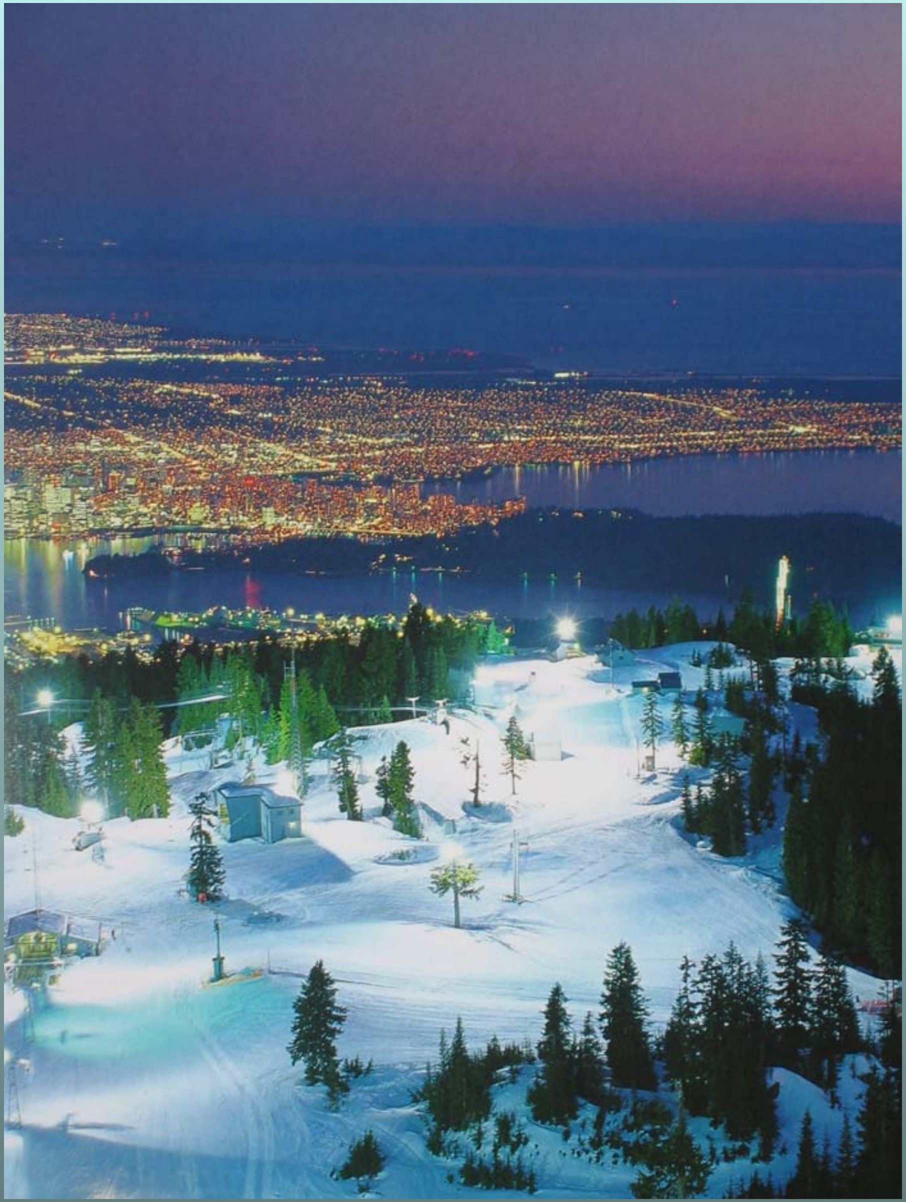
PCE STAMP

Physics & Astronomy
UBC
Vancouver



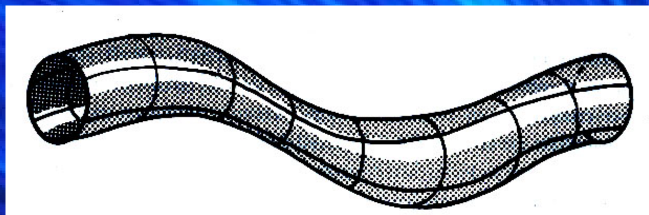
PiTP

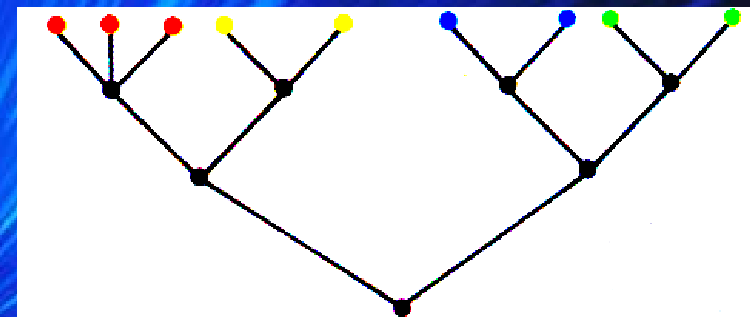
Pacific Institute
for
Theoretical Physics

EFFECTIVE HAMILTONIANS & EFFECTIVE VACUA

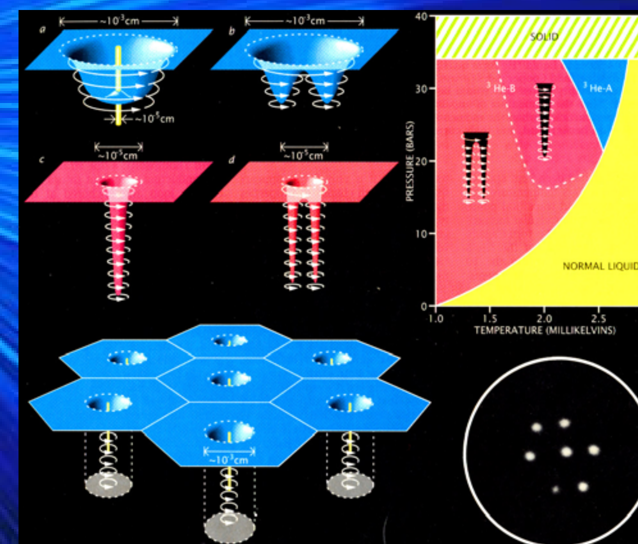
P.C.E. Stamp (UBC & PiTP)

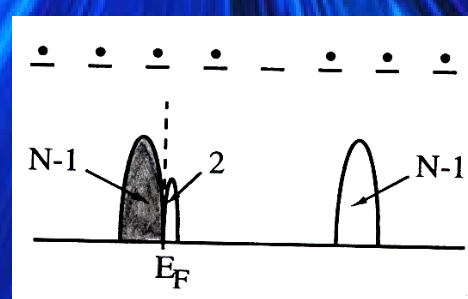
PSA Meeting (Austin TX, Nov 2004)



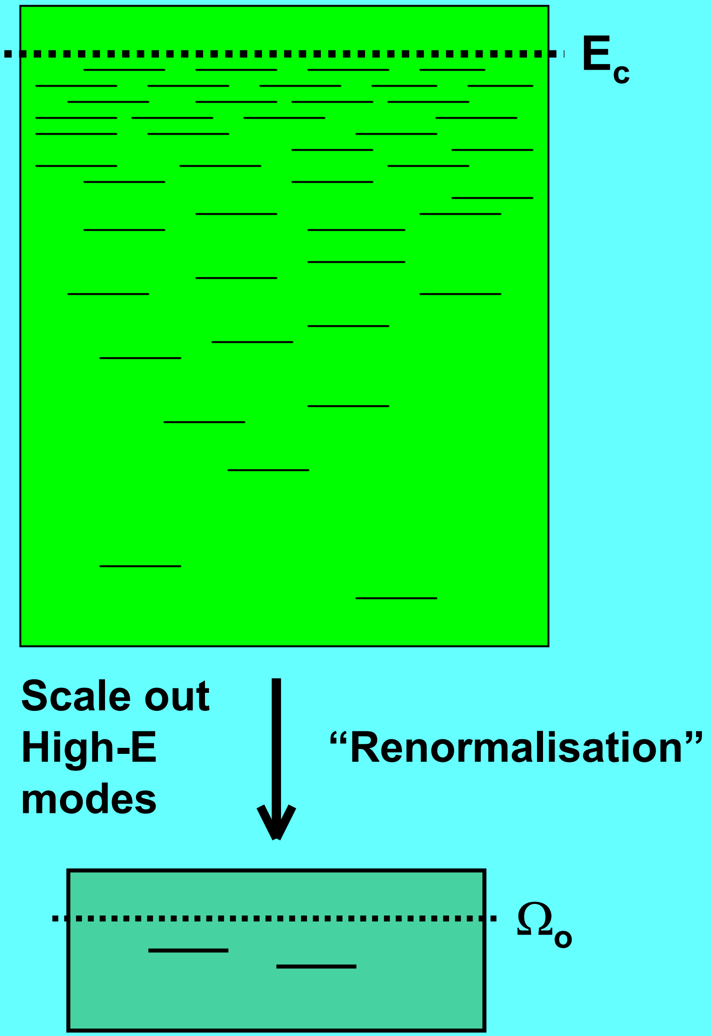


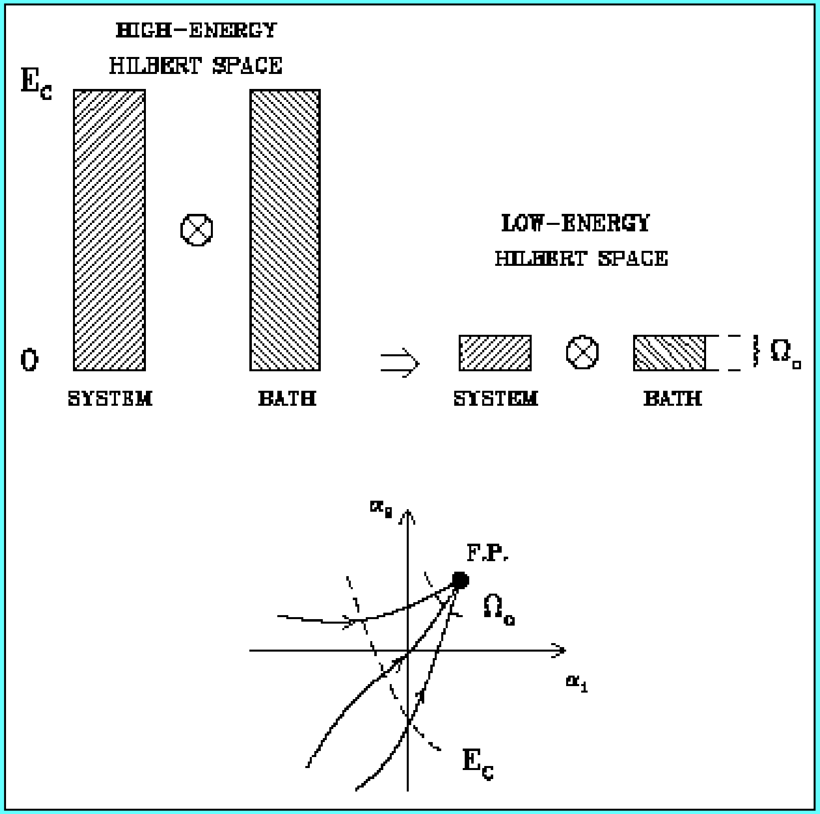
$\mathcal{H}_{eff}$





Orthodox view of $\mathcal{H}_{\mathbf{eff}}$



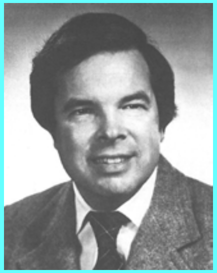


$\mathcal{H}_{\mathbf{eff}}\,(E_c\,) \rightarrow \mathcal{H}_{\mathbf{eff}}\,(\Omega_o)$

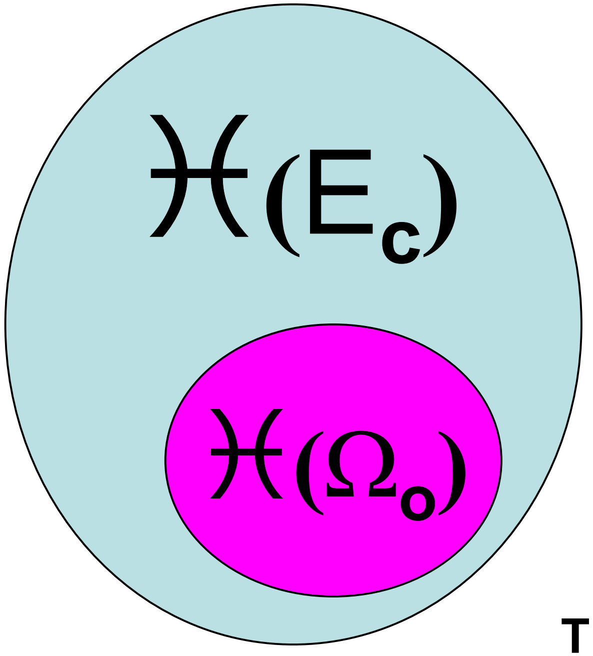
$|\psi_i\rangle\ \ H_{ij}(E_c)\ \ \langle\psi_j| \quad \rightarrow \quad |\phi_\alpha\rangle\ \ \mathcal{H}_{\alpha\beta}(\Omega_o)\ \langle\phi_\beta|$

Flow of Hamiltonian & Hilbert space with UV cutoff

The RG mantra is: **RG flow**
fixed points
low-energy $\mathcal{H}_{eff}$
universality classes



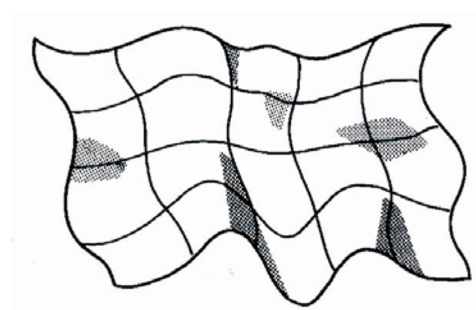
MORE ORTHODOXY



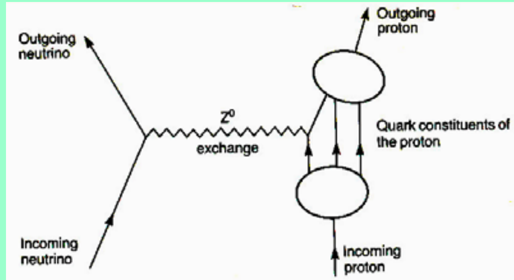
Continuing in the orthodox vein, one supposes that for a given system, there will be a sequence of Hilbert spaces, over which the effective Hamiltonian and all the other relevant physical operators (NB: these are **effective operators**) are defined.

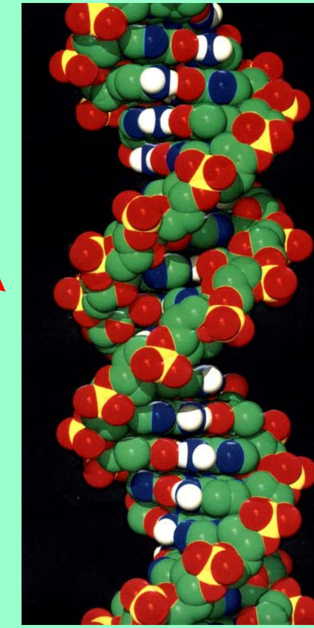
Then, we suppose, as one goes to low energies we approach the '*real vacuum'*; the approach to the fixed point tells us about the excitations about this vacuum. This is of course a little simplistic- not only do the effective vacuum and the excitations change with the energy scale (often discontinuously, at phase transitions), but the effective Hamiltonian is in any case almost never one which completely describes the full N-particle states.

Nevertheless, it is believed that the basic structure is correct - that the effective Hamiltonian (***& note that ALL Hamiltonians or Actions are effective)*** *captures all the basic physics*



REMARKS on 'EMERGENCE' vs 'REDUCTIONISM'





The reductionist view is that all matter can be understood in terms of its 'basic constituents'. It is an atomistic point of view.

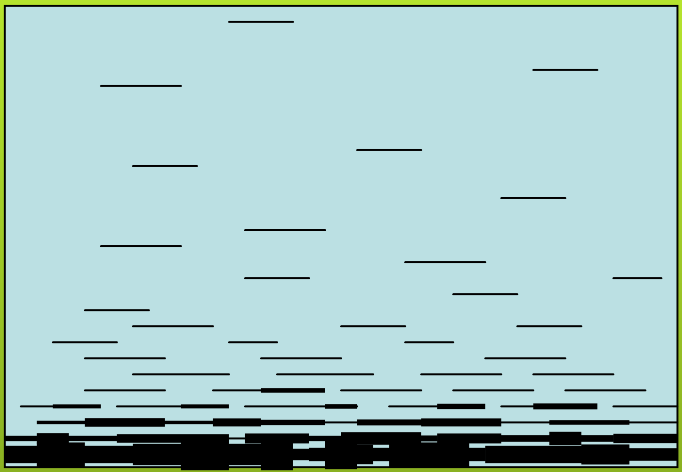
The 'emergence' point of view says that structures of matter at higher levels, & in more complex systems, CANNOT be understood in terms of basic constituents- that they have properties that are ineluctably 'complex' & which cannot ever be understood in terms of elementary constituents, even in principle.

NB1: ***Many if not most 'emergence' believers still nevertheless assume that matter is composed of 'bits'***

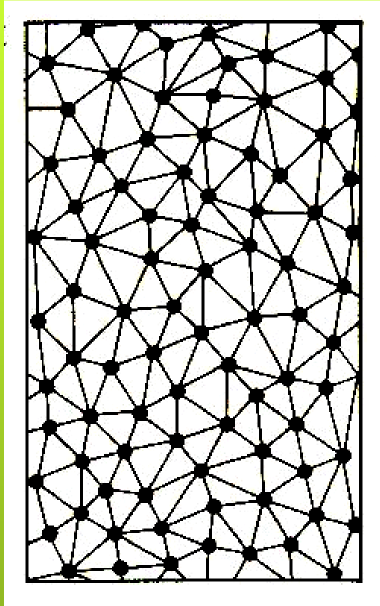
NB2: ***In fact there is no obvious end in sight to the long road towards 'elementary constituents'. Nature may just be 'wheels with in wheels..'.***

1ST CONUNDRUM- the 'GLASS'

States in a glass- piled up at low E



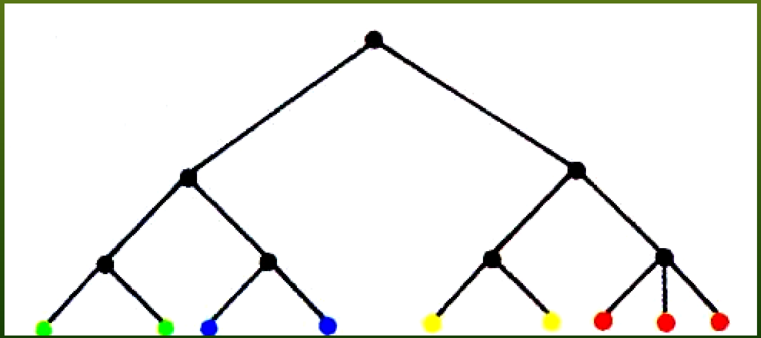
The simple picture of excitations perched above a vacuum gets a rude shock when we consider Glasses - systems with disorder & 'frustrating interactions'. We are surrounded by these! States pile up at low energy, but these can't communicate with each other.



Frustrating interactions

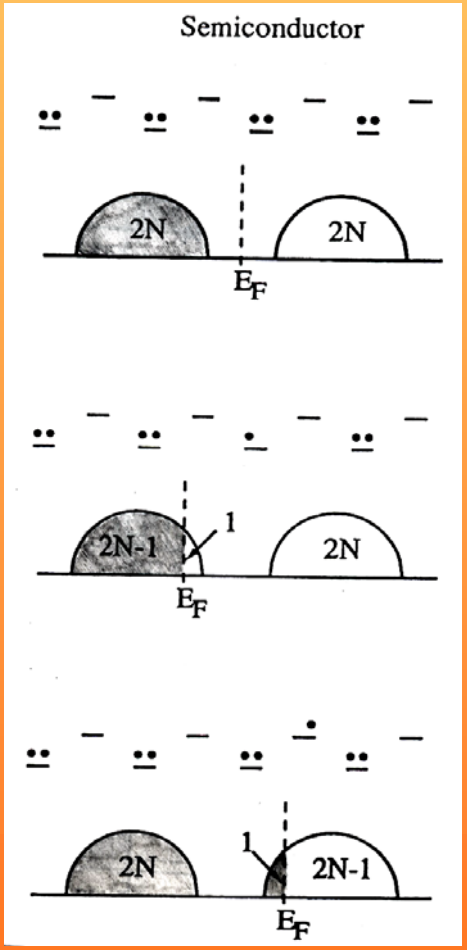
The 'frustration' means that at low energy, any change of state must re-organize simultaneously a vast number of states. This forces the Hilbert space of the effective Hamiltonian to have an ultrametric' geometry.

What this means is that no matter what energy or temperature one is working at, the ground state of the spin glass effective Hamiltonian is **meaningless.** At finite T, the system can never reach the ground state, and the finite-T Hilbert space is disconnected from any ground state. At zero-T, the system splits into subspaces that can never communicate with each other. Thus the effective vacuum & its structure are physically meaningless. A glass can only be defined by its dynamic (non-equilibrium) properties.



'Ultrametric geometry' of a glass Hilbert space

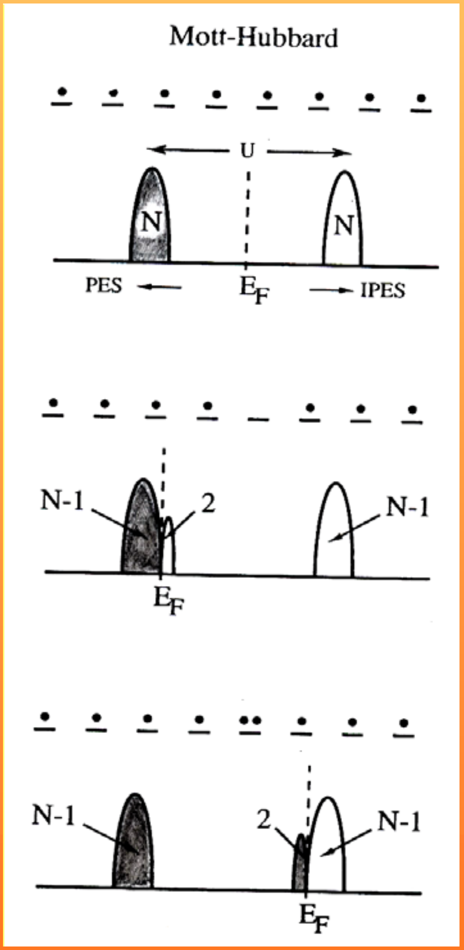
2ND CONUNDRUM- the HUBBARD MODEL



The 'standard model' of condensed matter physics for a lattice system is the 'Hubbard model', having effective Hamiltonian at electronic energy scales given by

$$H = -t \sum_{\langle i,j \rangle} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

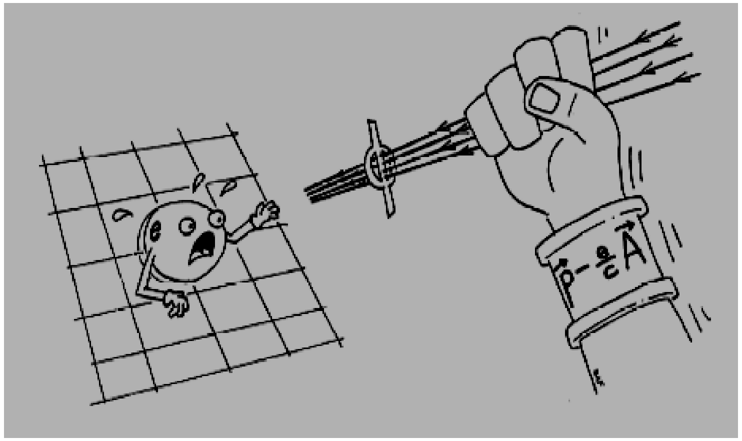
This apparently simple Hamiltonian has some very bizarre properties. Suppose we try to find a low energy effective Hamiltonian, valid near the Fermi energy- eg., when the system is near "half-filling". We therefore assume a UV Cutoff much smaller than the splitting U between the Mott-Hubbard sub-bands (we assume that U > t).



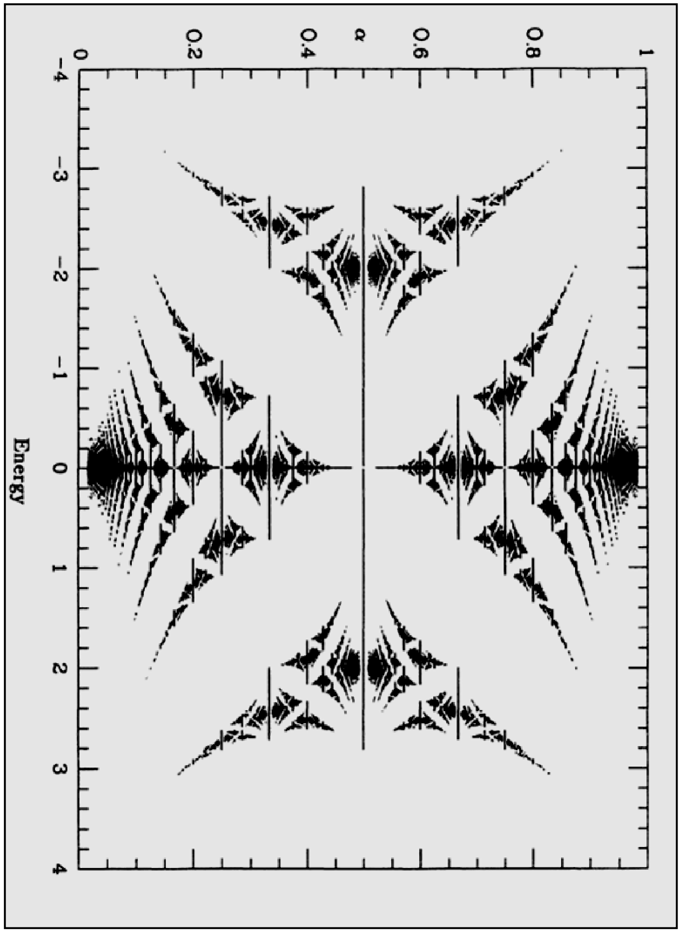
The problem is that this appears to be impossible. Any attempt to write an effective Hamiltonian around the Fermi energy must deal with 'spectral weight transfer' from the other Hubbard sub-band- which is very far in energy from the Fermi energy. Thus we cannot disentangle high- and low-energy states. This is sometimes called UV/IR mixing.

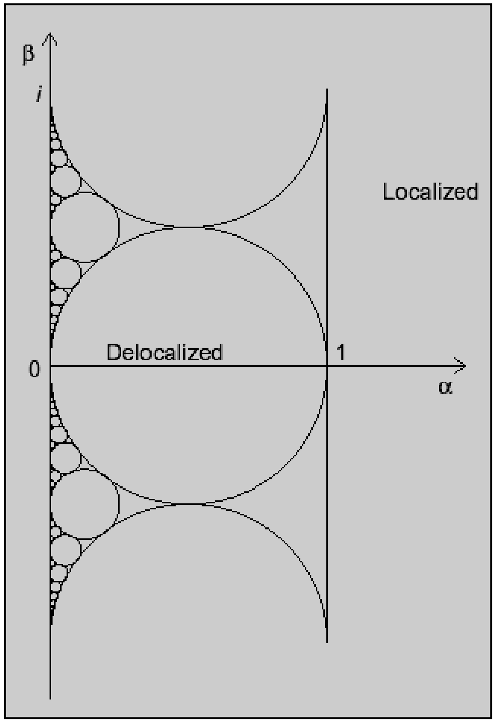
3RD CONUNDRUM- TOPOLOGICAL FIELD THEORIES

RIGHT: A statistical flux attaches itself to an electron to make an 'anyon'- here on a lattice



It is now apparent that the basic theories required in string theory & quantum field theory will be topological in nature. Theories like Chern-Simons theories have anyonic excitations & topologically different but degenerate vacua. In string theory it is hard to get rid of tachyons, which create the analogue of a lattice potential for the strings, leading to the complexity of the famous 'WAH' butterfly (left); once fluctuations & coupling to bosons are added, we get a fractal phase diagram →.



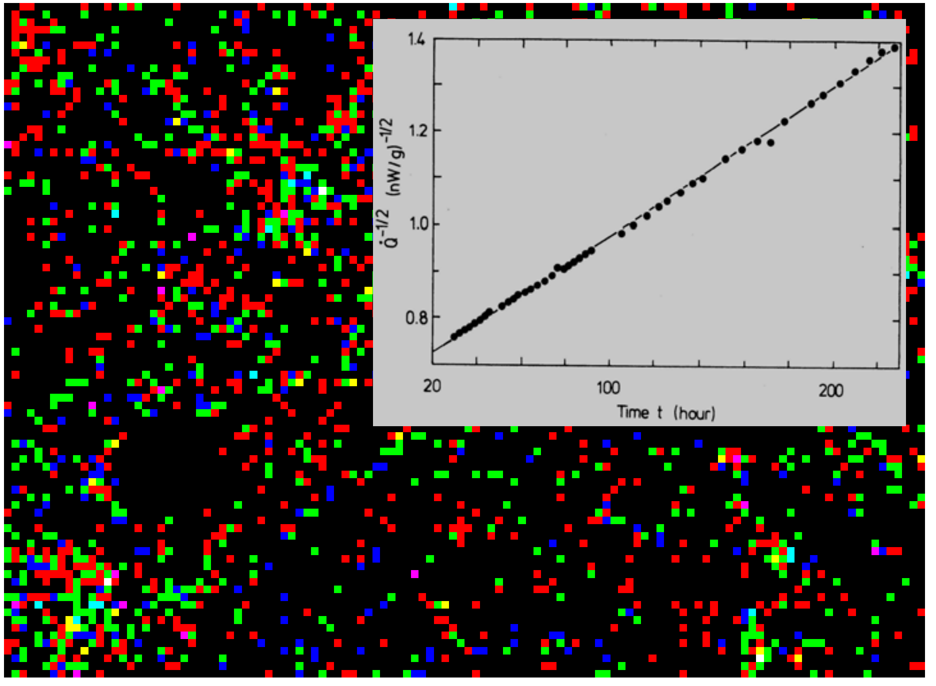


A key feature of all of these theories, and of any non-commutative gauge theory, is the same UV/IR mixing we saw in the Hubbard model- ie., no well-defined effective low-E action or Hamiltonian.

4^{TH} CONUNDRUM- ENTANGLEMENT, DECOHERENCE, etc

At this point it is useful to note that quantum entanglement makes the situation more complex. If we start off with 2 systems (A & B) that have once interacted, but are now separated, then their states are still entangled. It then makes no sense to write down 2 separate effective Hamiltonians, one for each- the complete description of A can not be given by a Hamiltonian which operates only in the Hilbert space of the variables of A, no matter what the cut-offs may be.

This problem takes on interesting features if one starts looking at the entangled system *par excellence,* a system of qubits (2-level systems) operating as a quantum computer. Not only are these tangled with each other, they are inevitably entangled with the rest of the universe (their 'environment'). Now it is an important result of recent research that that part of the environment that causes all the decoherence at low E is basically low-energy 'junk modes', which are themselves described in QM as a set of interacting 2-level systems, which have entangled or are entangling with the computer. The amusing feature of this is that the junk is itself largely immobile (essentially another Glass!) UNTIL it entangles with the computer. Then its phase dynamics is driven by the computer (with little or no energy dissipation), & it is irrevocably entangled with it.



REMARKS

R1: **One could argue that despite all this, the idea that we can still think of matter as made of 'elementary constituents' (the lego philosophy) is nevertheless intact.**

If so, one would like to know how to formulate this in physical theory- at the present time the fundamental formulation of the properties of any physical system is in terms of an effective Hamiltonian or effective action

R2: **Despite the literature and the fond beliefs of reductionists in the particle physics community, this is not just a problem of condensed matter physics- it arises in high energy physics as well.**

Notice that whereas the IR / UV mixing comes in in condensed matter systems typically in the presence of a lattice, this is not necessary- in non-commutative gauge theory or open string theory there is no lattice.

R3: **Some of the problems discussed so far exist in a classical theory. However features like IR / UV mixing seem to be quantum mechanical. And of course, the ineluctable role of entanglement is entirely a QM feature.**

Philosophers of physics (and also many theoretical physicists) might want to stop pretending that one can understand high-energy physics questions without reference to the low energy physics- and also stop imagining that 'complexity' & 'dirt' are somehow irrelevant to these questions. The basic message- high and low energy physics can't be separated, and also we can't ignore entanglement in the most basic formulation of our description. Many unsolved problems here!

TALK: see http://physics.ubc.ca/~stamp

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