Physics 410

Assignment #3: Due Friday, September 27, 2013

1) The equations of motion for projectile motion are

$$m\frac{d^2\vec{x}}{dt^2} = -m\vec{g}$$

where $\vec{g}$ is the gravitational acceleration.

a) Write a code which solves these equations using the second order Runge Kutta method. Set initial conditions corresponding to launching an object with an initial velocity of 20 m/s at an angle of 30° from vertical. Take the coordinate origin to be the position of the launch and $|\vec{g}| = 10$ m/s^2. Attach a printout of your code for credit for this part.

b) Perform calculations with different numbers of mesh points. Find the time of flight, maximum height and landing position. How accurate is your numerical code? Discuss.

Hint: It may be useful to first get the code working with the Euler algorithm. Once it works, you can easily switch to the second order Runge Kutta method by switching the integrating function.

Now modify your program to solve the equation for projectile motion including air resistance:

$$m\frac{d^2\vec{x}}{dt^2} = -\vec{v}(c_1 + c_2|\vec{v}|) - m\vec{g}$$

where $\vec{v} = \frac{d\vec{x}}{dt}$ is, of course, the velocity. For air resistance acting on a sphere, $c_1 = 1.55 \times 10^{-4}D$ and $c_2 = 0.22D^2$ in SI units where D is the diameter of the sphere in meters. Assume $m = .75kg$ and $D = .15$ m (approximately the mass and diameter of a rubber chicken).

c) Rewrite the equations of motion as a set of coupled first order equations and write a code which solves these equations using the second order Runge Kutta method. Again take the coordinate origin to be the position of the launch, an initial velocity of 20 m/s at an angle of 30° from vertical and $|\vec{g}| = 10$ m/s^2. Attach a printout of your code for credit for this part.

d) Study the error in your code using different stepsizes as a function of time. Comment on your results. For what stepsizes is the behavior of the error in your results what you expected? (Note: you should first discuss what you will use as the measure of the error and why you have chosen this.)

e) Determine the corrections to the trajectory due to air resistance. In particular, how is the time of flight, maximum height and landing position changed by the inclusion of air

resistance? Discuss your results; in particular justify your choice of stepsize, the error in the results and how you estimated this error.