Physics 503 Problem Set 6

Official due date: Thursday, April 5. An automatic extension is granted until Thursday, April 12.

1. Include the electron spin in the Shankar approach to interacting electrons in two dimensions with a circular Fermi surface.
a) Assuming that the effective action is invariant under SU(2) electron spin rotation, show that there are only two marginally relevant coupling functions in this case, are V_s and V_t, with:

$$S_{int} = \sum_{\alpha\beta\gamma\delta} \int_{K\omega\theta} \bar{\psi}_\alpha(1)\bar{\psi}_\beta(2)\psi_\gamma(3)\psi_\delta(4)$$
$$\cdot[\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}V_s(\theta_1-\theta_3) - (\sigma_2\vec{\sigma})_{\alpha\beta}\cdot(\vec{\sigma}\sigma_2)_{\gamma\delta}V_t(\theta_1-\theta_3)]. \tag{1}$$

Here $\int_{K\omega\theta}$ is defined in Shankar, Eq. (336) and we only keep the integration region where $\vec{K}_1 \approx -\vec{K}_2$ and $\vec{K}_3 \approx -\vec{K}_4$. $\epsilon_{\alpha\beta} = i(\sigma_2)_{\alpha\beta}$ where the σ_a are the Pauli matrices. (The subscripts s and t stand for spin singlet and triplet.)
b) What restrictions on the functions V_s and V_t follow from Fermi statistics?

2a) Calculate these functions by starting with the Hubbard model:

$$S_{int} = U\int d\tau \sum_i \bar{\psi}_{i,\uparrow}(\tau)\bar{\psi}_{i,\downarrow}(\tau)\psi_{i,\uparrow}(\tau)\psi_{i,\downarrow}(\tau), \tag{2}$$

and then eliminating all Fourier modes except those in the usual narrow band. (You can assume here that $k_F << 1/a$ where a is the lattice spacing, so that the Fermi surface is approximately circular.) You can assume two dimensions with a square lattice, but since the interactions are purely on-site, the choice of lattice doesn't play much role.
b) Calculate the one-loop RG equations for V_s and V_t. (Don't assume anything about the bare values of these couplings here; calculate the RG equations in general.)