Physics 410
Assignment #2: Due Friday, September 20, 2013

1) Compute the first derivative of

$$f(x) = \sin(\pi x)$$

for $x = 0$, .25 and 2.5 using forward, central and extrapolated-difference algorithms (i.e. that one line below 3.2 in Horst-Jensen notes).

a) Find the mathematical expressions for the total error due to loss of precision and due to truncation error. Find the step length that gives the smallest value. Perform the analysis for double precision.

b) Next write a program that computes the first derivative using the 3 algorithms as a function of various step lengths $h$ and let $h \to 0$. Your program should contain the following elements:

- A vector that contains the step lengths. Use dynamic memory allocation.
- Vectors for the computed derivatives for double precision. Again use dynamic memory allocation.
- A function which computes the derivative.
- A function that writes the results to a file.

Attach a printout of your code for credit for this part.
c) Compute thereafter

$$\epsilon = \log_{10}\left(\left|\frac{f'_{computed} - f'_{exact}}{f'_{exact}}\right|\right)$$

as a function of $\log_{10}(h)$ for each method and plot the results. Are the regions for which the slope has the behavior expected if the error is dominated by truncation error? By precision error? Discuss these points and any other important features you observe.
2) Compute the second derivative of

$$f(x) = \sin(\pi x)$$

for $x = .25$ using both the algorithm

$$\frac{d^2f}{dx^2} \approx \frac{f(x+h) + f(x-h) - 2f(x)}{h^2}$$

and the algorithm

$$\frac{d^2f}{dx^2} \approx \frac{f(x+2h) + 2f(x-h) - 3f(x)}{3h^2}$$

a) Find the mathematical expressions for the total error due to loss of precision and due to truncation error. Find the step length that gives the smallest value. Perform the analysis for double precision.

b) Next write a program that computes the second derivative using these two algorithms as a function of various step lengths $h$ and let $h \to 0$. Your program should contain the same elements as that for problem 1).
Attach a printout of your code for the functions that compute the second derivative for credit for this part.
c) Compute thereafter the relative error as a function of $\log_{10}(h)$ for each method. Plot the results. Is there a difference in results between the two algorithms? Discuss.