

EXPERIMENT #4

AC and TRANSIENT RESPONSE OF A RESONANT LCR CIRCUIT

This experiment is intended to provide you with a working knowledge of some of the basic properties of a resonant LRC circuit, both its transient response and its frequency response. It also provides you with further practice in computer modeling and introduces you to non-linear least-squares fitting.

Introduction

The series LRC circuit shown in Fig. 4.1 is an example of the simplest damped resonant system, known as the damped harmonic oscillator. If the AC source produces a sinusoidal signal with amplitude V_0 and angular frequency ω , then the ratio of the voltage amplitude across the resistor to the voltage amplitude across the AC source is

$$\frac{V_R}{V_0} = \frac{1}{\sqrt{1 + \left(\omega \frac{L}{R} - \frac{1}{\omega CR} \right)^2}} \quad (4.1)$$

The resonant frequency and bandwidth of this harmonic oscillator are

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{and} \quad \gamma = R/L \quad (4.2)$$

You will find it convenient to rewrite the voltage ratio in terms of these two quantities, which will give you an important expression for the frequency response of a resonant system (also known as a Lorentzian lineshape).

Further details on the LRC circuit can be found in almost any basic text on AC circuits.

1. Transient Response

Assemble the following series LCR resonant circuit,

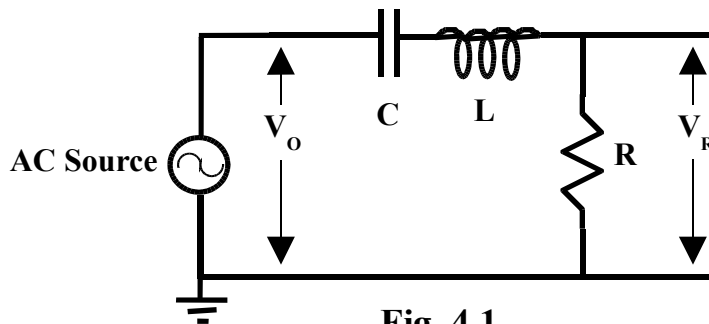


Fig. 4.1

using the component values $L = 20 \text{ mH}$, $C = 0.001 \text{ }\mu\text{F}$ and $R = 1000 \text{ }\Omega$.

Investigate the transient response by applying low frequency *square waves* from the function generator and using the oscilloscope to observe the voltage across the resistor V_R . The oscilloscope can be triggered with the **SYNC OUT** signal from the generator and you should then observe damped harmonic oscillations in $V_R(t)$. Measure both the frequency of the oscillations ω_0 and the time constant τ that governs the exponential damping of the amplitude of the oscillations. You should collect enough points in the exponential decay to perform a linear least squares fit to determine τ .

2. To investigate the AC frequency response of the resonant circuit, you should switch the generator to *sinusoidal output* and observe both V_0 and V_R on the two channels of the oscilloscope. Measure both the phase and amplitude of V_R relative to V_0 over a frequency range wide enough to clearly show the resonant behaviour. Pay particular attention to the frequencies in the vicinity of the resonance peak. There are 3 points of great importance: the resonant frequency where V_R reaches its maximum value, and the two points on either side of the resonance where $V_R/V_0 = \frac{1}{\sqrt{2}}$ and the phase shift is $\pm 45^\circ$.

Use your measurements to plot V_R/V_0 vs. ω and then determine, by fitting a Lorentzian curve to the data, the resonant frequency, f_0 , of this circuit (where $\omega_0 = 2\pi f_0$) and the bandwidth γ . The dependence of V_R/V_0 on ω is clearly non-linear. For this curve-fitting task (again weighted by the errors in the measured parameters), you will be using the non-linear least squares fitting function of gnuplot. Initially try to fit to the Lorentzian-shaped peak of Eq. 3.1. If the fit is unsatisfactory, you may need to include a third fitting parameter to adjust the height of the peak. Try to explain why this third parameter is needed. Compare the resonant frequency and damping to the results obtained in part 1 in order to determine the relationship between the transient response and the AC response.