Physics 410
Assignment #6: Due Monday, November 4, 2013

1) Simulate the process of cooling a nuclear fuel rod assembly by immersing it in a cooling pool. Assume that the cooling pool is water maintained at a temperature of 50 °C. The rod assembly is made of small rods of diameter of 15 mm and length of 493 mm bundled into a 28-element circular array of overall diameter 102 mm and length 495 mm. (i.e. it is a CANDU 28 element fuel rod). The rods consist of Uranium Oxide pellets with a thin Zircaloy-4 cladding.

Assuming that the system is isotropic, the temperature of a fuel rod evolves according to

$$\frac{1}{\kappa}\frac{\partial T(\vec{r},t)}{\partial t} - \nabla^2 T(\vec{r},t) = S(\vec{r},t)$$

where the diffusivity $\kappa = \sigma/(c\rho)$ where σ is the conductivity, c the specific heat and ρ the density of the material and the effective source $S = Q/\sigma$ where $Q(\vec{r},t)$ is the heat released into the system per unit volume per unit time at position $\vec{r}$ and time t. If one models this problem as a two dimensional system with circular symmetry, the laplacian reduces to

$$\nabla^2 T(r,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(r,t)}{\partial r}\right) \ .$$

Note that at $r = 0$, energy cannot flow into or out of the problem; consequently

$$\left.\frac{\partial T(r,t)}{\partial r}\right|_{r=0} = 0 \ .$$

For this symmetry $Q(\vec{r},t) = Q_0(1 - \frac{r^2}{R^2})\exp(-\lambda t)$ for $r < R$, R the outer radius of the fuel rod.

Specifically:

a) First compute the cooling of the rod assembly if it is initially at a uniform temperature of 500 C when immersed in the pool, **neglecting the decay heat**. How long does it take to cool to 100 C?

b) Further, now taking the decay heat into account, compute the equilibrium temperature profile in the rod assembly over short time periods.

c) Finally, again taking the decay heat into account, calculate how long it takes for the rod assembly to melt if all cooling water is suddenly removed.

Discuss all simplifying assumptions in modelling this problem including an explicit definition of needed scales, a discussion of your numerical method and its implementation, and a discussion of your results, including an analysis of error. Provide data and graphs as needed. Provide a bibliography listing all references you used in this problem and include a print-out of your codes.

Some useful numbers:

Thermal properties of Zircaloy-4: density 6570 kg/m^3 conductivity 13 W/mK specific heat 340 J/kgK melting point 1850 °C.

Uranium oxide: density 10.97 g/cm^3 conductivity .06 W/cm °C specific heat 300 J/kgK melting point 3120 K.

Q_0 is approximately 10 kW per tonne, decaying to 1 kW per tonne after 10 years..