Physics 410

Assignment #5: Due Monday, October 21, 2013 (Note change of date!)

1) Consider the one dimensional diffusion equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

with initial conditions that $u(0,x) = 20$, $0 < x < 1$ and boundary conditions $u(t,0) = 0$, $u(t,1) = 100$. This situation corresponds, for example to heating one end of a rod at room temperature by immersing it in boiling water while keeping the other immersed in ice water.

a) Implement this equation numerically using the three methods discussed in class: the forward difference method

$$\frac{u(t+\Delta t,x) - u(t,x)}{\Delta t} = \frac{1}{\Delta x^2}\left(u(t,x+\Delta x) - 2u(t,x) + u(t,x-\Delta x)\right)$$

the backward difference method (implicit method)

$$\frac{u(t,x) - u(t-\Delta t,x)}{\Delta t} = \frac{1}{\Delta x^2}\left(u(t,x+\Delta x) - 2u(t,x) + u(t,x-\Delta x)\right)$$

and the Crank-Nicholson method with $\theta = \frac{1}{2}$

$$\begin{aligned}\frac{u(t,x) - u(t-\Delta t,x)}{\Delta t} &= \frac{(u(t,x+\Delta x) - 2u(t,x) + u(t,x-\Delta x))}{2\Delta x^2} \\ &+ \frac{(u(t-\Delta t,x+\Delta x) - 2u(t-\Delta t,x) + u(t-\Delta t,x-\Delta x))}{2\Delta x^2} \qquad (1)\end{aligned}$$

Use the tridiagonal matrix function provided by Horst-Jensen in your implementation of the implicit and Crank-Nicolson methods. Test the solution for these three approaches using $\Delta x = .1$, $\Delta x = .01$, $\Delta x = .001$ and Δt chosen in accordance with the stability limit.

b) Compare the numerical results for all three methods at both short times and at long times, when the solution is approaching the stationary state. Which would you classify as best? Discuss the role of error versus other considerations in your classification.