Phys 523B: Fault tolerant quantum commputation

Homework 2

Posted: Sunday, February 14, 2021 • Due: Monday, March 1, 2021, 6PM.

Please email your scanned or typeset assignment solutions to the TA Xiruo Yan: xyan@phas.ubc.ca

Problem 1 (5 points): The so-called graph state $|K_3\rangle$ corresponding to the 3-vertex complete graph K_3 is a stabilizer state on 3 qubits, defined by the stabilizer relations

$$\begin{aligned} X_1 \otimes Z_2 \otimes Z_3 |K_3\rangle &= |K_3\rangle, \\ Z_1 \otimes X_2 \otimes Z_3 |K_3\rangle &= |K_3\rangle, \\ Z_1 \otimes Z_2 \otimes X_3 |K_3\rangle &= |K_3\rangle. \end{aligned}$$

Compute the expectation values $\langle K_3|A|K_3\rangle$ for the Pauli observables

(a) $A = X_1 \otimes X_2 \otimes I_3$,

(b) $A = Y_1 \otimes Y_2 \otimes I_3$,

(c) $A = X_1 \otimes X_2 \otimes X_3$.

Problem 2 (5 points): In class we discussed the 5-qubit stabilizer code (tensor product symbols "⊗" are from now on suppressed when the meaning is clear from the context)

$$\mathcal{S}_5 = \langle Z_1X_2X_3Z_4, Z_2X_3X_4Z_5, Z_1Z_3X_4X_5, X_1Z_2Z_4X_5 \rangle. \tag{1}$$

(a) We observe that the stabilizer generators are cyclic translates of one another, i.e., they can be obtained from one another by permuting the qubits $1 \to 2 \to 3 \to 4 \to 5 \to 1$. However, one translate is missing from the above list of generators, namely $X_1X_2Z_3Z_5$.

Show that this last translate is dependent on the above generators,

$$X_1X_2Z_3Z_5 \in \mathcal{S}_5.$$

(b) Show that the 5-qubit stabilizer code $\mathcal{S}_5$ specified by Eq. (1) is not a CSS code.

Problem 3 (5 points): The weight of a Pauli operator is the size of its support, i.e., the number of qubits it affects in a non-trivial manner. For example, the Pauli operator $Y_1Y_2I_3$ from Problem 1 has weight 2 even though it is defined for a three-qubit system. The distance of a stabilizer code is the minimum weight of a Pauli operator that commutes with the code stabilizer but is not in the code stabilizer (not even up to sign).

What is the distance of the 7-qubit Steane code? Justify your answer.

Problem 4 (5 points): For the surface codes without boundary, prove the formula given in class that relates the number of encoded qubits to the genus g of the orientable surface on which the code is defined, namely

$$\# \text{ encoded qubits} = 2g.$$

Hint: You may find the Euler characteristic useful.

Total: 20 points.