Phys 523B: Fault tolerant quantum commputation

Homework 1

Posted: Sunday, January 24, 2021 • Due: Monday, February 8, 2021, 6PM.

Please email your scanned or typeset assignment solutions to the TA Xiruo Yan: xyan@phas.ubc.ca

Problem 1 (5 points): Show that the error recovery map $\mathcal{R}$ defined in class (definition attached for reference below) is a completely positive trace-preserving (CPTP) map.

Provisional definition. *An error recovery operation $\mathcal{R}$ for a quantum code with code space C is a quantum channel of the form*

$$\mathcal{R} = \sum_i [C_i \Pi_i], \tag{1}$$

where the projections Π_i are orthogonal, $\Pi_i \Pi_j = \delta_{ij} \Pi_i$, and satisfy the completeness condition $\sum_i \Pi_i = I$. The correction operations C_i are all unitary, and satisfy $C_i \Pi_i = \Pi\, C_i$.

Problem 2 (5 points): Give an example of a 2-qubit Pauli error that the 9-qubit Shor code can correct, and explain why it does so.

Problem 3 (5 points): Compute the concatenation threshold for the 9-qubit Shor code, assuming the error correction procedure is as discussed in class and the decoherence is described by independent stochastic Pauli noise $\mathcal{T}$ on each qubit,

$$\mathcal{T} = (1-p)[I] + \frac{p}{3}\left([X] + [Y] + [Z]\right).$$

Remark: These five points are relatively hard to earn. Fractional points will be given for reasonable approximations.

Problem 4 (5 points): Eq. (44) of the lecture notes describes the code size needed to reach a target logical error rate ϵ. It applies to concatenating a code that can correct for a single error. How does Eq. (44) generalize to quantum codes that correct up to e local errors instead of one?

Remark: We did not treat the subject of error suppression in finite codes in class (we only looked at the thermodynamic limit), and therefore, before attempting this question it is advised to read Section 2.3.2 of the lecture notes.

Total: 20 points.