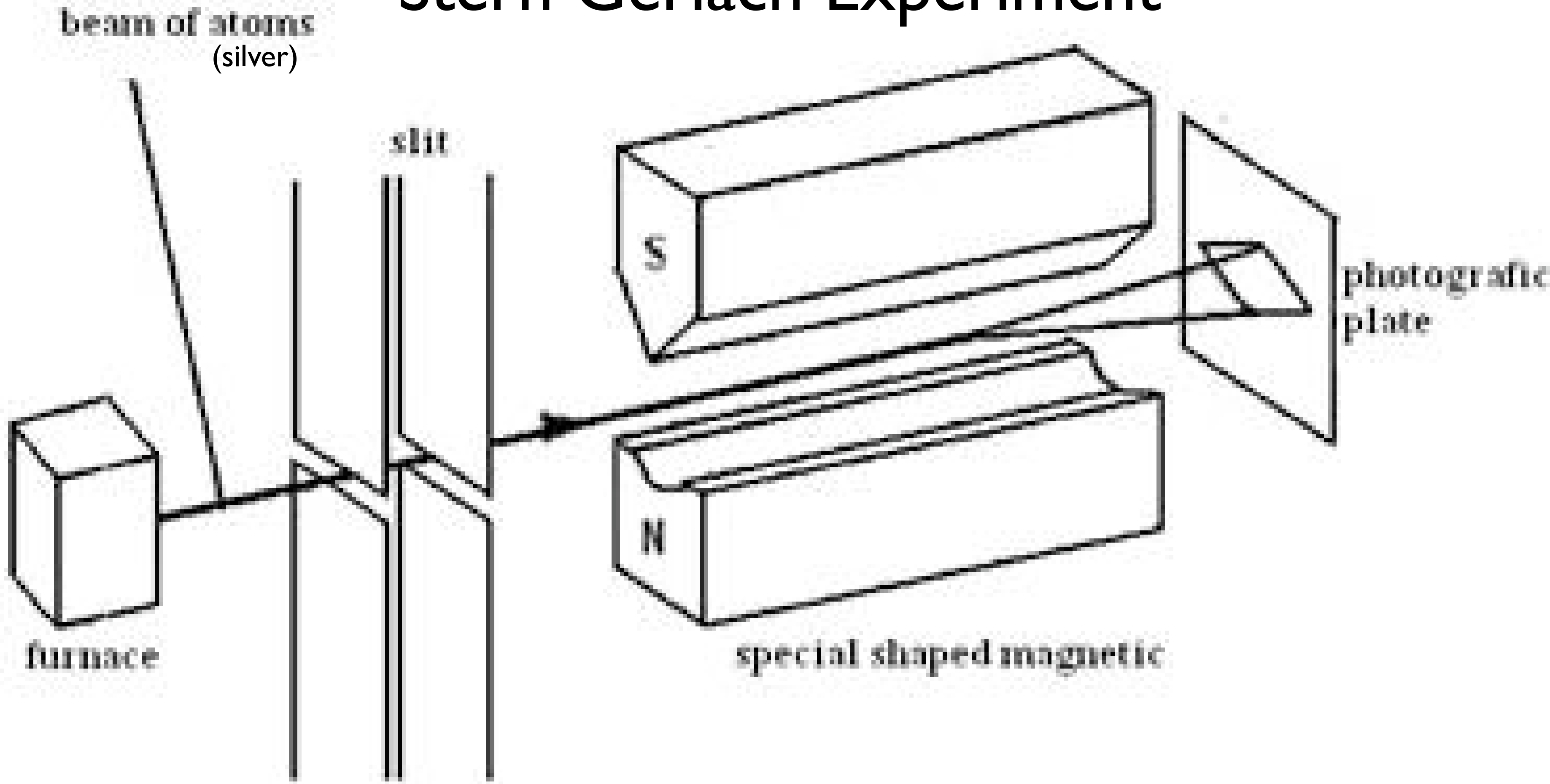
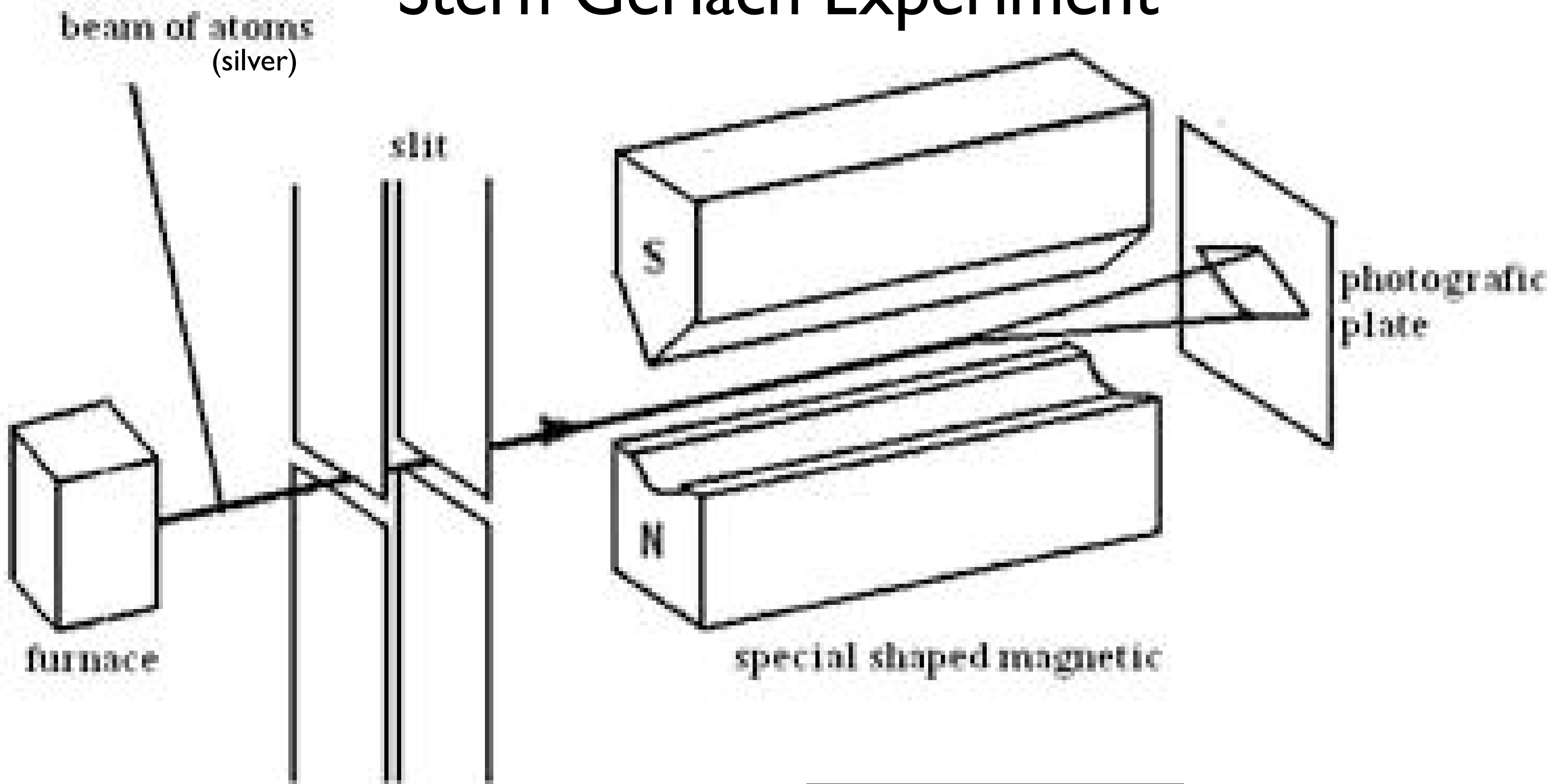
In 1922, Otto Stern and Walther Gerlach, performed their famous experiment (in Frankfurt Germany)

"The Stern-Gerlach Experiment"

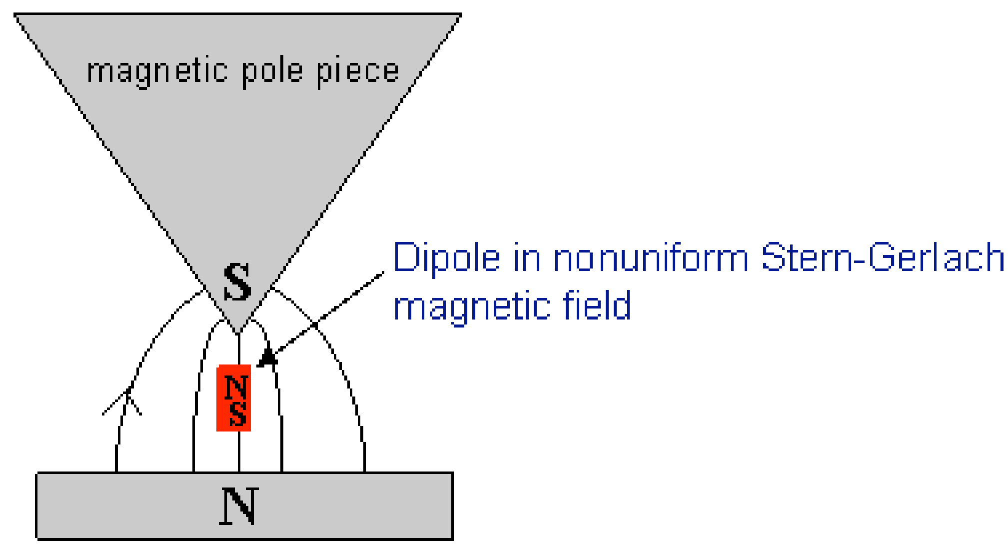
Stern Gerlach Experiment



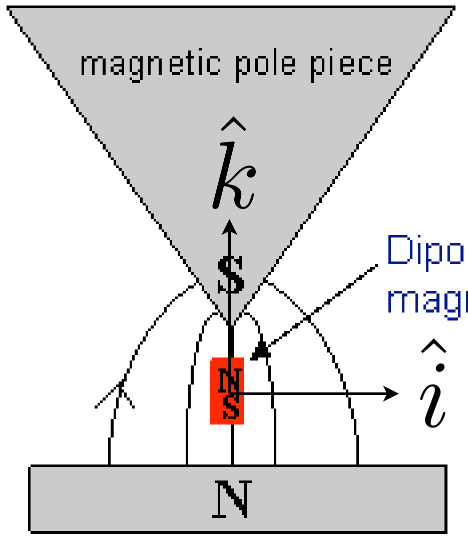
Stern Gerlach Experiment



Non-uniform magnetic field exerts a force on a magnetic dipole



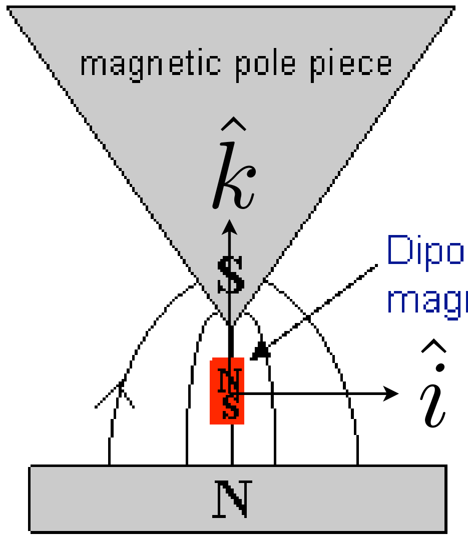
Force on a magnetic dipole



$$\vec{B} = -\alpha x \hat{i} + (B_0 + \alpha z)\hat{k}$$

$$\nabla \cdot \vec{B} = 0$$

Force on a magnetic dipole



$$\vec{B} = -\alpha x \hat{i} + (B_0 + \alpha z)\hat{k}$$

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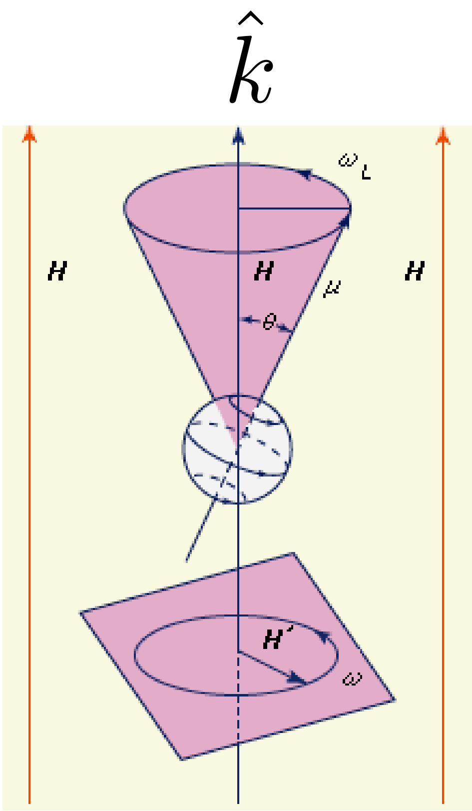
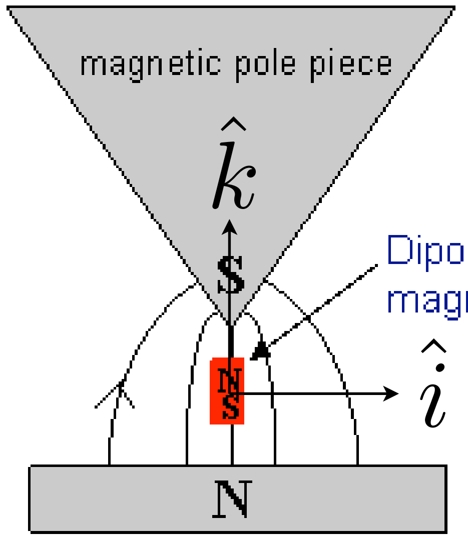


Larmor precession around magnetic field axis $\hat{k}$

Force on a magnetic dipole



$$\vec{B} = -\alpha x\hat{i} + (B_0 + \alpha z)\hat{k}$$

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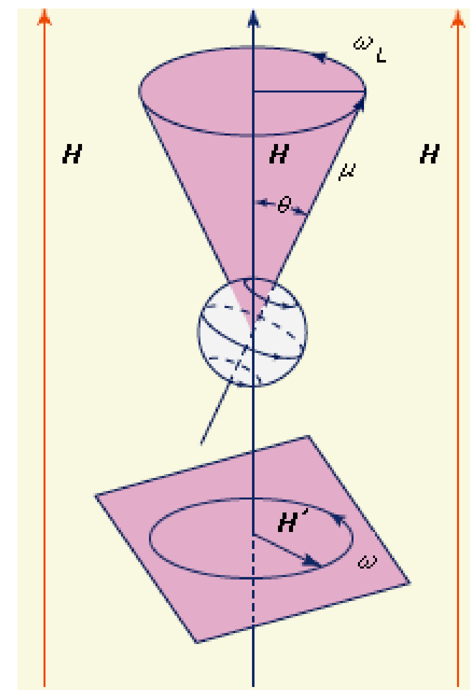


potential energy of dipole

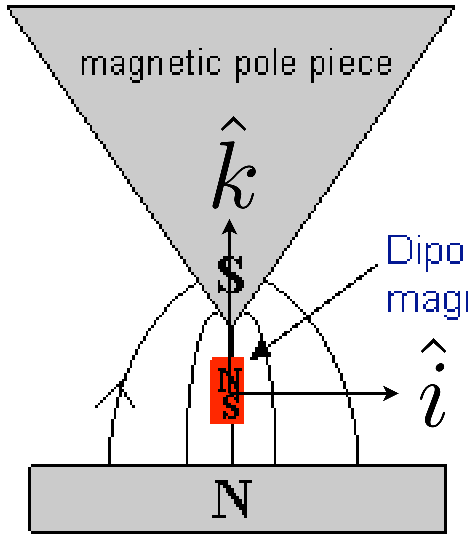
$$V = -\vec{\mu} \cdot \vec{B}$$

Force on dipole

$$\vec{F} = -\nabla V = \nabla(\vec{\mu} \cdot \vec{B})$$

$$\vec{F} = \gamma\alpha(-S_x\hat{i} + S_z\hat{k})$$

Force on a magnetic dipole



Dipole in nonuniform Stern-Gerlach magnetic field

$$V = -\vec{\mu} \cdot \vec{B}$$

$$\vec{B} = -\alpha x \hat{i} + (B_0 + \alpha z)\hat{k}$$

$$\nabla \cdot \vec{B} = 0$$

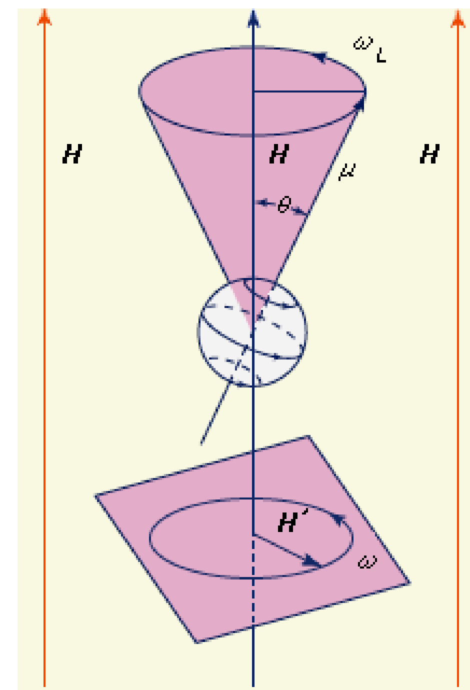
Force on dipole

$$\vec{F} = -\nabla V = \nabla(\vec{\mu} \cdot \vec{B})$$

$$\vec{F} = \gamma\alpha(-S_x \hat{i} + S_z \hat{k})$$

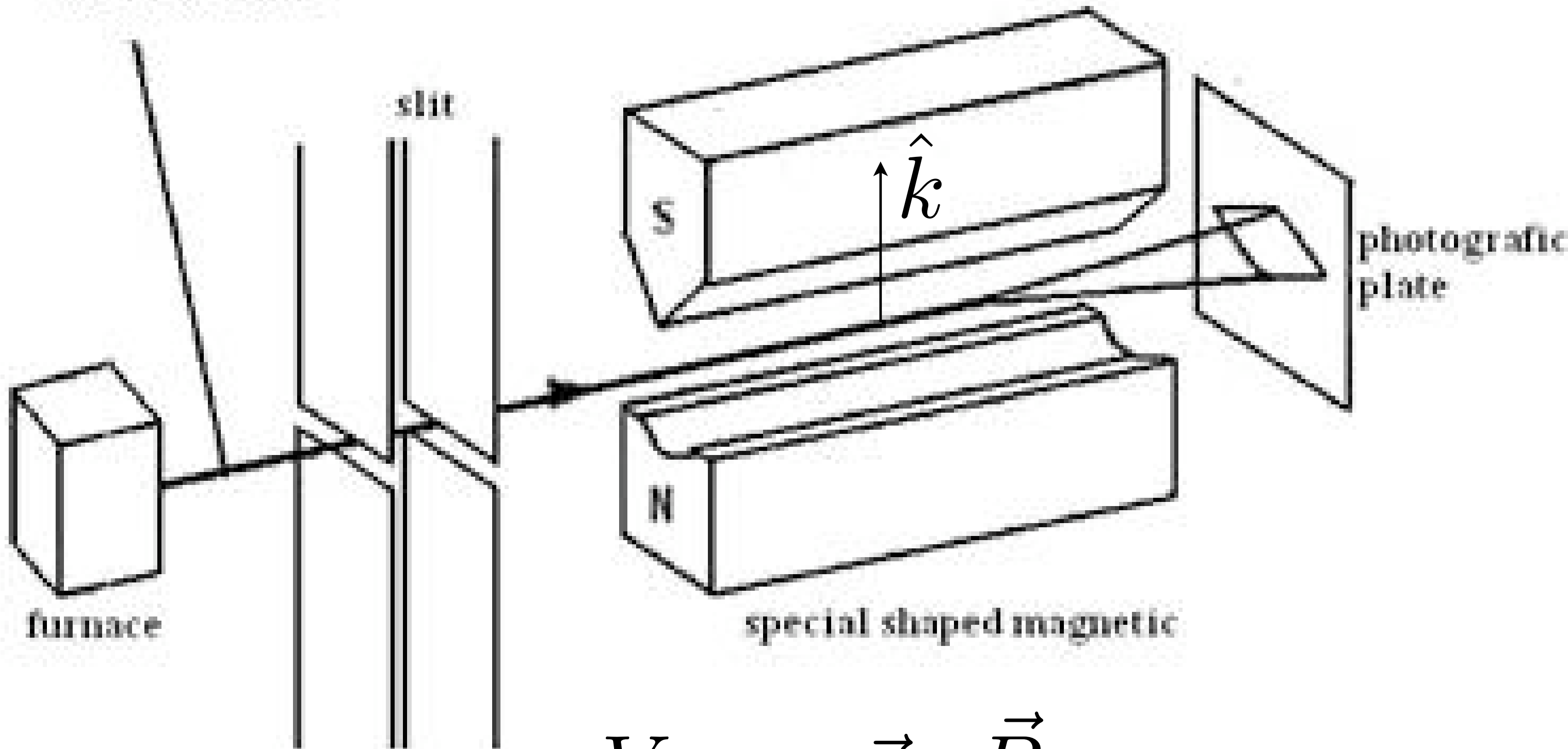
Larmor precession around $\hat{k}$

$\langle S_x \rangle$ oscillates rapidly... $\langle F_x \rangle \longrightarrow 0$



$$F_z = \gamma\alpha S_z$$ (net force along z)

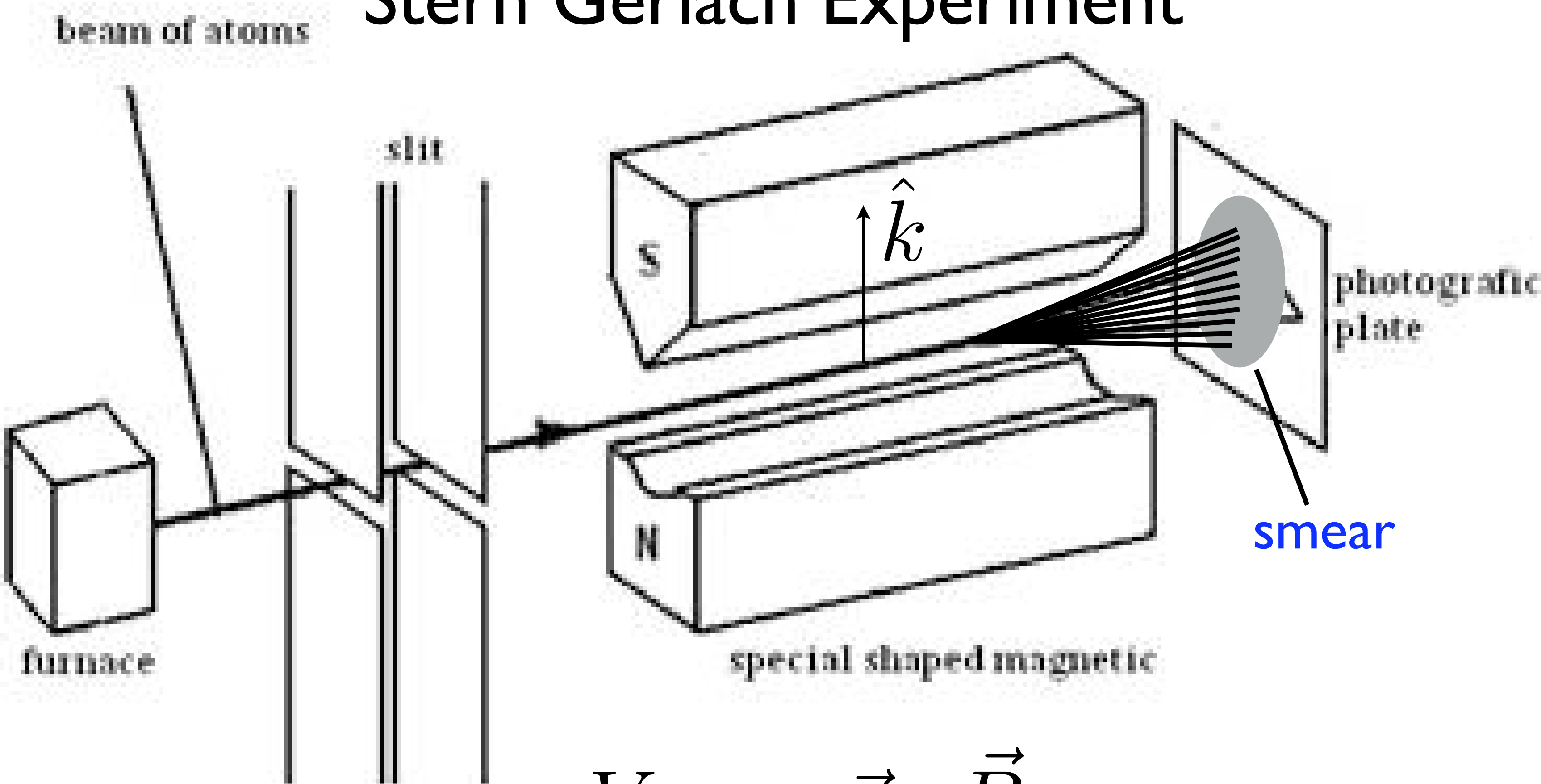
Stern Gerlach Experiment



$$V = -\vec{\mu} \cdot \vec{B}$$

$F_z = \gamma \alpha S_z$ (net force along z)

Stern Gerlach Experiment

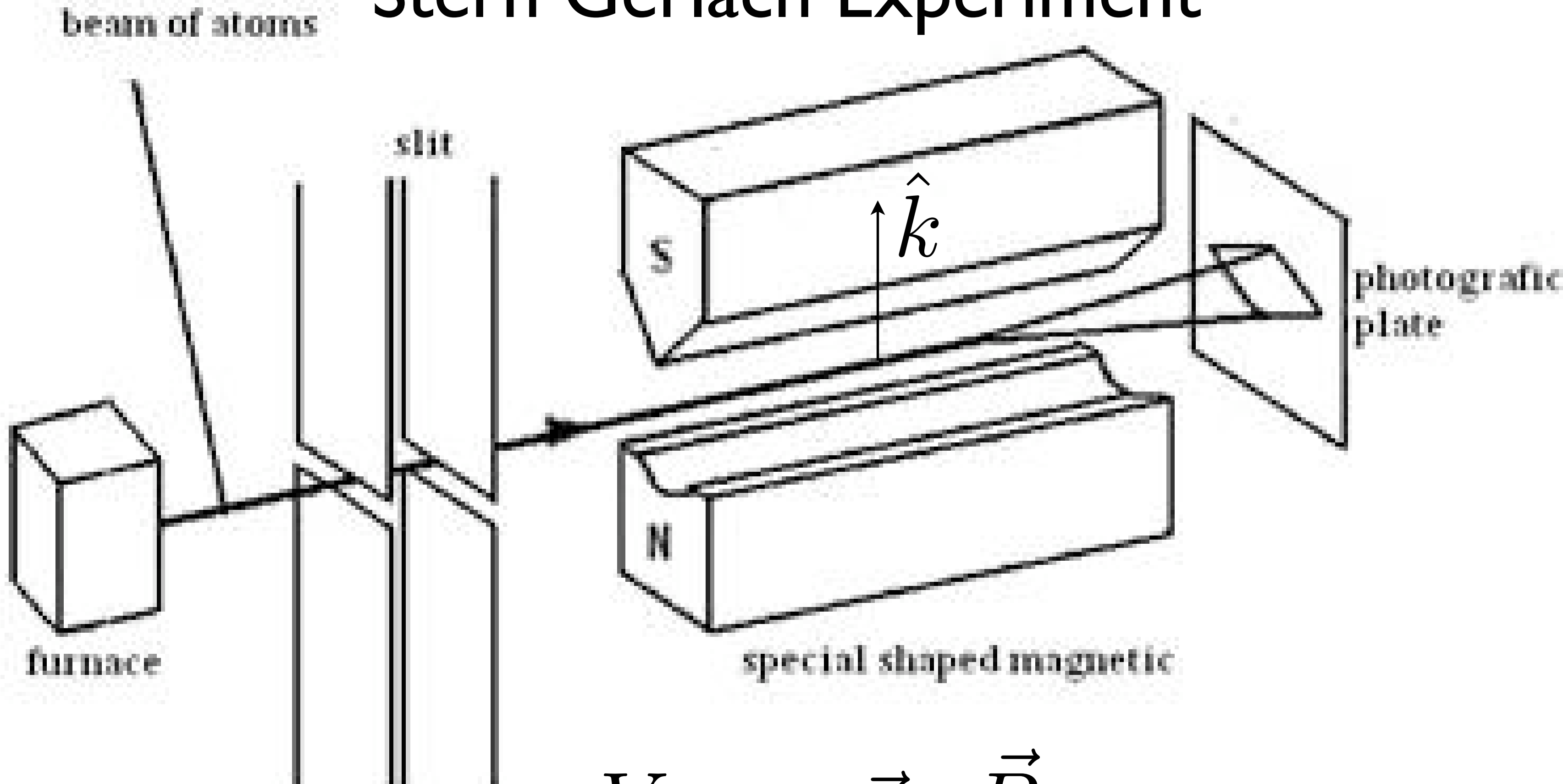


$$V = -\vec{\mu} \cdot \vec{B}$$

$$F_z = \gamma \alpha S_z$$ (net force along z)

Classically you would expect a smear along z because (classically) Sz is not quantized and dipole can point with any angle w.r.t B

Stern Gerlach Experiment



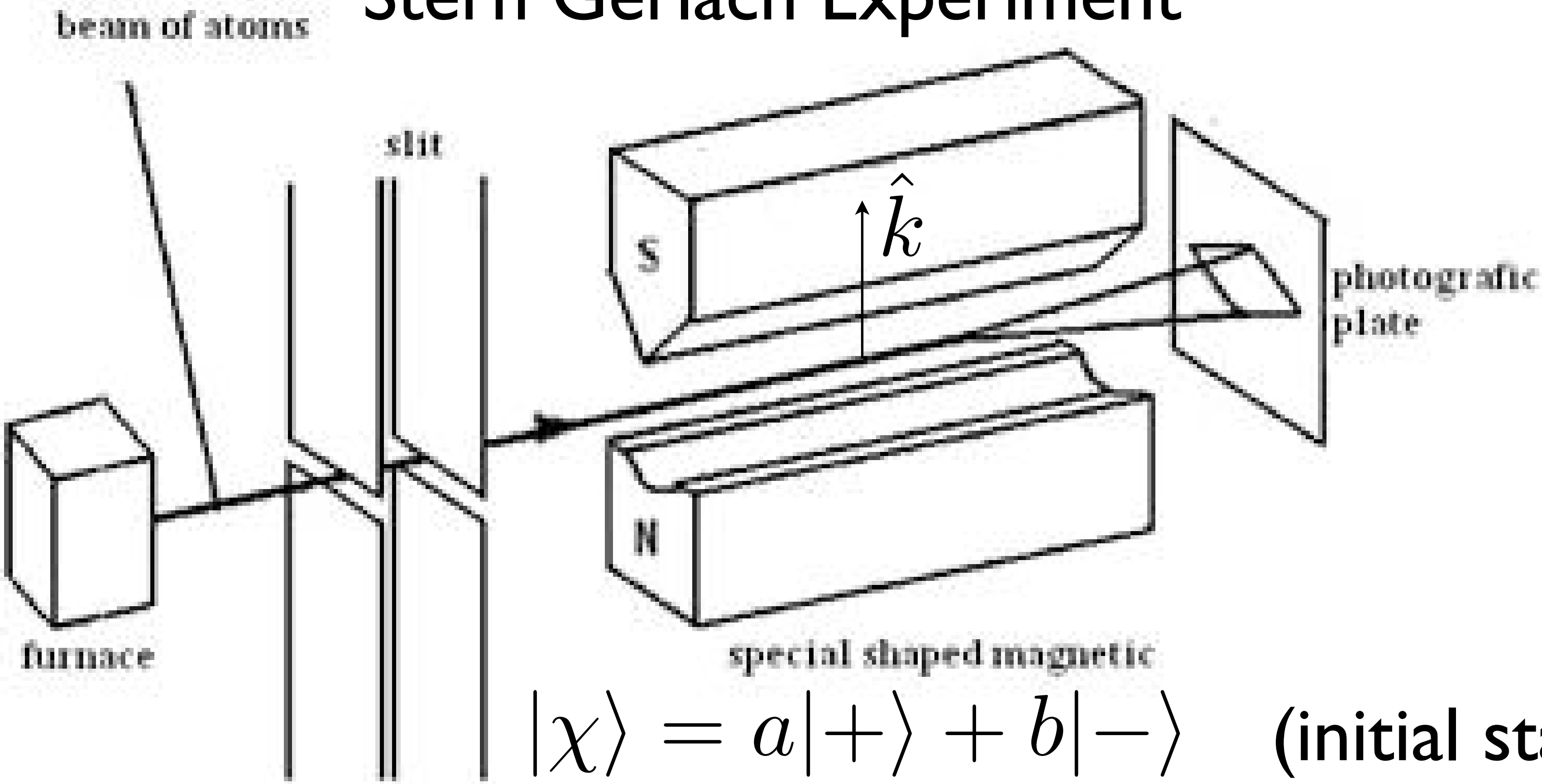
$$V = -\vec{\mu} \cdot \vec{B}$$

$F_z = \gamma \alpha S_z$ (net force along z)

$$E_{\pm} = \mp \gamma (B_0 + \alpha z) \frac{\hbar}{2}$$

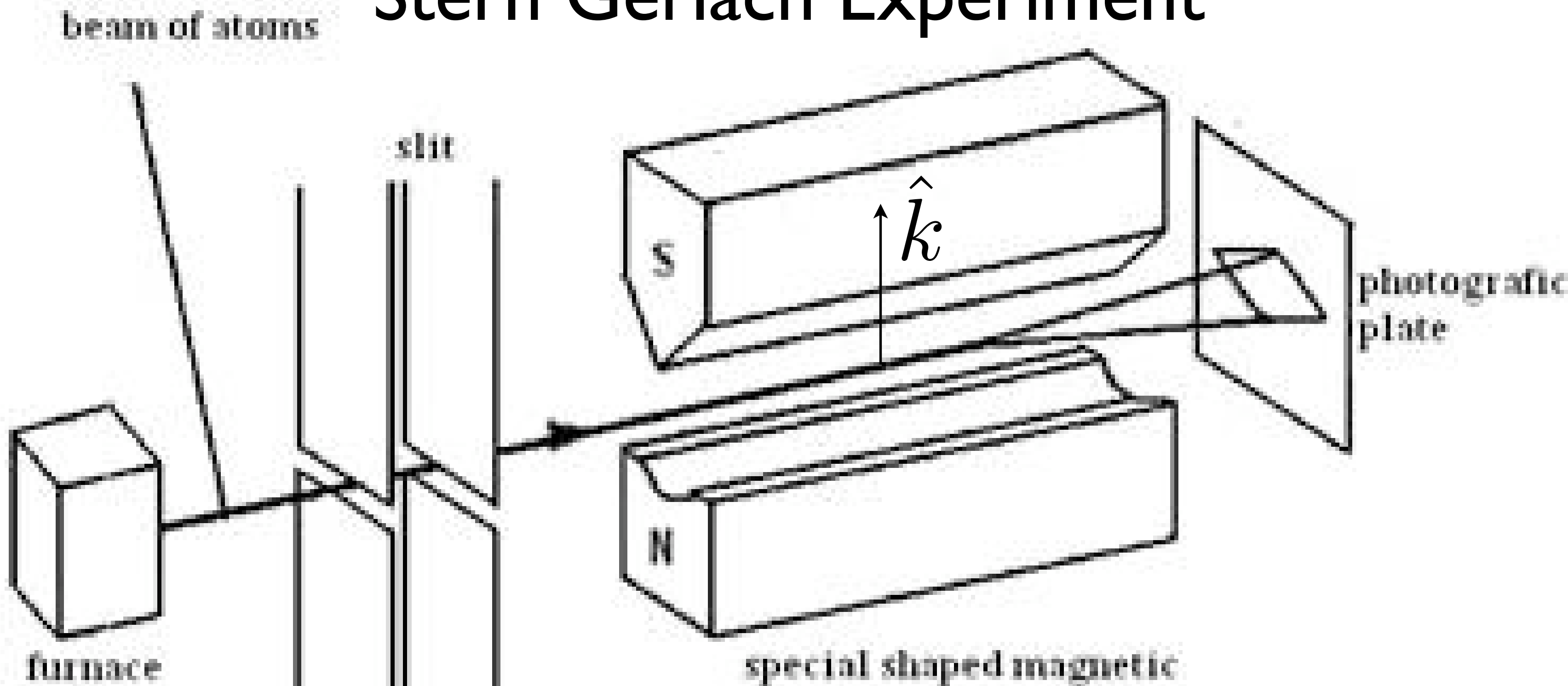
In reality you see only two possible z positions
corresponding to two possible values for Sz, $\pm\hbar$

Stern Gerlach Experiment



$$|\chi\rangle = a|+\rangle + b|-\rangle \quad \text{(initial state)}$$

Stern Gerlach Experiment

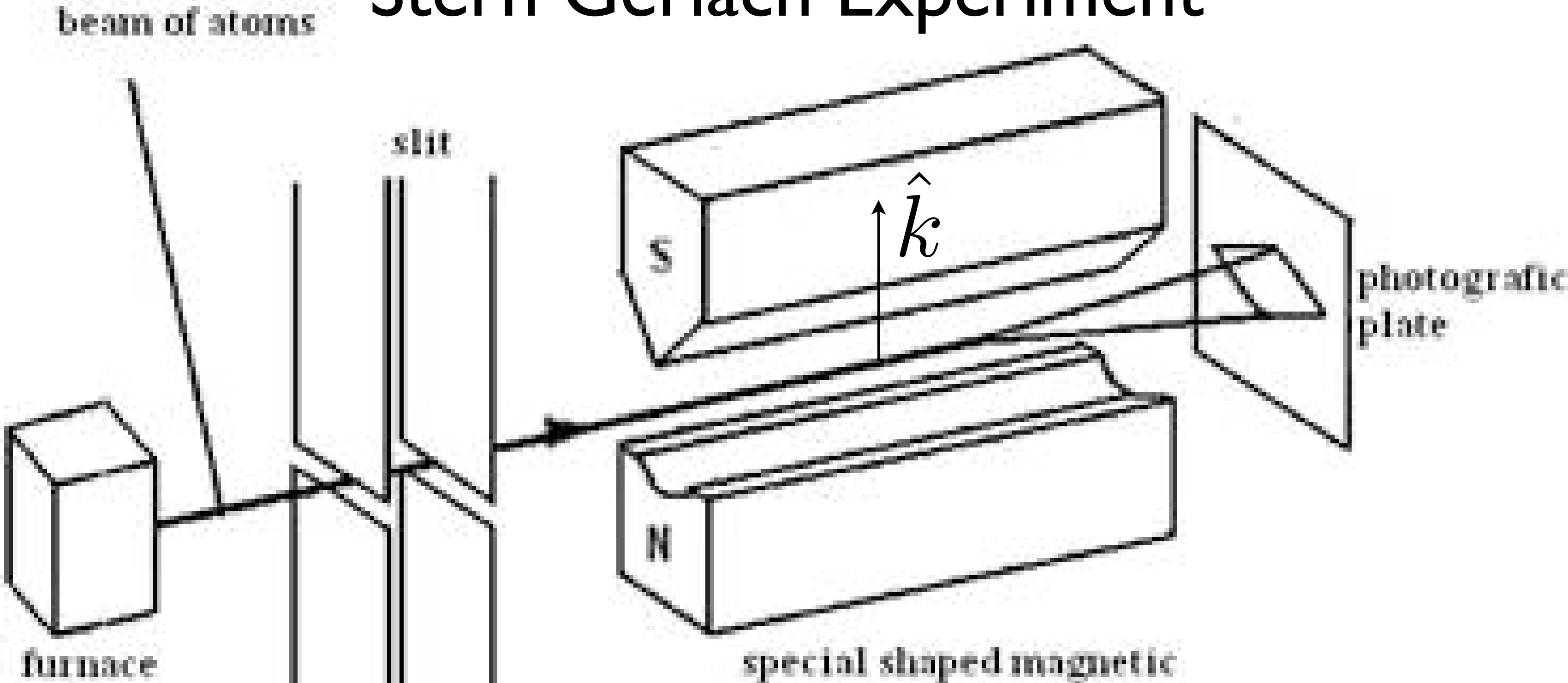


$$|\chi\rangle = a|+\rangle + b|-\rangle \quad \text{(initial state)}$$

$$|\chi(t)\rangle = a|+\rangle e^{-iE_+ t/\hbar} + b|-\rangle e^{-iE_- t/\hbar}$$

$$E_\pm = \mp\gamma(B_0 + \alpha z)\frac{\hbar}{2} \qquad V = -\vec{\mu}\cdot\vec{B}$$

Stern Gerlach Experiment



$$|\chi\rangle = a|+\rangle + b|-\rangle \quad \text{(initial state)}$$

$$|\chi(t)\rangle = a|+\rangle e^{-iE_+ t/\hbar} + b|-\rangle e^{-iE_- t/\hbar}$$

$$|\chi(t)\rangle = \left(ae^{+i\gamma T B_0/2}|+\rangle\right) e^{i(\alpha\gamma T/2)z} + \left(be^{-i\gamma T B_0/2}|-\rangle\right) e^{-(i\alpha\gamma T/2)z}$$

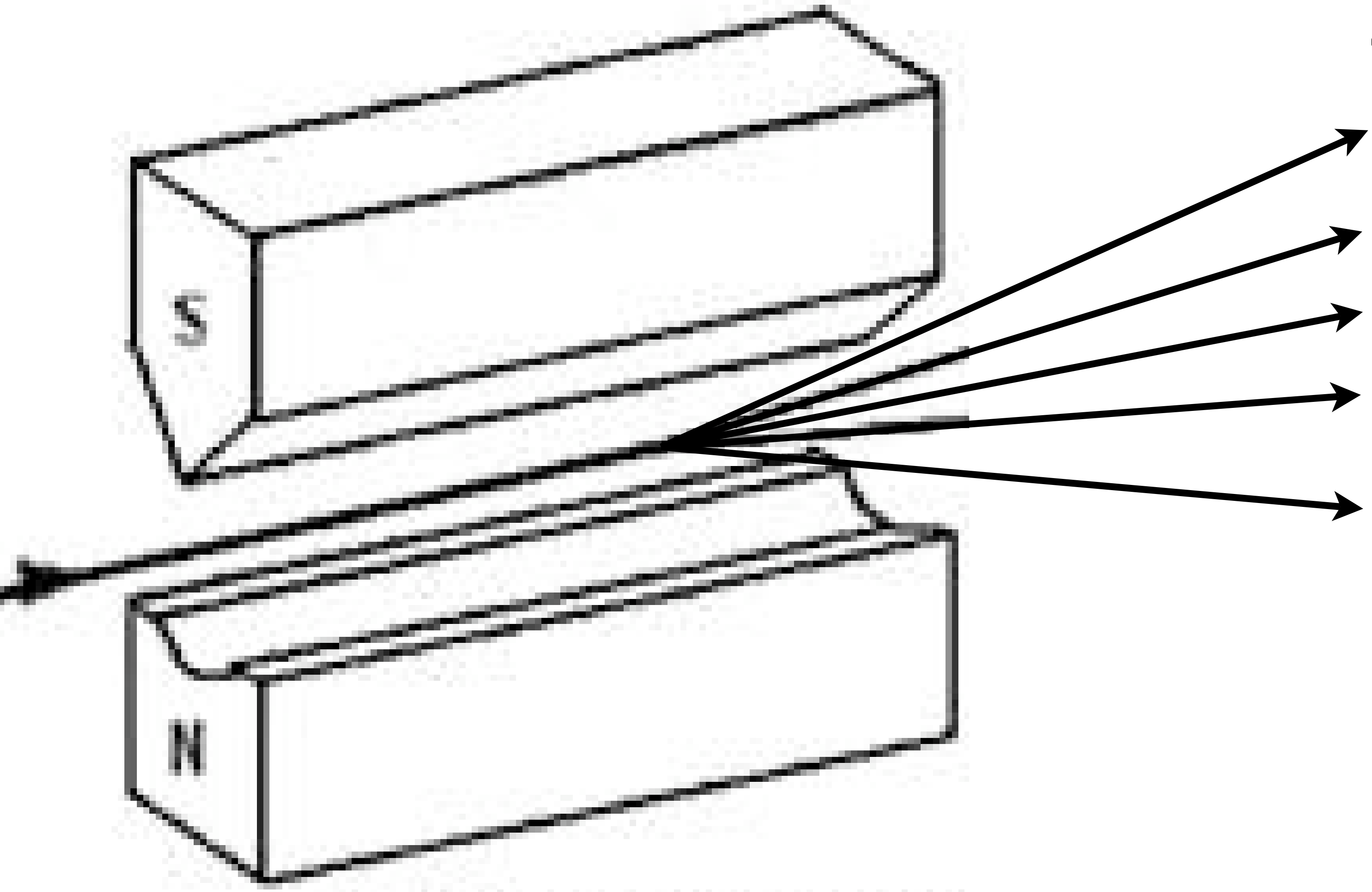
$$e^{ip_0 z/\hbar} \qquad\qquad e^{-ip_0 z/\hbar}$$

"+" state moving up "-" state moving down

$$P_0 = \hbar(\alpha\gamma T/2)$$

$$f_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$$

Stern Gerlach Experiment
beam is split into 2s+1 separate streams



$\vert l, m\rangle$	S_z
$\vert 2, 2\rangle$	$2\hbar$
$\vert 2, 1\rangle$	$\hbar$
$\vert 2, 0\rangle$	0
$\vert 2, -1\rangle$	$-\hbar$
$\vert 2, -2\rangle$	$-2\hbar$