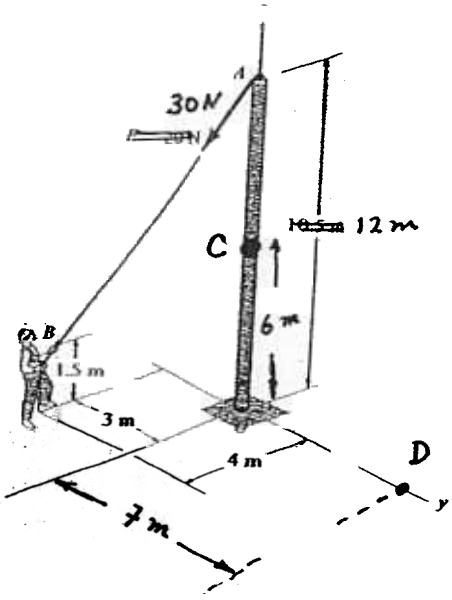
SOLUTIONS & MARK DISTRIBUTION

Name:_____ Student Number:_____

Note: Very little credit for correct answers without a clear explanation of reasoning.

Question #1: The end of the rope (point-B) is pulled by a student with a force, $\vec{F}$, of magnitude 30 N.

(a) Express the force, $\vec{F}$, in terms of $\hat{i}$, $\hat{j}$, and $\hat{k}$ components.
(b) Determine the unit vector pointing from point-D to point-C.
(c) Determine the moment that the force, $\vec{F}$, exerts about point-D.
(d) Determine the magnitude of the above moment about the axis going through points D and C.
(e) The pole is held in the sidewalk with cement and, thereby, restrained from moving. The mass of the pole is 200 kg. Draw the free body diagram for the pole. (Do not solve for the reaction forces and couples; just give the free body diagram).



Mark Distribution

Part	Answer	Reasoning	Totals
a)	1	1	2
b)	1	1	2
c)	1/2	1 1/2	2
d)	1/2	1 1/2	2
e)	2	0	2
			10

Solution: Question-1

a) $\vec{F} = (30\,N)\left[\dfrac{(4-0)\hat{i} + (-3-0)\hat{j} + (10.5-12)\hat{k}}{\sqrt{(4)^2+(3)^2+(10.5)^2}}\right]$

$= +10.3\,\hat{i} - 7.74\,\hat{j} - 27.1\,\hat{k}\ N$ ✓

b) $\hat{u}_{DC} = \left[\dfrac{0\,\hat{i} + (0-7)\hat{j} + (6-0)\hat{k}}{\sqrt{(0)^2+(7)^2+(6)^2}}\right]$

$= 0\,\hat{i} - 0.759\,\hat{j} + 0.651\,\hat{k}$ ✓

c) $\vec{M}_D = \vec{r}_{DA} \times \vec{F} =$

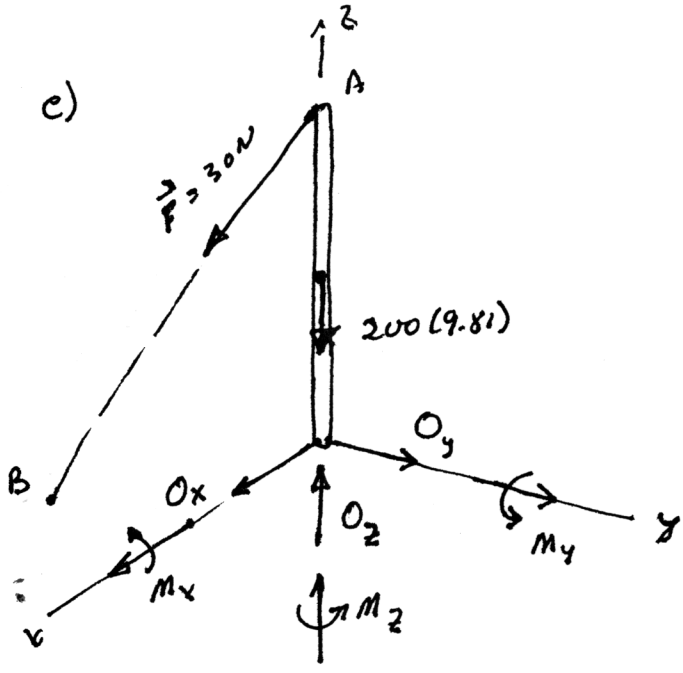
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -7 & 12 \\ 10.3 & -7.74 & -27.1 \end{vmatrix} \quad \begin{matrix} \hat{i} & \hat{j} \\ 0 & -7 \\ 10.3 & -7.74 \end{matrix}$$

$= 283\,\hat{i} + 124\,\hat{j} + 72.1\,\hat{k}\ N.m$

d) $M_{DC} = \vec{M}_D \cdot \hat{u}_{DC} = (283)(0) + (124)(-0.759) + (72.1)(0.651)$

$= -46.9\ N.m$ (= 46.9 N.m. Don't worry about sign)

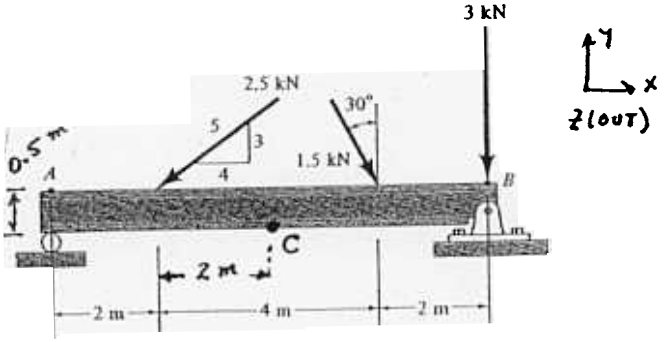
e)



Name:_____ Student Number:_____

Note: Very little credit for correct answers without a clear explanation of reasoning.

Question #2: Replace the three forces (2.5 kN, 1.5 kN, and 3 kN) acting on the beam by an equivalent force and couple moment acting at point-C. Use the x-y-z axes shown below to express your final answer in terms of $\hat{i}$, $\hat{j}$, and $\hat{k}$ components.



Mark Distribution

	Answer	Reasoning	Totals
Resultant Force	2	3	5
Couple	2	3	5
			10

Solution : Question-2

$$\vec{R} = 2.5\left[-\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right] + 1.5\left[+\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j}\right] + 3\left[-\hat{j}\right]$$

$$= \hat{i}\left[2.5\left(-\tfrac{4}{5}\right) + 1.5(\sin 30)\right] + \hat{j}\left[2.5\left(-\tfrac{3}{5}\right) + 1.5(-\cos 30) + (3)(-1)\right]$$

$$= \hat{i}\left[-1.25\right] + \hat{j}\left[-5.799\right] = \underline{\underline{-1.25\,\hat{i} - 5.80\,\hat{j}\ \text{kN}}}$$

$$+\circlearrowleft \Sigma M_C = +2.5\left(\tfrac{4}{5}\right)(0.5) + 2.5\left(\tfrac{3}{5}\right)(2) + 1.5(\sin 30^\circ)(0.5) - 1.5(\cos 30^\circ)(2) - 3(4)$$

$$= -10.97\ \text{kN.m}$$

$\vec{M}_C = 10.97\ \text{kN.m}\ \circlearrowright$

or $= 10.97(-\hat{k})\ \text{kN.m}$

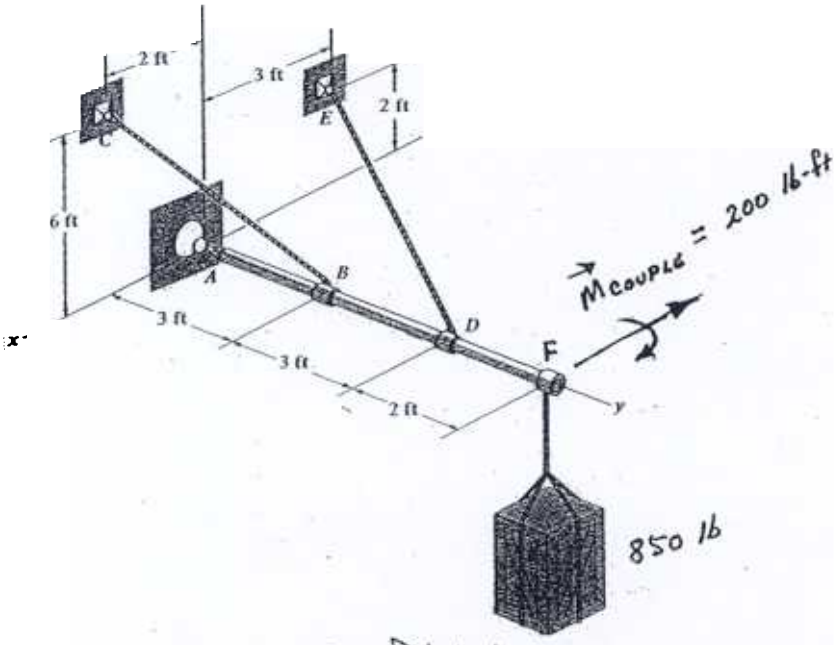
Name:_____ Student Number:_____

Note: Very little credit for correct answers without a clear explanation of reasoning.

Question #3: The boom supports a 850 lb weight and a couple $\vec{M}_{couple} = -200(\hat{i})\ lb - ft.$ at point-F.

(a) Draw the free body diagram.

(b) Determine the tension in cables BC and DE. (Do not have to solve for reaction forces at point-A).

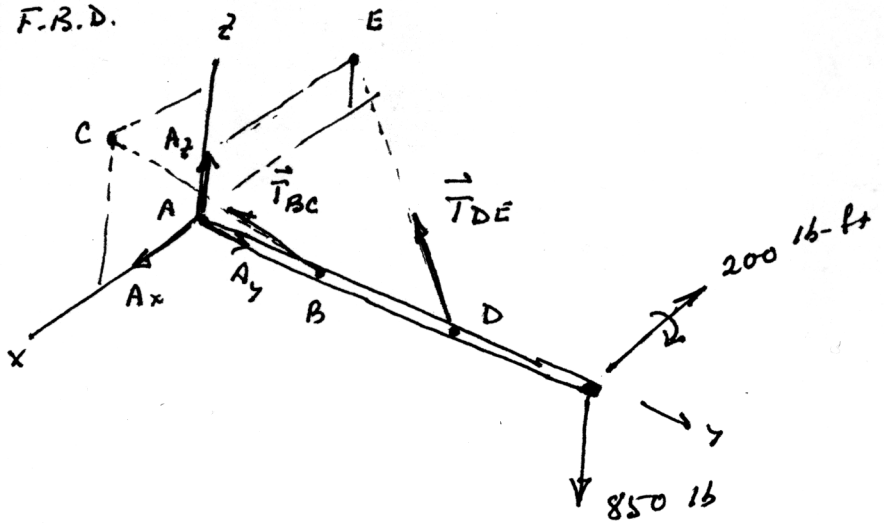


Mark Distribution

Part	Answers	Reasoning	Totals
a)	2	0	(2)
b)	2	6	(8)
		Total	10

Solution: Question - 3

a) F.B.D.



b) $\Sigma \vec{M}_A = 0 = ?$

$$\vec{T}_{BC} = T_{BC}\left[\frac{(2-0)\hat{i} + (0-3)\hat{j} + (6-0)\hat{k}}{\sqrt{(2)^2+(3)^2+(6)^2}}\right]$$

$$= T_{BC}\left[0.2857\,\hat{i} - 0.4286\,\hat{j} + 0.8571\,\hat{k}\right]$$

$$\vec{T}_{DE} = T_{DE}\left[\frac{(-3-0)\hat{i} + (0-6)\hat{j} + (2-0)\hat{k}}{\sqrt{(3)^2+(6)^2+(2)^2}}\right]$$

$$= T_{DE}\left[-0.4286\,\hat{i} - 0.8571\,\hat{j} + 0.2857\,\hat{k}\right]$$

$$\Sigma M_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ T_{BC}(0.2857) & -T_{BC}(0.4286) & T_{BC}(0.8571) \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 3 \\ T_{BC}(0.2857) & -T_{BC}(0.4286) \end{matrix}$$

$$+ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 6 & 0 \\ T_{DE}(-0.4286) & T_{DE}(-0.8571) & T_{DE}(0.2857) \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 6 \\ T_{DE}(-0.4286) & T_{DE}(-0.8571) \end{matrix}$$

$$+ 200(-\hat{i})$$

$$+ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & 0 \\ 0 & 0 & -850 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 8 \\ 0 & 0 \end{matrix}$$

Question-3 Continue

$$= \hat{i}\left[(3)T_{BC}(0.8571) - 0 + (6)T_{DE}(0.2857) - 0 - 200 + (8)(-850) - 0\right] +$$

$$\hat{j}\left[0 - 0 + 0 - 0 + 0 - 0\right]$$

$$\hat{k}\left[0 - (3)T_{BC}(0.2857) + 0 - (6)T_{DE}(-0.4286) + 0 - 0\right] +$$

$$= \hat{i}\left[T_{BC}(2.571) + T_{DE}(1.714) - 7000\right] +$$

$$\hat{j}\left[0\right] +$$

$$\hat{k}\left[T_{BC}(-0.8571) + T_{DE}(2.572)\right]$$

$\hat{k}$'s = 0 $\quad T_{BC} = T_{DE}(2.572)/0.8571 = 3.000\,T_{DE}$

$\hat{i}$'s = 0 $\quad (3.000\,T_{DE})(2.571) + T_{DE}(1.714) = 7000$

$$T_{DE} = \frac{7000}{9.427} = 742.5$$

$T_{DE} = 743$ lb

$T_{BC} = 3.00(743) = 2228$

$T_{BC} = 2230$ lb

Name:_____ Student Number:_____

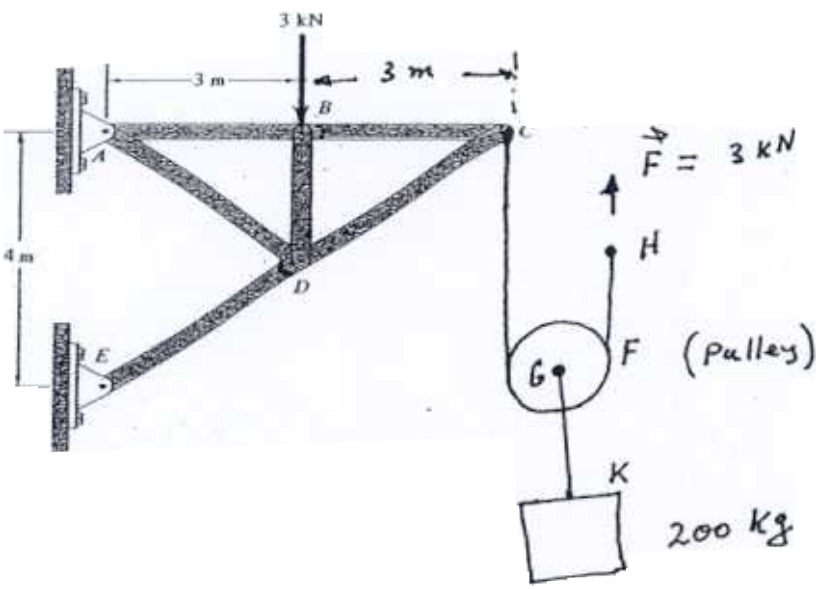
Note: Very little credit for correct answers without a clear explanation of reasoning.

Question #4: A cable is attached to point-C on the truss. This cable goes under the pulley and is being pulled vertically at point-H by the force, $\vec{F}$ = 3 kN. A 200 kg mass is attached to the cable G-K. The pulley has negligible mass and no friction.

(a) Show that the acceleration of 200 kg mass is equal to half the acceleration of point-H (the end of the cable).

(b) Determine the tension in the cable G-K (i.e. the cable attached to the 200 kg mass).

(c) Determine the force in members DC, BC, and DB, and state if the members are in tension or compression.

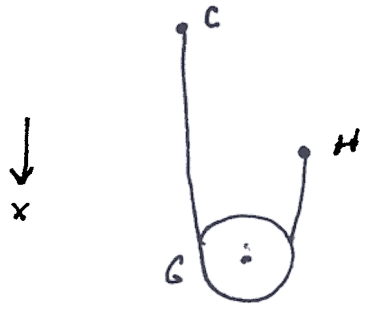


Mark Distribution

	Answers	Reasoning	Totals
a)	0	2	(2)
b)	1	1	(2)
c)	2	4	(6)
		Total	[10]

Solution: Question #4

a)



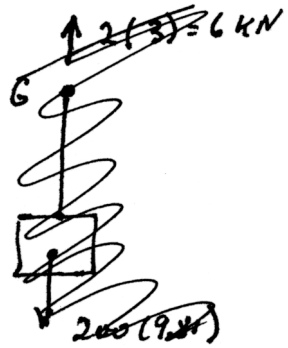
$$l = x_G - x_C + x_G - x_H$$

$$0 = \frac{dl}{dt} = v_G - \cancel{v_C}^{0} + v_G$$

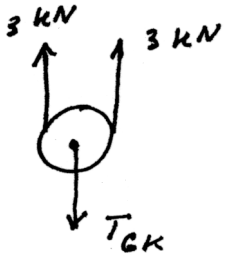
$$\therefore \quad v_G = \frac{1}{2} v_H$$

$$\frac{dv_G}{dt} = \frac{1}{2}\frac{dv_H}{dt} \qquad \therefore \quad a_G = \frac{1}{2} a_H$$

b)



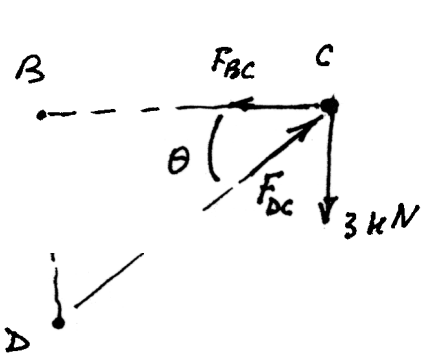
F.B.D. fn pulley:



$\Sigma F_x = ma_x \qquad m_{pulley} = 0$

$\therefore \Sigma F_x = 0 \Rightarrow T_{CK} = 2(3) = 6\ kN$

c) Pin – C



$$\theta = \text{Arctan}\left(\frac{4}{6}\right) = 33.69^\circ$$

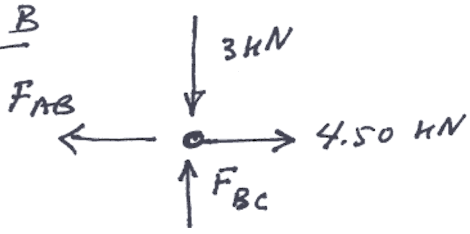
$$\Sigma F_y = 0 = -3 + F_{DC}\sin(33.69) = 0$$

$$F_{DC} = 5.41\ kN \text{ (Compression)}$$

$$\Sigma F_x = 0$$

$$-F_{BC} + (5.41)\cos(33.69) = 0 \qquad F_{BC} = 4.50\ kN \text{ (Tension)}$$

Pin – B



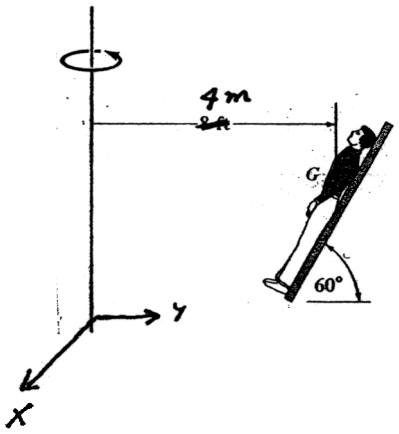
$$\Sigma F_y = 0 \Rightarrow F_{BC} = 3\ kN \text{ (Compression)}$$

Name:_____ Student Number:_____

Note: Very little credit for correct answers without a clear explanation of reasoning.

Question #5: A 80 kg student lies against the cushion. The student and cushion rotate about the z-axis at speed v = 3 m/s, which is changing at rate dv/dt = 2 m/s^2. At the instant shown, the student and cushion are in the y-z plane. The student is not sliding on the cushion.

(a) Draw the free body diagram for the student. Neglect the size of the student.
(b) Determine the x-component of the friction force exerted by the cushion on the student.
(c) Determine the normal force exerted by the cushion on the student (i.e. the force perpendicular to the plane of the cushion).
(d) Determine the y-component of the friction force exerted by the cushion on the student.
(e) What is the minimum static coefficient of friction, μ_s, needed to keep the student from slipping on the cushion.



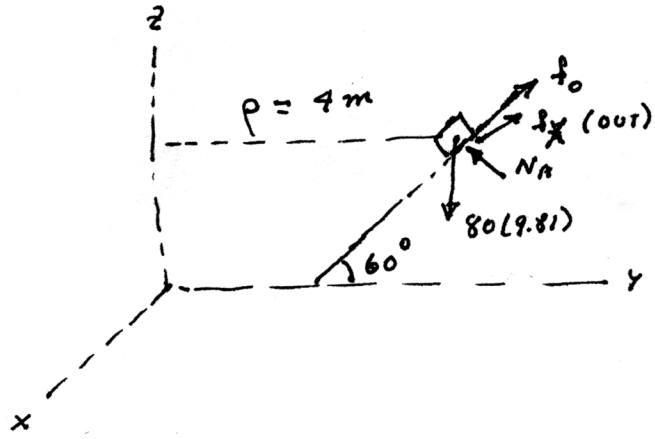
Mark Distribution

	Answer	Reasoning	Total
a)	2	ø	(2)
b)	1	1	(2)
c)	1	2	(3)
	1	ø	(1)
	0	2	(2)

Total: 10

Solution: Question #5

(a)



friction force, ~~$\vec{f} = \vec{f}_o + \vec{f}_x$~~

$\vec{f} = \vec{f}_o + \vec{f}_x$

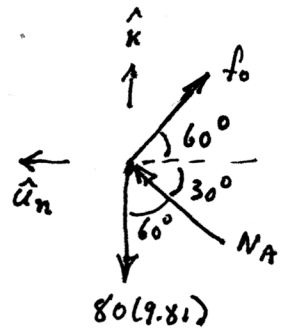
$v = 3\ m/s \qquad \frac{dv}{dt} = 2\ m/s^2$

(b) $\Sigma F_x = m a_{tx}$

$f_x = (80) a_t = (80) \frac{dv}{dt} = 80(2) = \underline{160\ N}$

$\vec{f}_x = 160(-\hat{i})\ N$

(c)



$\Sigma F_z = m a_z = 0$ $\quad a_z = 0$ because z = Constant

$(N_A \sin 30°) + (f_o \sin 60°) - 80(9.81) = 0$

$(0.5\, N_A) + (0.8660\, f_o) = 784.8 \quad (1)$ ✓

$\Sigma F_n = m a_n = m v^2/\rho$

$(N_A \sin 60°) - (f_o \cos 60°) = (80)(3)^2/4 = 180$

$N_A (0.8660) - (f_o (0.5)) = \cancel{240}\ 180$ ✓

$N_A = \frac{\cancel{240}^{180} + 0.5 f_o}{0.8660} = \cancel{277.1}^{207.9} + 0.5774 f_o$

sub (1) $0.5\left[\cancel{277.1}^{207.9} + 0.5774 f_o\right] + [0.8660 f_o] = 784.8$

$\cancel{138.6}^{103.9} + 0.2887 f_o + 0.8660 f_o = 784.8$

$(0.2887 + 0.8660) f_o = \cancel{646.2}\ 680.9$

$f_o = \cancel{559.1}\ 589.7$

$N_A = \cancel{277.1}^{207.9} + 0.5774 (\cancel{589.7}^{589.7}) = \cancel{600.3}^{548.3}$

$N_A = \cancel{600}^{548}\ N$

[Don't worry about Direction]

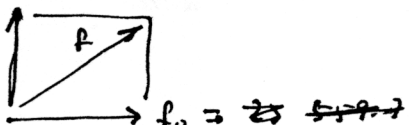
(d) $[f_o]_y = f_o \cos 60° = (\cancel{589.7}^{589.7}) \cos 60 = \underline{\cancel{280\ N}\ 274.2\ N}$

Solution: Question # 5 (Continued)

(e) $\mu_s = \dfrac{f}{N_A}$

$f = \left[(160)^2 + (589.7)^2 \right]^{1/2} = 611.0$ ~~611.1~~ N

$f_x = 160\ N$



$\mu_s = \dfrac{611.0}{600} = 1.018 = 1.02$ ~~0.970~~