Physics 410

Assignment #4: Due Friday, October 11, 2013 (note date!)

1) Project 8.3 (modified) in Horth-Jensen, namely

a) Rewrite Eqs. (8.111) and (8.112) as dimensionless equations.

b) Write then a code which solves Eq. (8.111) using the fourth-order Runge Kutta method. Choose an appropriate stepsize for this calculation and justify your choice. Perform calculations for ten periods for values of $\nu = 1$, $\nu = 5$ and $\nu = 10$. Set $l = 1$ m, $g = 1\text{m/s}^2$, $m = 1$ kg. Choose as initial conditions $\theta = 0.2$ (radians) and $v = 0$ (radians/s). Make plots of θ in radians as function of time and phase space plots of θ versus the velocity v. Which case corresponds to damped, underdamped and overdamped oscillatory motion? Comment your results.

c) Now we switch to Eq. (8.112) for the rest of the project. Add an external driving force and set $l = g = 1$, $m = 1$, $\nu = 1/2$ and $\omega = 2/3$. Choose as initial conditions $\theta = 0.2$ and $v = 0$. Carry out the simulation for both $A = 0.5$ and $A = 1.2$. Make plots of θ (in radians) as a function of time for at least 300 periods (think of how to usefully make this plot!) and phase space plots of θ versus the velocity v. Choose an appropriate time step. Comment and explain the results for the different values of A.

d) Keep now the constants from the previous exercise fixed but set now $A = 1.35$, $A = 1.44$ and $A = 1.465$. Plot θ (in radians) as function of time for at least 300 periods for these values of A and comment your results.

e) We want to analyse further these results by making phase space plots of phase space plots of θ versus the velocity v using only the points where we have $\omega t = 2n\pi$ where n is an integer. These are normally called the drive periods. This is an example of what is called a Poincare section and is a very useful way to plot and analyze the behavior of a dynamical system. Comment your results.