Now consider one of additional interactions generated by previous RG calculation

$$\int d\tau \, \bar{\psi} \, \vec{\sigma} \, \frac{d\psi}{dt} \cdot \vec{S}$$

$$= 2\pi \int \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} \frac{dk \, dk'}{(2\pi)^2} \; \bar{\psi} \, \vec{\sigma} \, \psi \cdot \vec{S} \; i(\omega' - \omega)$$

NB - extra factor of $(\omega' - \omega)$

- under RG rescaling $\omega \to \frac{\tilde{\omega}}{s}$

so this term in S_{eff} is not invariant under RG rescaling but instead gets multiplied by $\frac{1}{s} < 1$

- if we write corresponding coupling constant as λ_1

$$\lambda_1 \to \frac{\lambda_1^{(0)}}{s} = \lambda_1(s)$$

$$s \frac{d\lambda_1}{ds} = -\lambda_1(s)$$

- different than Kondo coupling

$$\frac{d\lambda}{ds} = \lambda(s)^2 \text{ - no linear term}$$

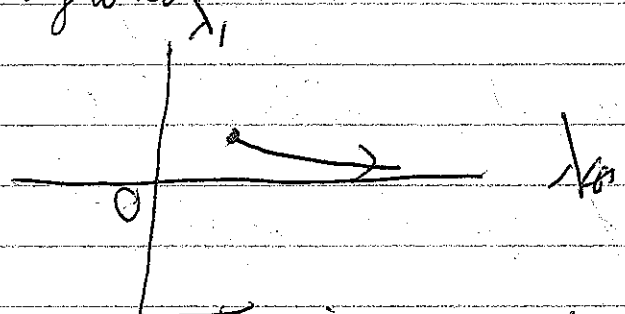
- similarly, higher order terms in S_{eff} have even more time derivatives $\Rightarrow$ powers of frequency, so they drop off with higher powers of $1/s$
- in general, at some stage in RG process we have ∞ number of coupling constants
- β-functions all coupled
- some coupling constants have linear terms in β-function

$$\frac{d\,\cancel{\beta_i}}{d\ln s} = f_i(\cancel{\beta_1, \beta_2, \beta_3, \ldots}) \quad \lambda_1 \lambda_2 \ldots$$

$$= X_i \lambda_i + O[\lambda^2]$$

- at least in regime where all couplings are small, couplings with negative linear terms in β-function flow to zero quickly [powers of $1/s$] whereas

effects of higher order terms are slower
[logarithmic]



- estimate of T_K is essentially unchanged.
- universal trajectory from zero coupling to strong coupling fixed points is followed in limit of very small bare couplings
- some models (not Kondo) have coupling constants with positive X_i

$X_i < 0$ — irrelevant
$X_i > 0$ relevant
$X_i = 0$ marginal

- relevant couplings become large quickly & dominate RG flow, if they exist
- marginal couplings also flow but more

slowly - we must look at quadratic terms in β - function to see if they are marginally relevant or irrelevant (depends on sign of bare coupling)

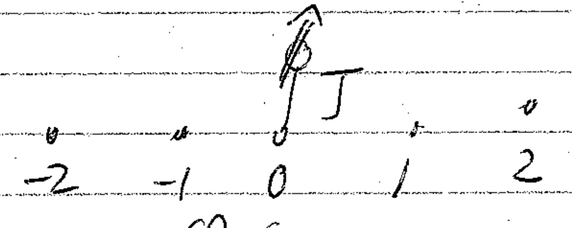
- we generally expect an ∞ number of coupling constants - all terms allowed by symmetry
- "anything that can happen will happen"

<u>Low Energy Fixed Point of Kondo model</u>

- for AF Kondo coupling $\lambda(D)$ becomes $O(1)$ at $D \sim T_K$
- perturbation theory fails at lower energies & perturbative RG isn't enough
- Naively we might guess $\lambda \to \infty$ at $E \to 0$
- what would this imply for resistivity?

$\rho \to \infty$ - seems unlikely (impossible) for dilute impurities

- to get some intuition it is very convenient to study a tight-binding model, eg. in 1D



$$H = -t \sum_{\substack{i=-\infty \\ \alpha}}^{\infty} \left(c^{\dagger}_{i\alpha} c_{i+1\,\alpha} + h.c. \right) + J\, c_0^{\dagger} \frac{\vec{\sigma}}{2} c_0 \cdot \vec{S}$$

- impurity couples to site 0 only
- we are really interested in case where bare coupling $J \ll t$, but since λ may flow to ∞ at low E it is interesting to consider model with <u>large</u> <u>bare</u> coupling $J \gg t$
- remarkably, we can easily solve it exactly in that limit!

(- for simplicity let's consider model at $\frac{1}{2}$-filling)

$J \gg t$ is equivalent to $t \to 0$

$$H = J \; c_0^\dagger \frac{\vec{\sigma}}{2} c_0 \cdot \vec{S}$$

$$= J \; \vec{S}_0 \cdot \vec{S}$$

- 8 states (4×2)

- $$[\hat{n}_0, H] = 0$$

- for $n_0 = 0$ or 2, $\vec{S}_0 = 0$ - spin-zero state

$E_0 = E_2 = 0$ - doublets

- with $n_0 = 1$, $S_0 = 1/2$

- H is Hamiltonian for 2 coupled $S = \frac{1}{2}$'s

$$H = J \left[\frac{1}{2} (\vec{S} + \vec{S}_0)^2 - 3/4 \right]$$

$$= J \left[\frac{1}{2} S_T (S_T + 1) - 3/4 \right]$$

$$= -\frac{3J}{4}, \quad S_T = 0$$

$$= \frac{J}{4}, \quad S_T = 1$$

$\Rightarrow$ ground state has 1 electron at origin forming spin singlet with impurity

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left[C^\dagger_{0\uparrow}|0\rangle\,|\downarrow\rangle - C^\dagger_{0\downarrow}|0\rangle|\uparrow\rangle \right]$$

- other states have energies higher by $O(J)$
- what do other electrons do?
- at $t=0$, anything they like except enter site 0
- for small $t \ll J$, can do degenerate pert. theory
- other electrons form a free electron ground state [filled Fermi sea] with wave-functions vanishing at $j=0$
- even parity wave-functions modified from $\phi_i^{(e)} \propto \cos ki$, $(J=0)$ to

$\sin k|i|$, $J \to \infty$

- odd parity $\phi_i^{(o)} = \sin ki$ unchanged

- phase shifts

$$\varphi_i^{(e)} = \cos[ki + \delta^{(e/o)}] \qquad \boxed{\delta^{(e)} = \pi/2}$$

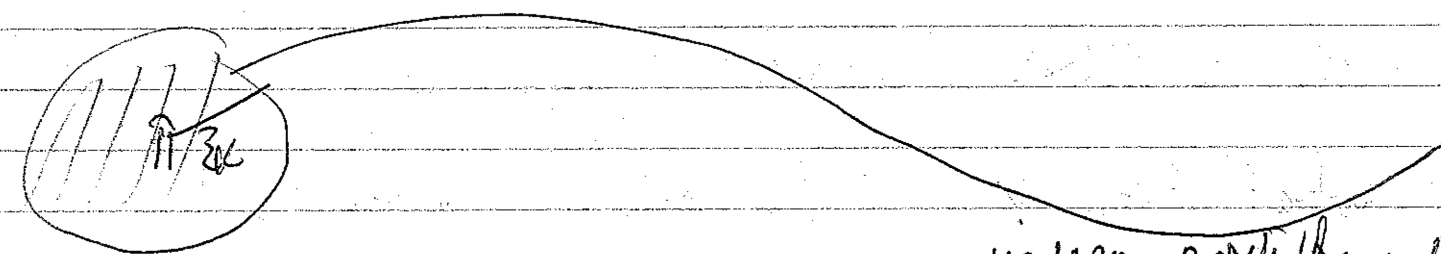
$$\delta^{(o)} = 0$$

- how might we apply this intuition to interesting case $J \ll t$?
- ie bare ~~to~~ dimensionless coupling small but renormalized coupling at energy scale $\lesssim T_K$ large
- perhaps lower energy behavior is similar to above description
- we might imagine an electron forms a spin singlet with impurity – "Kondo screening" but there is no reason to think that electrons sits at $i = 0$
- only low energy electrons, $|\epsilon| \lesssim T_K$ are strongly perturbed by Kondo interaction

- these have wave-vector $|k-k_F| \lesssim \cancel{\frac{V_F}{T_K}}\ \frac{T_K}{V_F}$
- a screening wave-function constructed from such a small band of wave-vectors would be spread out over a distance of order $\xi_K \equiv \frac{V_F}{T_K}$ [e would oscillate at k_F]

$\xi_K \sim a e^{1/J} \gg a$ is a very large scale [could be microns in expt.]

we might speculate that studying a Kondo impurity with a very long wavelength probe $\lambda \gg \xi_K$ would see the $\pi/2$ phase shift



- free electrons with $\pi/2$ phase shift in even parity channel could be universal description of low energy, long distance physics

- but more complicated physics must occur at intermediate energy and/or lengthscales
i.e. a conjecture for low energy fixed point Hamiltonian is free electrons with impurity spin removed and $\pi/2$ phase shift in even parity channel (corresponding to a vanishing boundary condition at origin)
- let's assume this is correct and explore its consequences
- we can then check if they agree with expts. & other theoretical calculations
- what does this assumption predict for resistivity at $T=0$?
- should be same as that of a random array of potential scatterers with very

short-range repulsion, corresponding to a $\pi/2$ phase shift in s-wave channel (only)

N.B. - extending strong coupling solution to 3 D model we only get $\pi/2$ phase shift in s-wave - short-range interaction doesn't affect other harmonics since wave functions vanish at origin

- such a model gives largest possible resistance from potential scatterers of given (low) density n_{imp} - "unitary limit"

- T-matrix for potential scattering is

$$T = +\frac{i}{2\pi\nu}\left(e^{2i\delta}-1\right)$$ for s-wave

phase shift δ [see Mahan, Chap 4]

$$T = -\frac{i}{\pi\nu} \quad \text{for } \delta = \pi/2.$$

- ~~conductivity~~ resistivity can be expressed in terms of

Im (T) as before

- in general $\frac{1}{2\tau} = -\mathrm{Im}(T) = \frac{1}{\pi \nu} \sin^2 \delta$

- largest scattering rate for $\delta = \pi/2$

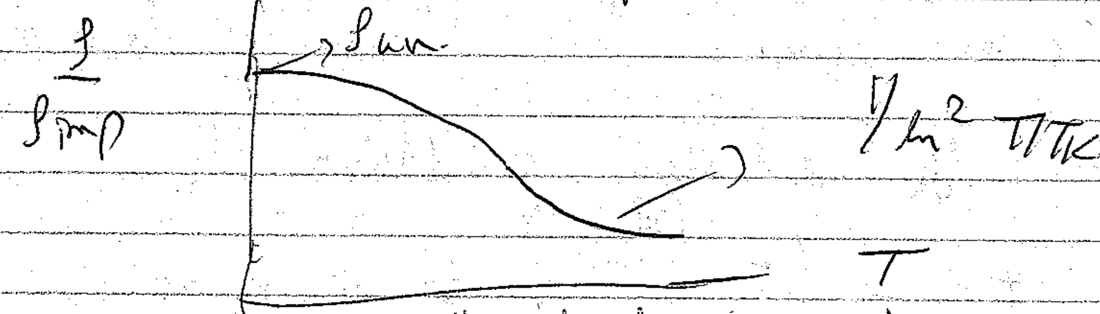
"unitary limit" $T \sim -i(1-S)$

S-matrix $e^{2i\delta}$ must be unitary

- from $\frac{1}{\tau}$ we get resistivity from ~~Bltz~~ Kubo formula as before

$\rho = \frac{3}{2\pi e^2 v_F^2 \nu} \rho_{imp}$ - finite value at $T \to 0$

$= \rho_{un} \cdot \rho_{imp}$



- agrees with other ~~results~~ theoretical approaches & experiments