Majorana Zero Modes in a Proximity-induced Superconducting Quantum Wire

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Outline

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2. Background Theory
   a. Cooper pairing in S-wave superconductors
   b. Majorana Zero Modes
   c. Non-Abelian Braiding of Majorana
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Motivation

- Advantage of Quantum Computers
  - Decoherence in conventional quantum computers

- Topological Insulators as **topologically-protected** quantum computers
  - Physical fault-tolerance vs. correction coding

- Quasiparticle excitations in lower dimensional topological insulators
  - **Majorana Zero Modes**
  - **Non-abelian Statistics**

Motivation

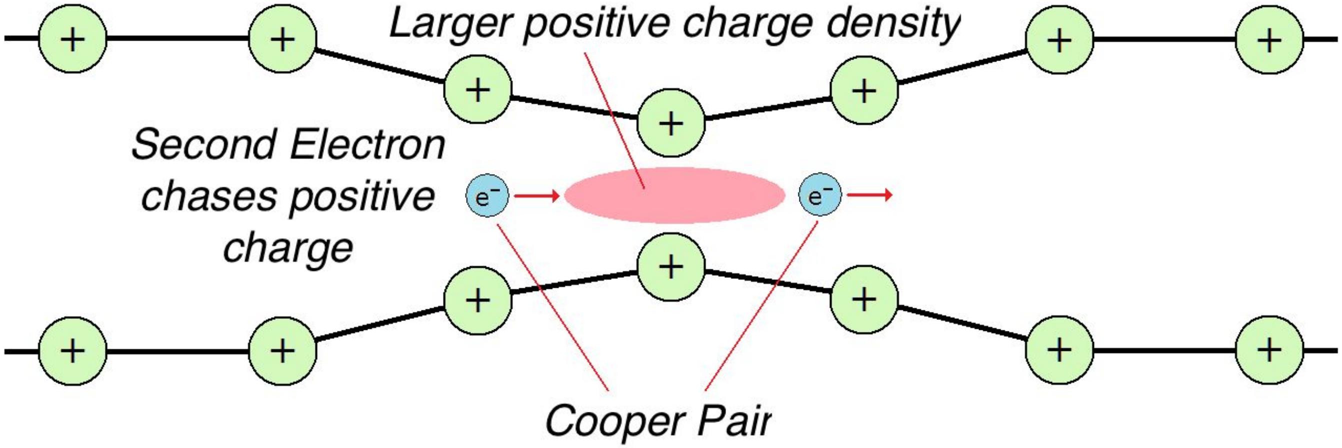
- Previous successful theoretical work
  - Kitaev (2001)
    - Majorana zero modes in a p-wave superconducting wire
  - Fu and Kane (2007)
    - Majorana bound states in a chiral p_x+ip_y 2D superconductor

- What's the problem?
  - P-wave superconductors theoretically exist, but are either extremely rare or non-existent

- Can we find a similar system using regular s-wave superconductors?

Theory: Cooper Pairing

- BCS Theory of Superconductivity
  - Long-range **attractive** force between electrons near Fermi surface



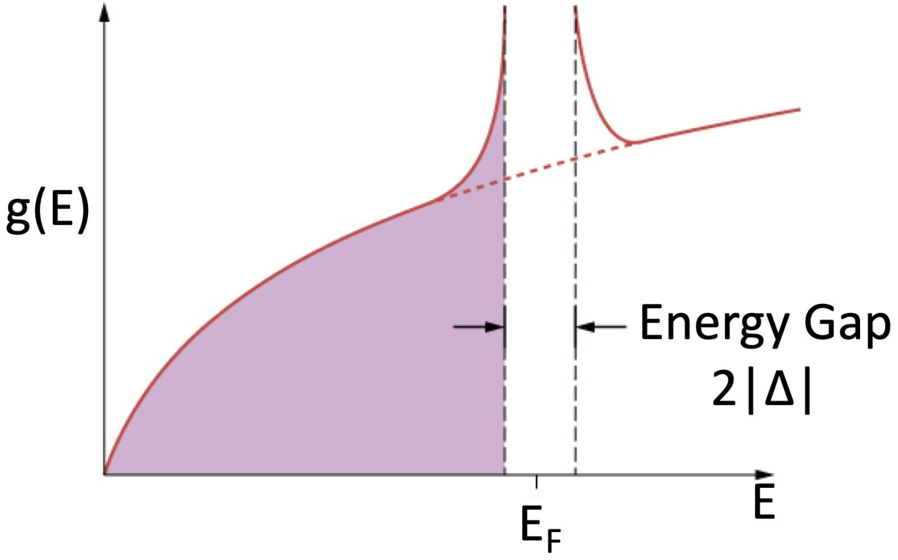
Theory: Cooper Pairing

- S-wave: Cooper pair in spin-singlet state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$

Spin 0

- Form Bose-Einstein condensate
  - Energetically favourable
  - **superconducting gap Δ**



Theory: Majorana Zero Modes

- Excitations in superconducting systems
  - Coherent superpositions of particles and holes

$$\mathcal{H} = \int d\mathbf{r} \sum_{\sigma=\uparrow,\downarrow} (H_o(\mathbf{r})\alpha_\sigma^\dagger(\mathbf{r})\alpha_\sigma(\mathbf{r})) + \Delta\alpha_\uparrow^\dagger(\mathbf{r})\alpha_\downarrow^\dagger(\mathbf{r}) + \Delta^*\alpha_\downarrow(\mathbf{r})\alpha_\uparrow(\mathbf{r})$$

Single particle H (KE + PE)

Creates electron at **r** with spin up (or annihilates a hole)

Annihilates electron at **r** with spin up (or creates a hole)

Theory: Majorana Zero Modes

- Is there an easier way?
  - Described by **Bogoliubov-de Gennes (BdG)** equation

$$\boxed{E_n \Phi_n = \mathcal{H}_{\mathrm{BdG}} \Phi_n}$$

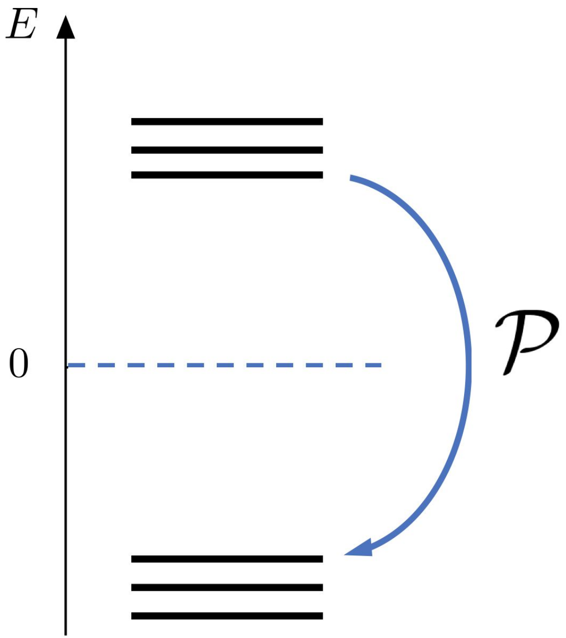
Related to $\alpha / \alpha^\dagger$

$$\mathcal{H} = \begin{bmatrix} H_o & -i\sigma^2 \Delta \\ i\sigma^2 \Delta^* & -H_o^* \end{bmatrix} \qquad \Phi_n = \begin{pmatrix} a_\uparrow \\ a_\downarrow \\ a_\uparrow^\dagger \\ a_\downarrow^\dagger \end{pmatrix}$$

Theory: Majorana Zero Modes

- **Particle-hole symmetry**: Unitary mapping between states with positive energy E and negative energy -E

$$\Phi_{-E_n} = \mathcal{P}\Phi_{E_n} \qquad \mathcal{P}\mathcal{P}^\dagger = \mathbb{I}$$

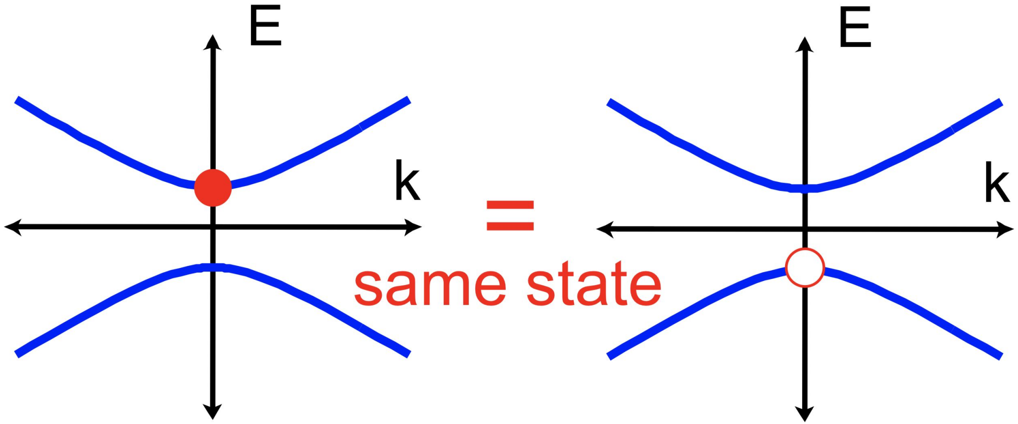


- Excitations must come in pairs!

Theory: Majorana Zero Modes

- What about E = 0?
  - Special state possible: **Majorana Zero Mode**

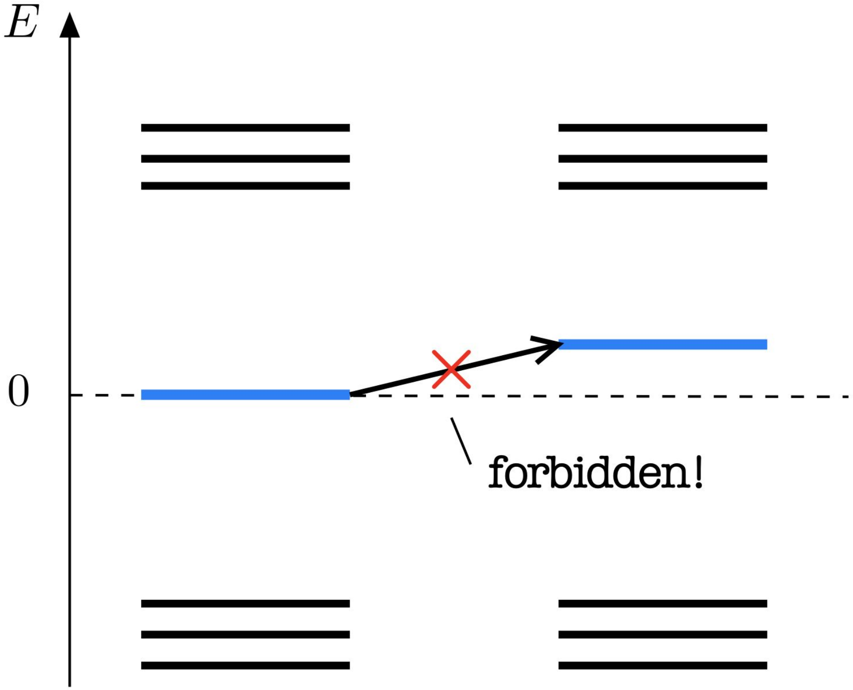
$$\alpha_\sigma^\dagger \equiv \alpha_\sigma$$



- Does not support an energy pair as before

Theory: Majorana Zero Modes

- These modes are **topologically protected** from decoherence
  - Local perturbations cannot influence E = 0 state
  - Change in topology of system required



Theory: Non-abelian Braiding

- In three dimensional systems, particles obey either Fermi-Dirac or Bose-Einstein Statistics

$$|\psi_1, \psi_2, ..., \psi_N\rangle = \pm|\psi_2, \psi_1, ..., \psi_N\rangle$$

- In lower dimensional systems, particles can obey different statistics

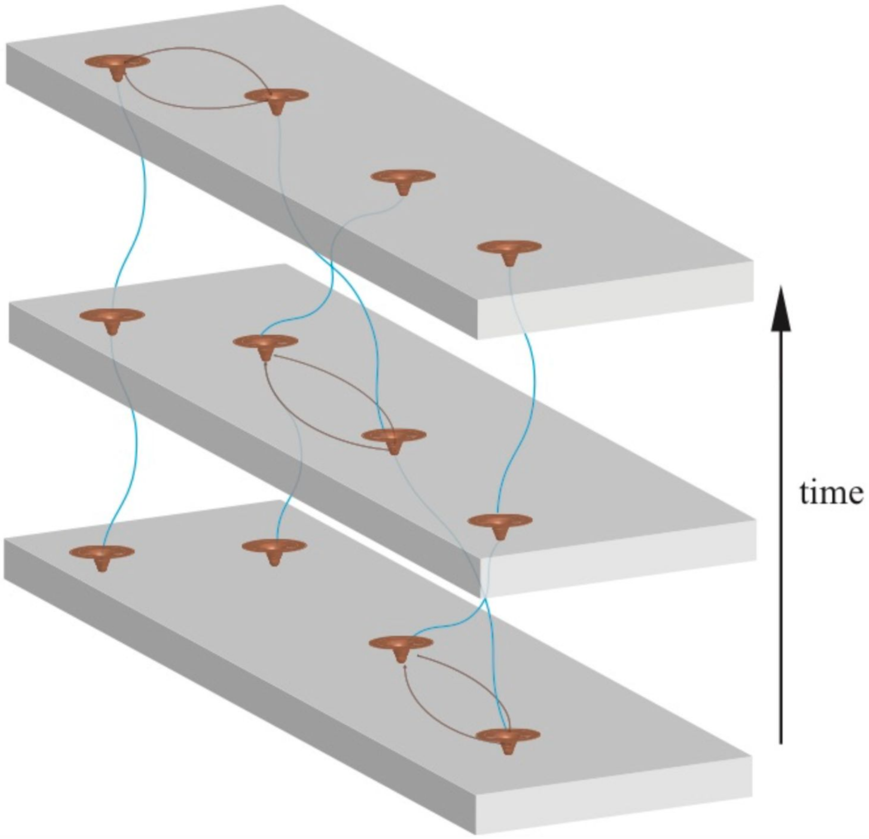
$$|\psi_1, \psi_2, ..., \psi_N\rangle = e^{i\theta}|\psi_2, \psi_1, ..., \psi_N\rangle$$

  - Arbitrary phase change

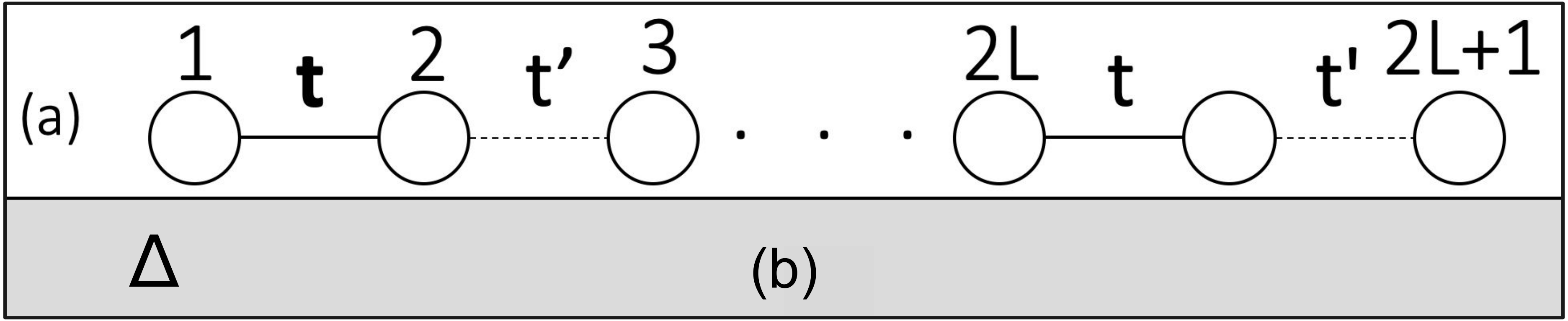
Theory: Non-abelian Braiding

- If these systems have degeneracy, particle exchanges can lead to **state exchanges**
  - Non-commuting unitary operations! **Order of operation matters**

$$U_1 U_2 |\Psi\rangle \neq U_2 U_1 |\Psi\rangle$$



Project Details



(a) 1D lattice with an odd number of sites (Tight-binding Model)

(b) 3D s-wave superconductor

Project Details

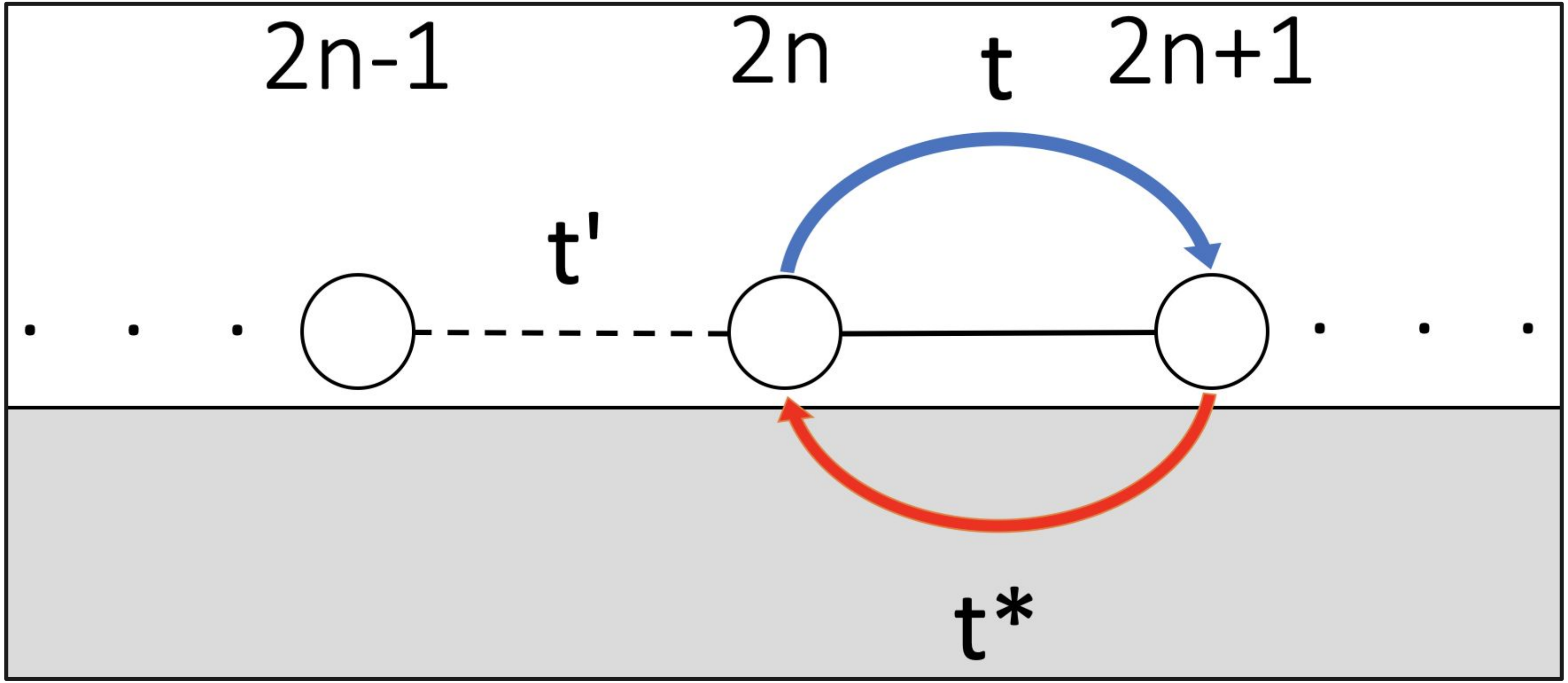
$$\begin{aligned}\mathcal{H} &= \sum_{\sigma=\uparrow,\downarrow} \sum_{n=1}^{L} \left(t\alpha_{2n+1,\sigma}^{\dagger}\alpha_{2n,\sigma} + t^{*}\alpha_{2n,\sigma}^{\dagger}\alpha_{2n+1,\sigma} \right) \\ &+ \sum_{\sigma=\uparrow,\downarrow} \sum_{n=1}^{L} \left(t'\alpha_{2n,\sigma}^{\dagger}\alpha_{2n-1,\sigma} + t'^{*}\alpha_{2n-1,\sigma}^{\dagger}\alpha_{2n,\sigma} \right) \\ &+ \sum_{n=1}^{2L+1} \left(\Delta\alpha_{n,\uparrow}\alpha_{n,\downarrow} + \Delta^{*}\alpha_{n,\downarrow}^{\dagger}\alpha_{n,\uparrow}^{\dagger} \right)\end{aligned}$$

$$\{\alpha_{n,\sigma}, \alpha_{m,\rho}^{\dagger}\} = \delta_{nm}\delta_{\sigma\rho} \qquad \alpha_{0,\sigma} = \alpha_{2L+2,\sigma} = 0$$

$$\{\alpha_{n,\sigma}, \alpha_{m,\rho}\} = 0 = \{\alpha_{n,\sigma}^{\dagger}, \alpha_{m,\rho}^{\dagger}\}$$

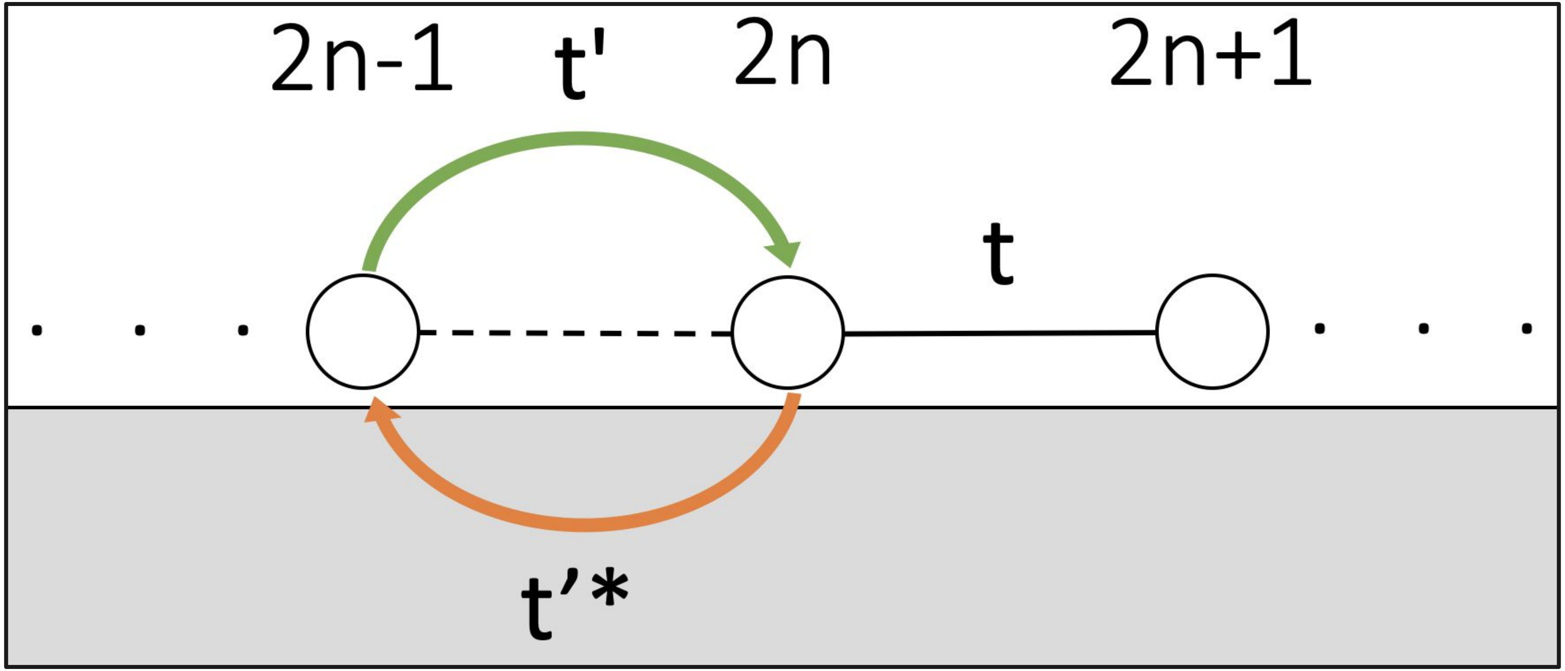
Project Details

$$\mathcal{H} = \sum_{\sigma=\uparrow,\downarrow} \sum_{n=1}^{L} \left(t\alpha^{\dagger}_{2n+1,\sigma}\alpha_{2n,\sigma} + t^{*}\alpha^{\dagger}_{2n,\sigma}\alpha_{2n+1,\sigma} \right)$$



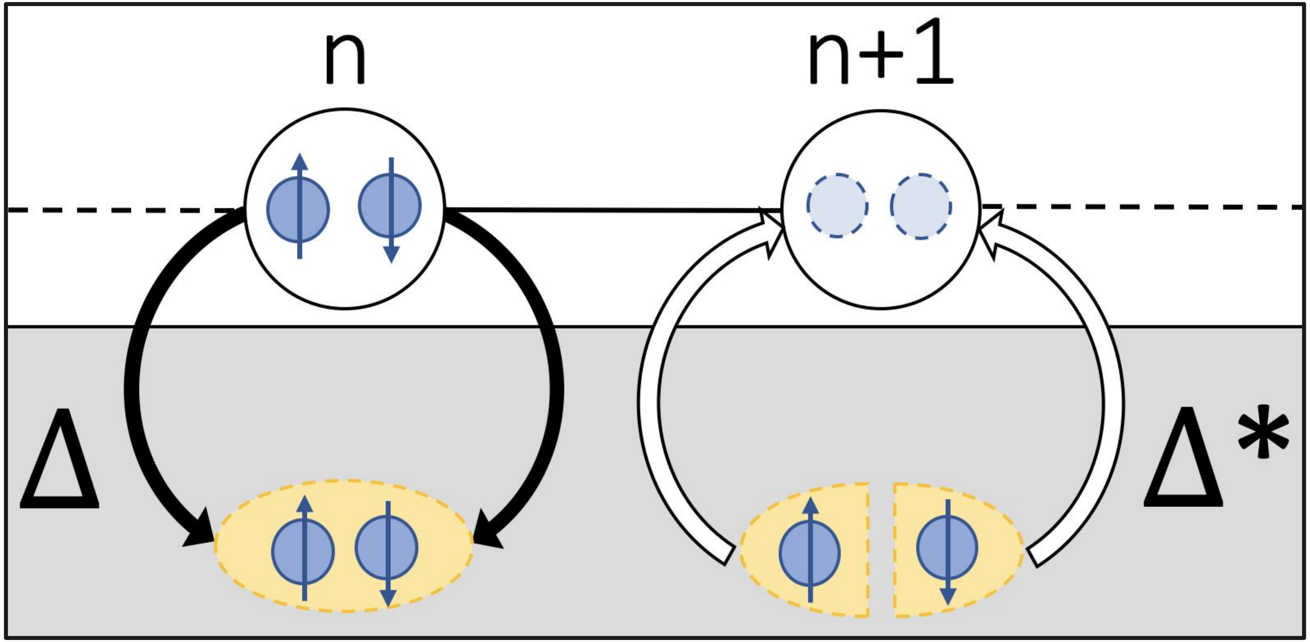
Project Details

$$+\sum_{\sigma=\uparrow,\downarrow}\sum_{n=1}^{L}\left(t'\alpha^{\dagger}_{2n,\sigma}\alpha_{2n-1,\sigma}+t'^{*}\alpha^{\dagger}_{2n-1,\sigma}\alpha_{2n,\sigma}\right)$$



Project Details

$$+\sum_{n=1}^{2L+1}\left(\Delta a_{n,\uparrow}a_{n,\downarrow}+\Delta^{*}a_{n,\downarrow}^{\dagger}a_{n,\uparrow}^{\dagger}\right)$$



Project Details

- Three important features of the system
  1. Odd-dimensional Hilbert space (single particle states)
  2. Single particle positive and negative energy states
  3. Particle-hole symmetry

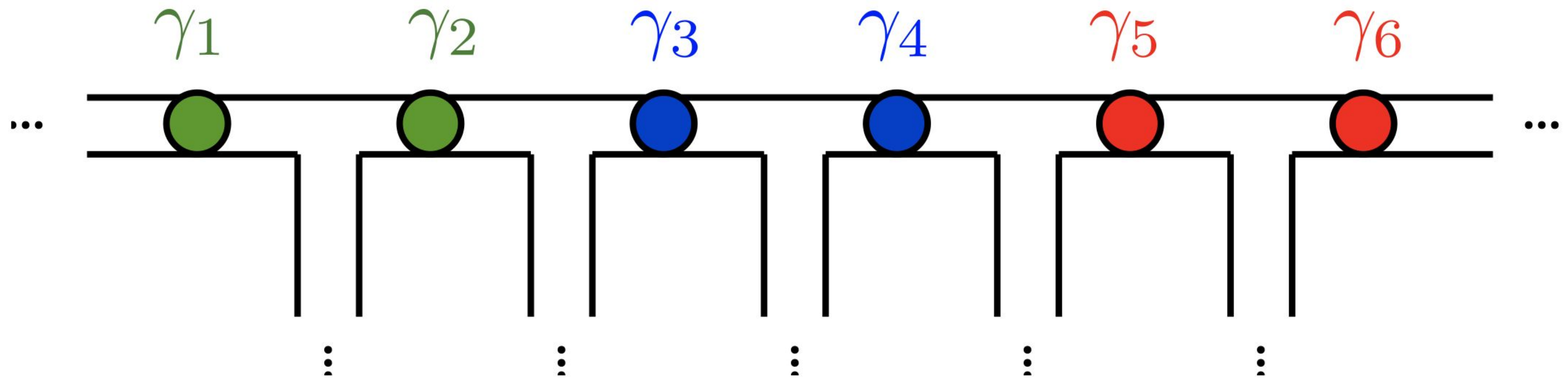
  $\Rightarrow$ Unpaired zero energy state

- ‘Topological’ attribute of the system

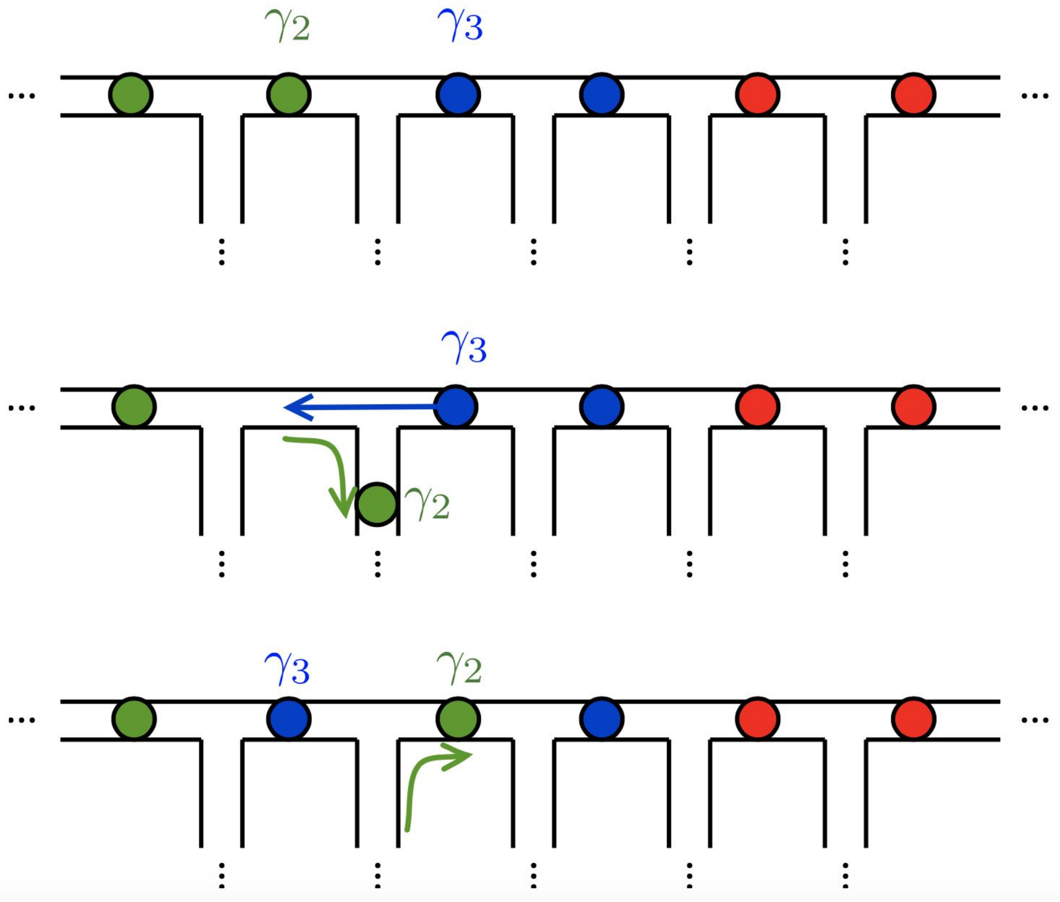
Project Details

- What will we do with this system?
  - Solve for energy eigenstates via a BdG equation
    - Analyze Majorana character of E = 0 state

- Braiding operations between Majorana of neighbouring wires



Project Details



Summary

- Topological Insulators may form the basis for topologically-protected quantum computing
  - Braiding operations of Majorana Zero Modes

- Physically-realizable system in which Majorana modes are possible
  - Analyze energy eigenstates of system + majorana character
  - Develop computation algorithms