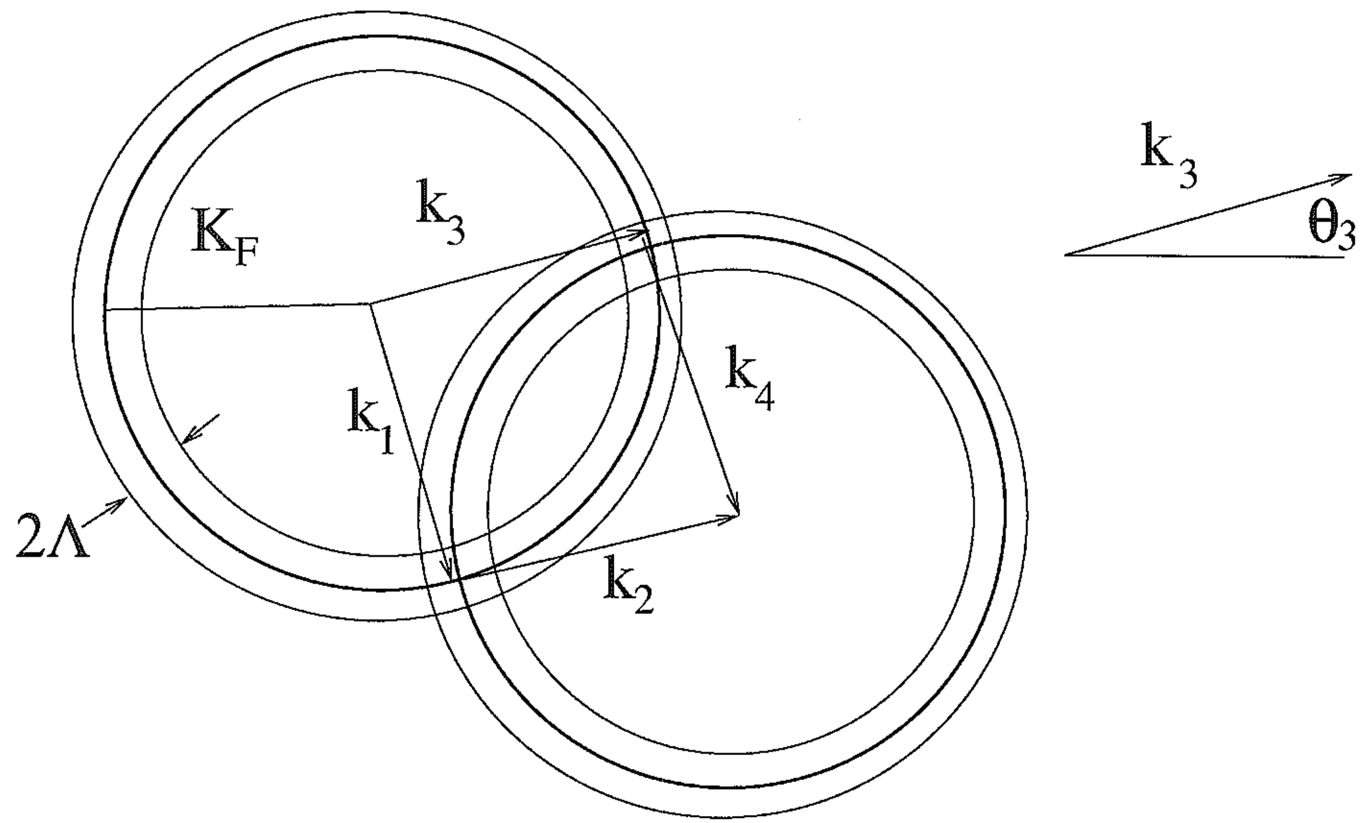
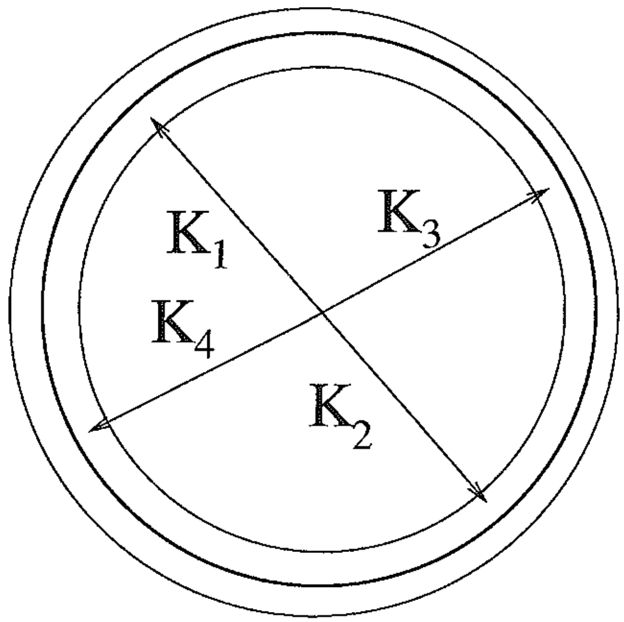
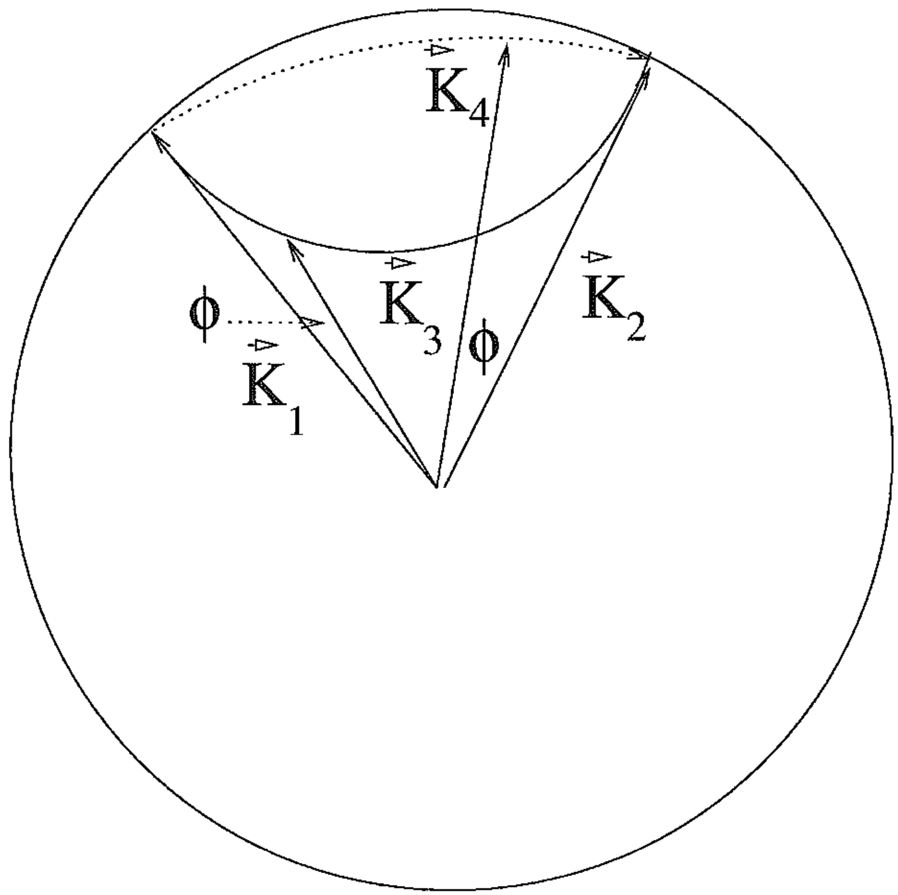
F- process, D=2



V-process, $D=2$



F- process, $D=3$



D=3 Case - Spherical Fermi Surface

again conditions $\vec{K}_1 + \vec{K}_2 = \vec{K}_3 + \vec{K}_4$ and $|\vec{K}_i| = K_F$ are very restrictive

- F type term can be more general
- $\theta_{34} = \theta_{12}$ as before but $\vec{K}_3$ and $\vec{K}_4$ could be rigidly rotated about axis of $\vec{K}_1 + \vec{K}_2$ by an angle ϕ as shown
- $\phi \equiv \phi_{12;34}$ is angle between plane of $(\vec{K}_1, \vec{K}_2)$ and plane of $(\vec{K}_3, \vec{K}_4)$

Coupling function F, can now depend on $Z_{12} \equiv \hat{\Omega}_1 \cdot \hat{\Omega}_2 = \cos\theta_{12}$ (angle between $\vec{K}_1, \vec{K}_2$) and $\phi_{12,34}$ $\quad F = F(Z_{12}, \phi_{12;34})$

- other possibility $\vec{K}_1 = -\vec{K}_2$; $\vec{K}_3 = -\vec{K}_4$

is essentially unchanged

V can depend on angle between $\vec{K}_1$ and

$\vec{K}_3$ only: $V(z_{13})$, $z_{13} = \hat{k}_1 \cdot \hat{k}_3 = \cos\theta_{13}$

- we can again calculate F and V

for cubic lattice tight-binding

model in limit of small K_F so

Fermi surface is approximately spherical

$u(\hat{k}_1, \hat{k}_2, \hat{k}_3, \hat{k}_4)$

$= u_0 (\hat{k}_1 - \hat{k}_2) \cdot (\hat{k}_3 - \hat{k}_4)$ as before

~~Fermi~~ F terms: $|\hat{k}_1 - \hat{k}_2| = |\hat{k}_3 - \hat{k}_4|$

$(\hat{k}_1 - \hat{k}_2) \cdot (\hat{k}_3 - \hat{k}_4) = |\hat{k}_1 - \hat{k}_2|^2 \cos\phi_{12;34}$

$= 2(1 - z_{12}) \cos\phi_{12;34}$

$F \propto (1 - z_{12}) \cos\phi_{12;34}$

V term: $\hat{\Omega}_1 = -\hat{\Omega}_2$, $\hat{\Omega}_3 = -\hat{\Omega}_4$

$(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot(\hat{\Omega}_3 - \hat{\Omega}_4) = 4\hat{\Omega}_1\cdot\hat{\Omega}_3 = 4 Z_{13}$

as before $V \propto Z_{13}$

- an important calculation that we can do with this effective Hamiltonian is the decay rate (at $T=0$) for a particle of momentum $\vec{K}_1$, slightly above the Fermi surface to decay into 2 particles and a hole

- let hole have momentum $\vec{K}_2$, particles have momenta $\vec{K}_3$ and $\vec{K}_4$

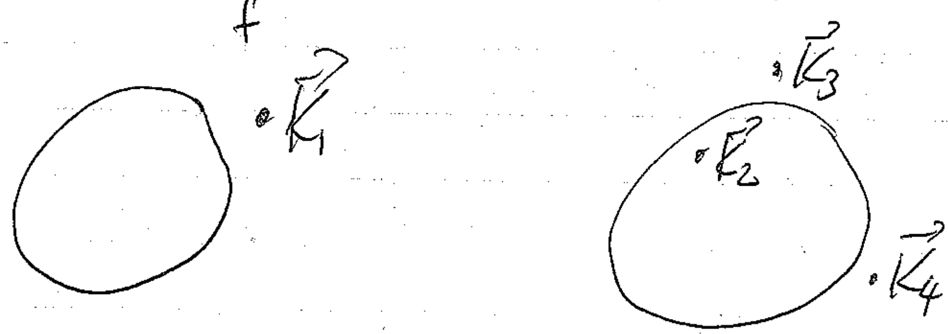
- $|k_2|$, $k_3, k_4 < k_1$ by energy conservation $V\cancel{(k_1 + k_2) = V(k_3 + k_4)}$

$$V k_1 = V(-k_2) + V k_3 + V k_4$$

$(k_2 < 0, \quad k_{3,4} > 0)$

- so we can do calculation using our low energy effective Hamiltonian
- let's assume $V = 0$ for now as it is for a Fermi liquid
- we calculate $\frac{1}{\tau}$ - decay rate to second order in F using Fermi's Golden Rule

$$\frac{1}{\tau} = 2\pi \sum_f |\langle f | H_{INT} | i \rangle|^2 \delta(E_f - E_i)$$



- this was probably 1^{st} step in Landau's development of Fermi liquid theory
- restricted phase space at low ϵ_1 makes $\frac{1}{T} \propto \epsilon_1^2$ so particle becomes infinitely long lived as $\epsilon_1 \to 0$ like a non-interacting electron

$$\frac{1}{T} = 2\pi \frac{1}{2!} \int \frac{d^3K_2\, d^3K_3\, d^3K_4}{(2\pi)^9\, K_F^4} \left|F(\theta, \phi)\right|^2$$

$$\delta^3(\vec{K}_1 + \vec{K}_2 - \vec{K}_3 - \vec{K}_4)\, \delta(\mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_3 - \mathcal{E}_4)$$

θ = angle between $\vec{K}_1, \vec{K}_2$; $\phi = \phi_{12;34}$

NB - this process is equivalent to an excited particle of momentum $\vec{K}_1$ colliding with a particle inside Fermi sea, of momentum $\vec{K}_2$, with final 2 particles, above Fermi sea, at

$\vec{K}_3$ and $\vec{K}_4$

- we must have $K_2 < K_F$; $K_3, K_4 > K_F$ at $T=0$ (to lowest order in interactions) due to "Pauli blocking"

$\frac{1}{2!}$ factor arises due to 2 final particles being identical

- $\frac{1}{K_F^4}$ due to the way we defined interactions in terms of fields rescaled by K_F

- I am ignoring spin here but result is very similar including it

- if we do $\vec{K}_4$ integral we get

$$\frac{1}{T} = \frac{1}{(2\pi)^5} \frac{1}{2!} \frac{1}{K_F^4} \int d^3K_2 \, d^3K_3 \; F(\theta,\phi)^2$$
$$\delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4)$$

~~θ~~ is angle between $\vec{K}_1$ (excited particle) and $\vec{K}_2$ (hole)

$\phi = \phi_{12;34}$

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$\int d^3\vec{K}_2 \to K_F^2 \int dk_2 \, d\theta \sin\theta \cdot 2\pi$

where 2π comes from integrating over azimuthal ~~polar~~ angle of $\vec{K}_2$ with axis $\vec{K}_1$

$\int d^3K_3 = K_F^2 \int dk_3 \; d\theta_3 \sin\theta_3 \; d\phi$

- it is convenient to measure θ_3 from direction of $\vec{K}_1 + \vec{K}_2$, $\theta_3 \simeq \theta/2$ where θ is angle between $\vec{K}_1$ and $\vec{K}_2$ [see figure]

$\int d\theta_3$ integral is strongly restricted by condition that $k_4 < k_1$

- it is convenient to change variables

$d\theta_3 \to dk_4$

- from figure $\delta k_4 = K_F \sin\theta \, \delta\theta_3$

$$d\theta_3 \to \frac{1}{K_F \sin\theta} dk_4$$

$$\frac{1}{T} = \frac{1}{(2\pi)^7} \frac{1}{2!} \int_{-k_1}^{0} dk_2 \int_0^{k_1} dk_3 \int_0^{k_1} dk_4$$

$$\delta\left[V(k_1+k_2-k_3-k_4)\right]$$

$$\int_0^{\pi} d\theta \int_0^{2\pi} d\phi \, \frac{\sin\theta}{2} \left|F(\theta,\phi)\right|^2$$

- we may do k-integrals exactly

$$\int_{-k_1}^{0} dk_2 \int_0^{k_1} dk_3 \int_0^{k_1} dk_4 \, \delta\left[V(k_1+k_2-k_3-k_4)\right]$$

$$= \frac{1}{V} \int_{-k_1}^{0} dk_2 \int_0^{k_1} dk_3 \, \Theta(k_1+k_2-k_3)$$

[recall $k_2 < 0$]

$$= \frac{1}{V} \int_{-k_1}^{0} dk_2 \int_0^{k_1+k_2} dk_3 = \frac{1}{V} \int_{-k_1}^{0} dk_2 \, (k_1+k_2)$$

$$= \frac{1}{2V} k_1^2$$