagator. Without writing down the formulas, it is obvious what we will get.

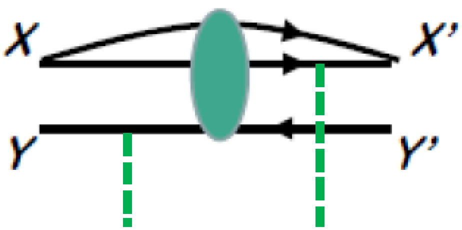
Thus, eg., if we have 2 external insertions, and the 2 particles are distinguishable, we have

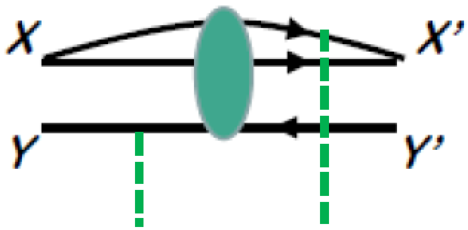
(a) Conventional QFT:





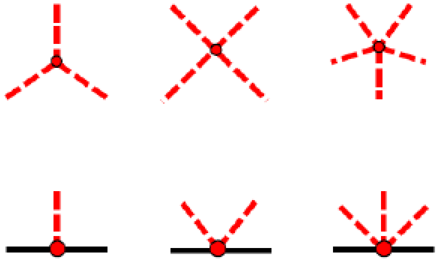
(b) CWL corrections: The lowest-order terms are:





It is fairly obvious where one goes on from here.

V. GRAVITON EXPANSIONS

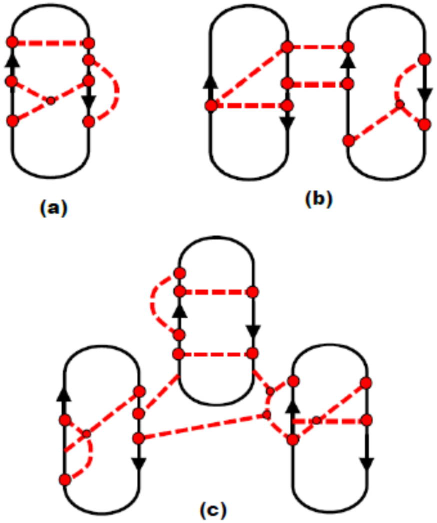


Suppose we make an expansion about a background spacetime – in this case flat space. Then:

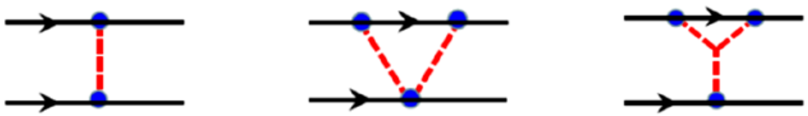
$$\tilde{\mathfrak{g}}^{\mu\nu}(x) = \eta^{\mu\nu} + \lambda h^{\mu\nu}(x)$$

The Lagrangian is written as a graviton expansion:

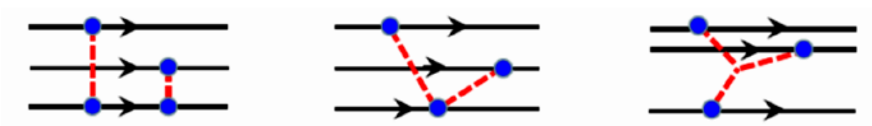
$$L_G = L_o - \int d^4x U(h^{\mu\nu})$$



The CWL generating functional then has the form shown at right, and the correlators have terms like those shown below.



$$\kappa_2[q_1(\tau), q_2(\tau)]$$



$$\kappa_3[q_1(\tau), q_2(\tau), q_3(\tau)]$$

EXAMPLE: DENSITY MATRIX PROPAGATOR

Define $\hat{h}(x) = h^{\mu\nu}(x)$

and $\hat{D}(x) \doteq D^{\mu\nu\lambda\rho}(x)$

Then

$$\mathcal{K}_{2,2';1,1'} = lim_{\hat{h}=0} \{ e^{\frac{i}{2\hbar}(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'})} \times e^{\frac{-i}{\hbar}\int U(\hat{h})} \mathcal{K}_{2,1}[\hat{h}(x)]\, \mathcal{K}_{1',2'}[\hat{h}(x')] \}$$

where

$$(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'}) = \int d^4x d^4x' \frac{\delta}{\delta\hat{h}(x)} \hat{D}(x,x') \frac{\delta}{\delta\hat{h}(x')}$$

and where $\mathcal{K}_{2,1}[\hat{h}(x)]$ is the CWL propagator in a field $\hat{h}(x)$