Phys 523B: Fault tolerant quantum commputation

Homework 3

Posted: Monday, March 8, 2021 • Due: Monday, March 22, 2021, 6PM.

Please email your scanned or typeset assignment solutions to the TA Xiruo Yan: xyan@phas.ubc.ca

Problem 1 (5 points): (a) Work out the conjugation relations for Pauli observables under the 1-qubit Clifford gate $S = e^{-i\frac{\pi}{4}Z}$. (b) Show that the S-gate can be realized transversally in the 7-qubit Steane code.

Problem 2* (5 points): In the reference solution for Problem 3 of Homework 1 (Solution B) we used the fact that, for concatenation of the Shor code, the recursion relations for $q := p_x + p_y$, $\tilde{q} := p_z + p_y$ are decoupled. I.e., we arrived at two recursion relations that each involved only a single parameter.

Show that the same decoupling holds for the recursion relations of all concatenated CSS codes.

Problem 3 (5 points): Given a qubit encoded with the 7-qubit Steane code, an error-correcting measurement of the encoded Pauli operator Z can be achieved by measurement of the seven physical qubits in the Z-basis, and classical post-processing of the outcomes.

Describe the required classical post-processing of the 7 measurement outcomes.

Problem 4 (5 points): Describe the effect of the encoded circuits (a) and (b) drawn below, on the unknown quantum state $|\Psi\rangle$. Justify your answer. (The line striking through the qubit world lines means that the qubits are encoded. Here, the encoding is with a CSS code such as the Steane code.)

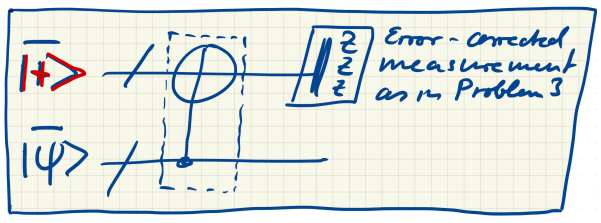

Figure A

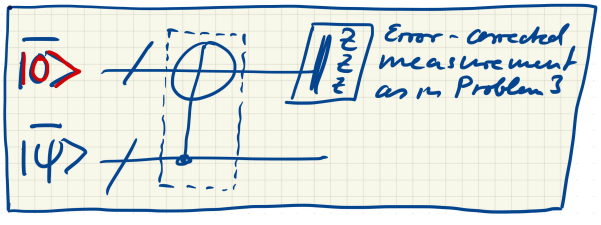

Figure B

Total: 20 points.