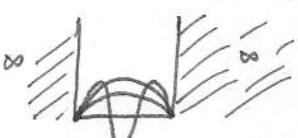
Multi-particle states — Ch. 5
Quantum Entanglement (Bell's Theorem / Inequalities) and the end of local realism — Ch. 12

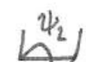
2 particle state

$\Psi(x_a, x_b, t)$ function of the coordinate of particle a
" of particle b
and time



example: consider 2 particles in the infinite square well

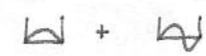
- let's limit the total energy $E_T \lessgtr 2E_2$



  so that each particle can only occupy the ground or 1st excited state

  Single particle states

$$\Psi_a(x_a) = c_1 \Psi_1(x_a) + c_2 \Psi_2(x_a) \qquad \Psi_b(x_b) = d_1 \Psi_1(x_b) + d_2 \Psi_2(x_b)$$



- let's assume the particles dont interact (no attraction/repulsion)

  ∴ ~~joint 2 particle state can be constructed~~
  we can construct joint 2 particle state from single particle states

Type I: product of the ~~single~~ single particle states

$$\Psi(x_a, x_b, t) = \Psi_a(x_a, t)\, \Psi_b(x_b, t) \qquad \text{"Separable state"}$$

$$= c_1 d_1\, \psi_1(x_a)\psi_1(x_b) + c_1 d_2\, \psi_1(x_a)\psi_2(x_b) + c_2 d_1\, \psi_2(x_a)\psi_1(x_b) + c_2 d_2\, \psi_2(x_a)\psi_2(x_b)$$

short-hand $\quad |\psi\rangle = c_1 d_1 |1,1\rangle + c_1 d_2 |1,2\rangle + c_2 d_1 |2,1\rangle + c_2 d_2 |2,2\rangle$

particle a in state $n=1, \psi_1, E_1$
" b " $n=2, \psi_2, E_2$

General state

$$|\Psi(x_a,x_b,t)\rangle = c_{11}|1,1\rangle + c_{12}|1,2\rangle + c_{21}|2,1\rangle + c_{22}|2,2\rangle$$

Normalization

$$\Psi^*\Psi \rightarrow \langle\Psi(x_a,x_b,t)|\Psi(x_a,x_b,t)\rangle = \underbrace{|c_{11}|^2}_{} + |c_{12}|^2 + |c_{21}|^2 + |c_{22}|^2$$

prob. of finding particle a in state with energy E_1 $(\psi_1(x_a))$
and of " " b " " " $(\psi_1(x_b))$

Type II : Entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|1,1\rangle + \frac{1}{\sqrt{2}}|2,2\rangle$$

"non-seperable state"
cannot write as $|\Psi_a\rangle \otimes |\Psi_b\rangle$

each particle equally likely ~~[illegible]~~ of being in states ψ_1 and ψ_2

Prob. of finding a with energy E_1 is $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

* But if you find a with energy E_1, you are guerenteed to find b with energy E_1 $\rightarrow$ you collapse joint state to either $|1,1\rangle$ or $|2,2\rangle$

Bell Inequality and the end of local realism

example: entangled photons — Light has 2 $\perp$ polarization directions along which the $\vec{E}$ field can wiggle as the photon propagates

Summary: QM allows 1 photon to be in a state of indeterminant polarization

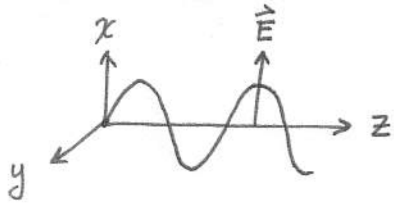
QM allows 2 photons to be in a joint state of having the same polarization but the pol. direction is indeterminant

if you measure one photon polarization along some axis and find it, the w.f. of the joint state collapses and the pol. of the other photon is known without measuring it

indeterminent pol. $\longrightarrow$ some definite pol. state (the same as photon #1)

$\rightarrow$ measurement collapse is non-local and "instantaneous"

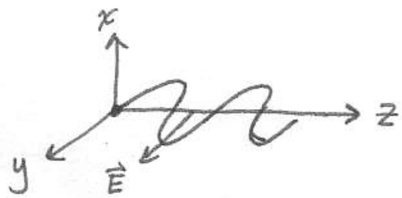
Polarization states of photons



pol. along x

Polarization state

$|\psi\rangle = |V\rangle$ "vertical" state vector

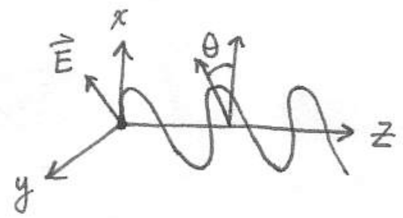


pol. along y $\quad |\psi\rangle = |H\rangle$ "horizontal" State vector

Properties of these state vectors

$|V\rangle$ and $|H\rangle$ are orthonormal states :

short-hand $|\psi|^2 = \langle\psi|\psi\rangle \qquad \langle V|V\rangle = 1,\ \langle H|H\rangle = 1,\ \langle V|H\rangle = 0$



pol. along a direction rotated by θ w.r.t. x $\rightarrow$ $|\psi\rangle = |\theta\rangle$

$|\psi\rangle = \cos\theta\,|V\rangle + \sin\theta\,|H\rangle$ expressed in terms of $|H\rangle, |V\rangle$ basis

QM state transforms like the $\vec{E}$ vector !

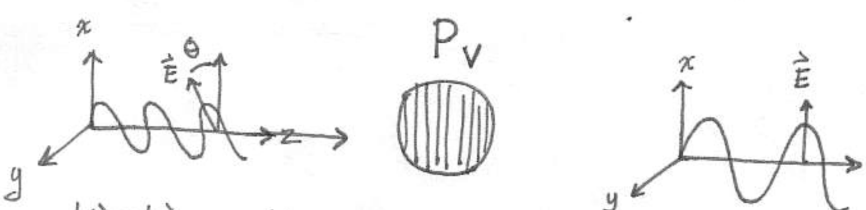
prob. of finding vertical pol. $\underline{\cos^2\theta}$

the $|\text{coefficient}|^2$

~~Polarizer = a device which will transmit light pol. along its axis and absorb light~~

Polarizer — will transmit light polarized along its axis and will absorb the other polarization and heat up

— represented by a projective operator —



$|\psi\rangle = |\theta\rangle = \cos\theta|V\rangle + \sin\theta|H\rangle \longrightarrow |\psi'\rangle = \cos\theta|V\rangle + 0\,|H\rangle$

$P_V|\psi\rangle$

$P_V = |V\rangle\langle V|$

no photon in $|H\rangle$ state

General Projective operator : $P_\psi = |\psi\rangle\langle\psi|$

projects on to an arb. state

$$|\psi\rangle = |V\rangle \xrightarrow{\quad\bullet\quad} |\psi\rangle = \left[\cos\theta\,|V\rangle + \sin\theta\,|H\rangle\right]\cos\theta$$
$$P_\theta|\psi\rangle$$

<u>Wave function collapse is non-local</u>

<u>Consider 2 photons created in an entangled polarization state but moving in opposite directions</u>

$$|\psi\rangle_e = \frac{1}{\sqrt{2}}|VV\rangle + \frac{1}{\sqrt{2}}|HH\rangle$$

polarizer acts just on photon a

vertical orientation: $$P_V^{(a)}|\psi\rangle_e = |V\rangle_a\,{}_a\langle V|\left[\frac{1}{\sqrt{2}}\left(|V\rangle_a \otimes |V\rangle_b + \cancel{\tfrac{1}{\sqrt{2}}}|H\rangle_a \otimes |H\rangle_b\right)\right]$$
$$= \frac{1}{\sqrt{2}}|V\rangle_a \otimes |V\rangle_b = \frac{1}{\sqrt{2}}|V\rangle_a|V\rangle_b$$

arb. orientation: $$P_\theta^{(a)}|\psi_e\rangle = |\theta\rangle_a\,\underbrace{{}_a\langle\theta|\psi\rangle_e}_{\left(\cos\theta\langle V|_a + \sin\theta\langle H|_a\right)\left(\frac{1}{\sqrt{2}}\left[|V\rangle_a|V\rangle_b + |H\rangle_a|H\rangle_b\right]\right) = \frac{1}{\sqrt{2}}\underbrace{\left[\cos\theta|V\rangle_b + \sin\theta|H\rangle_b\right]}_{|\theta\rangle_b}} = \frac{1}{\sqrt{2}}|\theta\rangle_a \otimes |\theta\rangle_b = \frac{1}{\sqrt{2}}|\theta\rangle_a|\theta_b\rangle$$

⟶ conclusion: no matter how you orient your polarizer
if photon a goes through (or not)
you <u>project</u> photon b onto the same particular pol. state even
if photon b is on the other side of the universe!

Orthodox view — before the measurement, neither photon had a pol. direction but the act of measuring a "produced" a pol. direction for b.

EPR Paradox

A Einstein, B. Podolsky and N. Rosen, Phys. Rev. <u>47</u>, 777 (1935)

this "spooky" action at a distance frustrated people (Einstein – preposterous)

Nature "should" be local and deterministic, but QM allows for <u>indeterminacy</u> and <u>non-locality</u>!

Bell Inequality : any local realist theory is incompatible with QM

2 polarizers can be oriented along a, b, c

- on average the Prob. of a "click" at A is $\frac{1}{2}$ } for any setting along a, b, c
  " " B is $\frac{1}{2}$
- but when both pol. along same direction, detectors always "agree"

e.g. for ~~any~~ pol setting a, a, if det A "clicks" then det B does also

Q. How often do detectors A + B agree for a random setting of the polarizers?

A. average over outcomes

settings	aa	ab	ac	ba	bb	bc	ca	cb	cc
agreement	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1

$$= \frac{3+\frac{6}{4}}{9} = \frac{\frac{18}{4}}{9} = \frac{1}{2}$$

$\cos^2 \cdot 120^\circ = \frac{1}{4}$

50 % agreement for avg. over random settings.

Local realism — local hidden variables

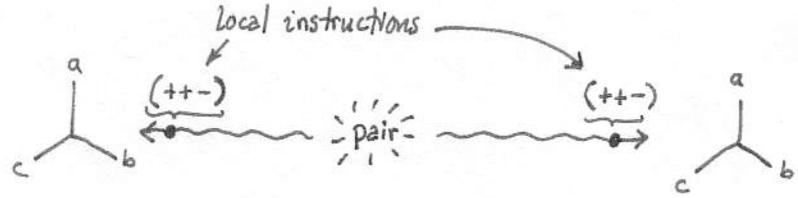
information about the outcome of ~~the~~ all possible measurements must be carried on the particles.

local instruction set

pass polarizer?

$+$ = yes

$-$ = no



8 possible instruction sets

orientation a	b	c	polarizer settings and detector agreement: aa	ab	ac	ba	bb	bc	ca	cb	cc	
+	+	+	1	1	1	1	1	1	1	1	1	perfect agreement
+	+	−	1	1	0	1	1	0	0	0	1	
+	−	+										5/9 agreement
−	+	+										
−	−	+										
−	+	−										
+	−	−	1	0	0	0	1	1	0	1	1	
−	−	−										perfect agreement

conclusion: no matter what distribution of these inst. sets the detectors will agree at least 5/9 of all runs

$$P^{\text{agree}}_{\text{Local realism}} \geq \frac{5}{9} \qquad \text{Bell Inequality}$$

$$P^{\text{agree}}_{\text{QM}} = \frac{1}{2} \qquad \text{QM violates Bell Inequality}$$

Nature (QM) is not compatible with local realism