

The following are excerpts from the Stellarium User Guide prepared by Matthew Gates on 11 March 2009. The full version can be found on the Stellarium web site: <http://www.stellarium.org/>

Stellarium is a software project that allows people to use their home computer as a virtual planetarium. It calculates the positions of the Sun and Moon, planets and stars, and draws how the sky would look to an observer depending on their location and the time. It can also draw the constellations and simulate astronomical phenomena such as meteor showers, and solar or lunar eclipses. Stellarium may be used as an educational tool for teaching about the night sky, as an observational aid for amateur astronomers wishing to plan a night's observing, or simply as a curiosity. (It's fun!). Because of the high quality of the graphics that Stellarium produces, it is used in some real planetarium projector products. Some amateur astronomy groups use it to create sky maps for describing regions of the sky in articles for newsletters and magazines.

Stellarium is under fairly rapid development, and by the time you read this guide, a newer version may have been released with even more features than those documented here. Check for updates to Stellarium at the Stellarium website. If you have questions and/or comments about this guide, please email the author. For comments about Stellarium itself, visit the Stellarium forums.

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Tour

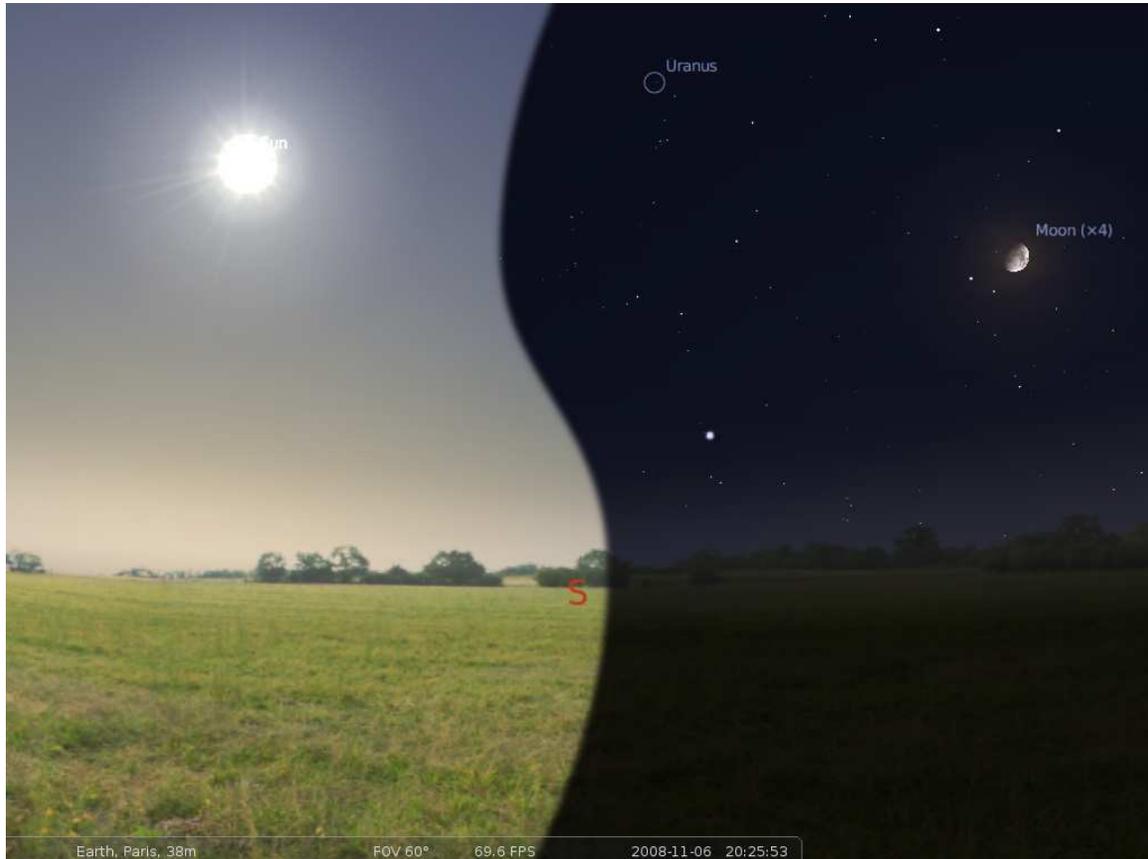


Figure 1 A composite screenshot showing Stellarium in both day time (left) and night time (right)

At the bottom left of the screen, you can see the status bar. This shows the current observer location, field of view (FOV), graphics performance in frames per second (FPS) and the current simulation date and time.

The rest of the view is devoted to rendering a realistic scene including a panoramic landscape and the sky. If the simulation time and observer location are such that it is night time, you will see stars, planets and the moon in the sky, all in the correct positions.

You can drag with the mouse on the sky to look around or use the cursor keys. You can zoom with the mouse wheel or the page up/page down keys. If you move the mouse over the status bar, it will move up to reveal a tool bar which gives quick control over the program.

Time Travel

When Stellarium starts up, it sets its clock to the same time and date as the computer's system clock. However, Stellarium's clock is not fixed to same time and date as the system clock, or indeed to the same speed. We may tell Stellarium to change how fast time should pass, and even make time go backwards! So the first thing we shall do is to travel into the future!

Let's take a look at the time control buttons on the right hand side of the tool-bar (shown in the table below).

<i>Button</i>	<i>Shortcut key</i>	<i>Description</i>
	j	Decrease the rate at which time passes
	k	Make time pass as normal
	l	Increase the rate at which time passes
	8	Return to the current time & date

If you hover the mouse cursor over the buttons, a short description of the button's purpose and keyboard shortcut will appear.

OK, so let's go see the future!

Click the mouse once on the increase time speed button. Not a whole lot seems to happen. However, take a look at the clock in the status bar. You should see the time going by faster than a normal clock! Click the button a second time. Now the time is going by faster than before. If it's night time, you might also notice that the stars have started to move slightly across the sky. If it's daytime you might be able to see the sun moving (but it's less apparent than the movement of the stars). Increase the rate at which time passes again by clicking on the button a third time. Now time is really flying!

Let time move on at this fast speed for a little while. Notice how the stars move across the sky. If you wait a little while, you'll see the Sun rising and setting. It's a bit like a time-lapse movie. Stellarium not only allows for moving forward through time - you can go backwards too!

Click on the real time speed button. The stars and/or the Sun should stop scooting across the sky. Now press the decrease time speed button once. Look at the clock. Time has stopped. Click the Decrease time speed button four or five more times. Now we're falling back through time at quite a rate (about one day every ten seconds!). Enough time travel for now. Wait until it's night time, and then click the Real time speed button. With a little luck you will now be looking at the night sky.

Moving around the sky

As well as travelling through time, Stellarium lets to look around the sky freely, and zoom in and out. There are several ways to accomplish this, listed in the table below

<i>Key</i>	<i>Description</i>
Cursor keys	Pan the view left, right, up and down
Page up / Page down	Zoom in and out
Backslash (\)	Auto-zoom out to original field of view
Left mouse button	Select an object in the sky
Right mouse button	Clear selected object
Mouse wheel	Zoom in and out
Space	Centre view on selected object
Forward-slash (/)	Auto-zoom in to selected object

Let's try it. Use the cursors to move around left, right, up and down. Zoom in a little using the Page Up key, and back out again using the Page Down. Press the backslash key and see how Stellarium returns to the original field of view (how "zoomed in" the view is), and direction of view.

It's also possible to move around using the mouse. If you left-click and drag somewhere on the sky, you can pull the view around. Another method of moving is to select some object in the sky (left-click on the object), and press the Space key to centre the view on that object. Similarly, selecting an object and pressing the forward-slash key will centre on the object and zoom right in on it.

The forward-slash and backslash keys auto-zoom in and out to different levels depending on what is selected. If the object selected is a planet or moon in a sub-system with a lot of moons (e.g., Jupiter), the initial zoom in will go to an intermediate level where the whole sub-system should be visible. A second zoom will go to the full zoom level on the selected object. Similarly, if you are fully zoomed in on a moon of Jupiter, the first auto-zoom out will go to the sub-system zoom level. Subsequent auto-zoom out will fully zoom out and return the initial direction of view. For objects that are not part of a sub-system, the initial auto-zoom-in will zoom right in on the selected object (the exact field of view depending on the size/type of the selected object), and the initial auto-zoom out will return to the initial FOV (field of view) and direction of view.

Main tool bar

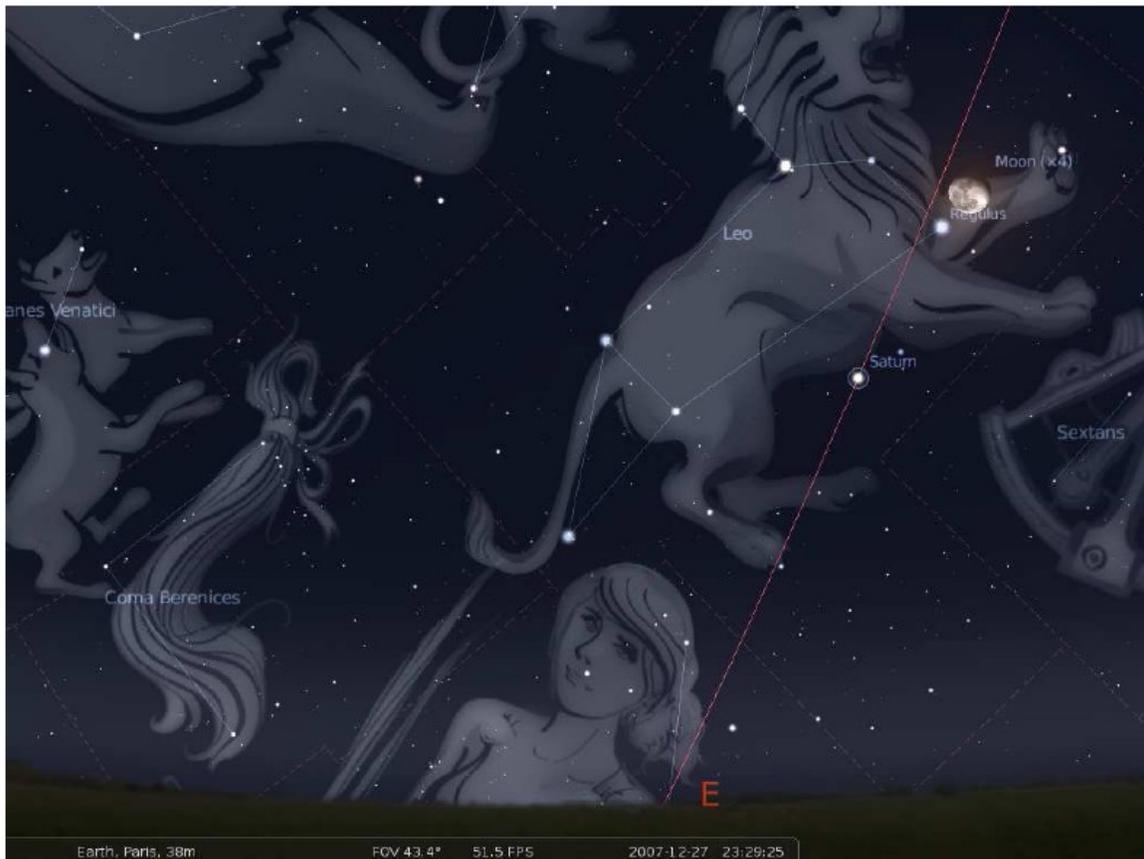


Figure 2 Screenshot showing some of Stellarium's visual effects

Stellarium can do a whole lot more than just draw the stars. Figure 2 shows some of Stellarium's visual effects including constellation line and boundary drawing, constellation art, planet hints, and atmospheric fogging around the bright Moon. The controls main tool-bar provides a mechanism for turning on and off the visual effects.

When the mouse is moved to the bottom left of the screen, a second tool-bar becomes visible. All the buttons in this side tool-bar open and close dialog boxes which contain controls for further configuration of the program.

The table below (continuing onto the next page) describes the operations of buttons on the main tool-bar and the side tool-bar, and gives their keyboard shortcuts.

<i>Feature</i>	<i>Tool-bar button</i>	<i>Key</i>	<i>Description</i>
Constellations		c	Draws the constellation lines
Constellation Names		v	Draws the name of the constellations
Constellation Art		r	Superimposes artistic representations of the constellations over the stars
Equatorial Grid		e	Draws grid lines for the RA/Dec coordinate system
Azimuth Grid		z	Draws grid lines for the Alt/Azi coordinate system
Toggle Ground		g	Toggles drawing of the ground. Turn this off to see objects that are below the horizon
Toggle Cardinal Points		q	Toggles marking of the North, South, East and West points on the horizon
Toggle Atmosphere		a	Toggles atmospheric effects. Most notably makes the stars visible in the daytime
Nebulae & Galaxies		n	Toggles marking the positions of Nebulae and Galaxies when the FOV is too wide to see them

Planet Hints		p	Toggles indicators to show the position of planets
Coordinate System		Enter	Toggles between Alt/Azi & RA/Dec coordinate systems
Goto		Space	Centres the view on the selected object
Night Mode		[none]	Toggle “night mode”, which changes the coloring of some display elements to be easier on the dark-adapted eye.
Full Screen Mode		F11	Toggle full screen mode.
Flip image (horizontal)		CTRL+SHIFT+h	Flips the image in the horizontal plane. Note this button is not enable by default. See section ??
Flip image (vertical)		CTRL+SHIFT+v	Flips the image in the vertical plane. Note this button is not enable by default. See section ??
Quit Stellarium		CTRL-Q	Close Stellarium. Note: the keyboard shortcut is COMMAND-Q on OSX machines
Help Window		F1	Show the help window, which lists key bindings and other useful information
Configuration Window		F2	Show the display of the configuration window

Object Search window

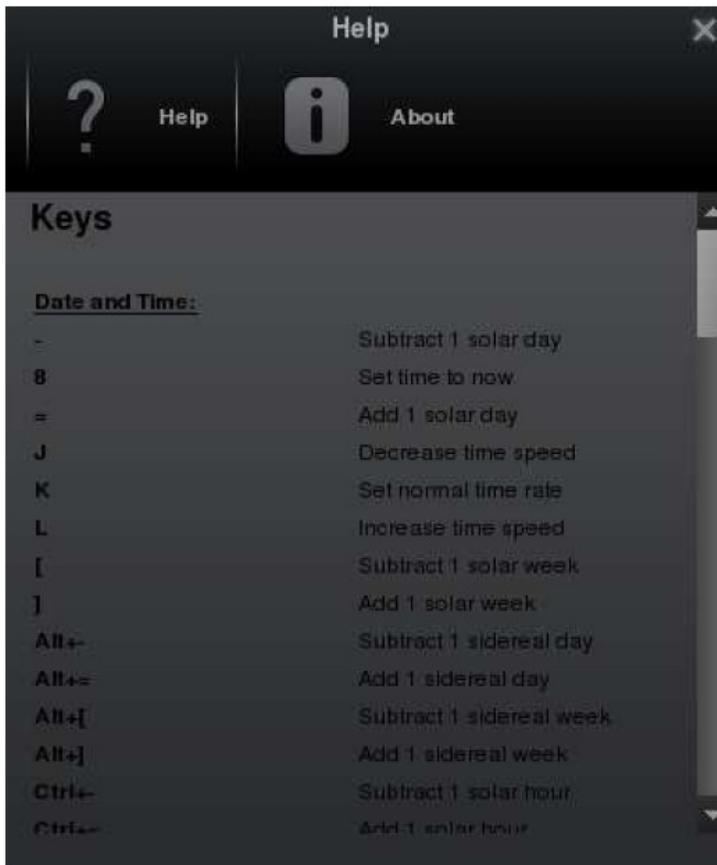
The Object Search window provides a convenient way to locate objects in the sky. Simply type in the name of an object to find, and then click the “go” button or press RETURN. Stellarium will point you at that object in the sky.

As you type, Stellarium will make a list of objects which begin with what you have typed so far. The first of the list of matching objects will be highlighted. If you press the TAB key, the selection will change to the next item in the list. Hitting the RETURN key will go to the currently highlighted object and close the search dialog.



For example, suppose we want to locate Mimas (a moon of Saturn). After typing the first letter of the name, “m”, Stellarium makes a list of objects whose name begins with M: Mars, Mercury, Mimas, Miranda, Moon. The first item in this list, Mars, is highlighted. Pressing return now would go to Mars, but we want Mimas. We can either press TAB twice to highlight Mimas and then hit RETURN, or we can continue to type the name until it is the first/only object in the list.

Help window



The Help window lists all of Stellarium’s key-strokes. Not that some features are only available as key strokes, so it’s a good idea to have a browse of the information in this window. The “About” tab in this window will show licensing information, and a list of people who helped to produce the program.

Configuration

Most of Stellarium's configuration is done using the configuration window and the view window. To open the configuration window, click the button on the left side tool-bar or press F2. To open the view window click the button if the left side tool-bar or press F4.

Setting the Date and Time

In addition to the time rate control buttons on the main tool-bar, you can use the date and time window to set the simulation time. The values for year, month, day, hour, minutes and seconds may be modified by typing new values, by clicking the up and down arrows above and below the values, and by using the mouse wheel.

Setting Your Location



Figure 3 Stellarium location window

The positions of the stars in the sky is dependent on your location on Earth (or other planet) as well as the time and date. For Stellarium to show accurately what is (or will be/was) in the sky, you must tell it where you are. You only need to do this once - Stellarium can save your location so you won't need to set it again until you move.

To set your location, you can press F6 or the proper icon on the menu bar to open the location window. There are a few ways you can set your location:

1. Just click on the map.
2. Search for a city where you live using the search edit box at the top right of the window, and select the right city from the list.
3. Enter a new location using the longitude, latitude and other data.

The time zone is taken from the operating system environment.

The View Settings Window

The View settings window controls many display features of Stellarium which are not available via the main tool-bar.

Sky Tab

The Sky tab of the View window contains settings for changing the general appearance of the main sky view. Some highlights:

Absolute scale is the size of stars as rendered by Stellarium. If you increase this value, all stars will appear larger than before.

Relative scale determines the difference in size of bright stars compared to faint stars. Values higher than 1.00 will make the brightest stars appear much larger than they do in the sky. This is useful for creating star charts, or when learning the basic constellations.

Twinkle controls how much the stars twinkle.

Dynamic eye adaptation When enabled this feature reduces the brightness of faint objects when a bright object is in the field of view. This simulates how the eye can be dazzled by a bright object such as the moon, making it harder to see faint stars and galaxies.

Light pollution In urban and suburban areas, the sky is brightened by terrestrial light pollution reflected in the atmosphere. Stellarium simulates light pollution and is calibrated to the *Bortle Dark Sky Scale* where 1 means a good dark sky, and 9 is a very badly light-polluted sky.

Planets and satellites This group of options lets you turn on and off various features related to the planets. Simulation of light speed will give more precise positions for planetary bodies which move rapidly against background stars (e.g. the moons of Jupiter). The *Scale Moon* option will increase the apparent size of the moon in the sky, which can be nice for wide field of view shots.

Labels and markers You can independently change the amount of labels displayed for planets, stars and nebulae. The further to the right the sliders are set, the more labels you will see. Note that more labels will also appear as you zoom in.

Shooting stars Stellarium has a simple meteor simulation option. This setting controls how many shooting stars will be shown. Note that shooting stars are only visible when the time rate is 1, and might not be visible at some times of day. Meteor showers are not currently simulated.



Figure 4 The “Sky” tab of the “View” window

Markings Tab

The Markings tab of the View window controls the following features:

Celestial sphere This group of options makes it possible to plot various grids and lines in the main view.

Constellations These controls let you turn on and off constellation lines, names, art and boundaries, and control the brightness of the constellation artwork.

Projection Selecting items in this list changes the projection method which Stellarium uses to draw the sky. Options are:

cylinder The full name of this projection mode is cylindrical equidistant projection. The maximum field of view in this mode is 233° .

equal area The full name of this projection method is, Lambert azimuthal equal area projection. The maximum field of view is 360° .

fisheye Stellarium draws the sky using azimuthal equidistant projection. In fisheye projection, straight lines become curves when they appear a large angular distance from the centre of the field of view (like the distortions seen with very wide angle camera lenses). This is more pronounced as the user zooms out. The maximum field of view in this mode is 180° .

Hammer-Aitoff The Hammer projection is an equal-area map projection, described by Ernst Hammer in 1892 and directly inspired by the Aitoff projection. The maximum field of view in this mode is 360° .

mercator Mercator projection preserves the angles between objects, and the scale around an object the same in all directions. The maximum field of view in this mode is 233° .

orthographic Orthographic projection is related to perspective projection, but the point of perspective is set to an infinite distance. The maximum field of view is 180° .

perspective Perspective projection keeps the horizon a straight line. The maximum field of view is 150° . The mathematical name for this projection method is “gnomonic” projection.

stereographic This mode is similar to fish-eye projection mode. The maximum field of view in this mode is 235° .



Figure 5 The “Markings” tab of the “View” window

Landscape Tab

The Landscape tab of the View window controls the landscape graphics (ground). To change the landscape graphics, select a landscape from the list on the left side of the window. A description of the landscape will be shown on the right. Note that while landscape can include information about where the landscape graphics were taken (planet, longitude, latitude and altitude), this location does not have to be the same as the location selected in the Location window, although you can set up Stellarium such that selection of a new landscape will alter the location for you.

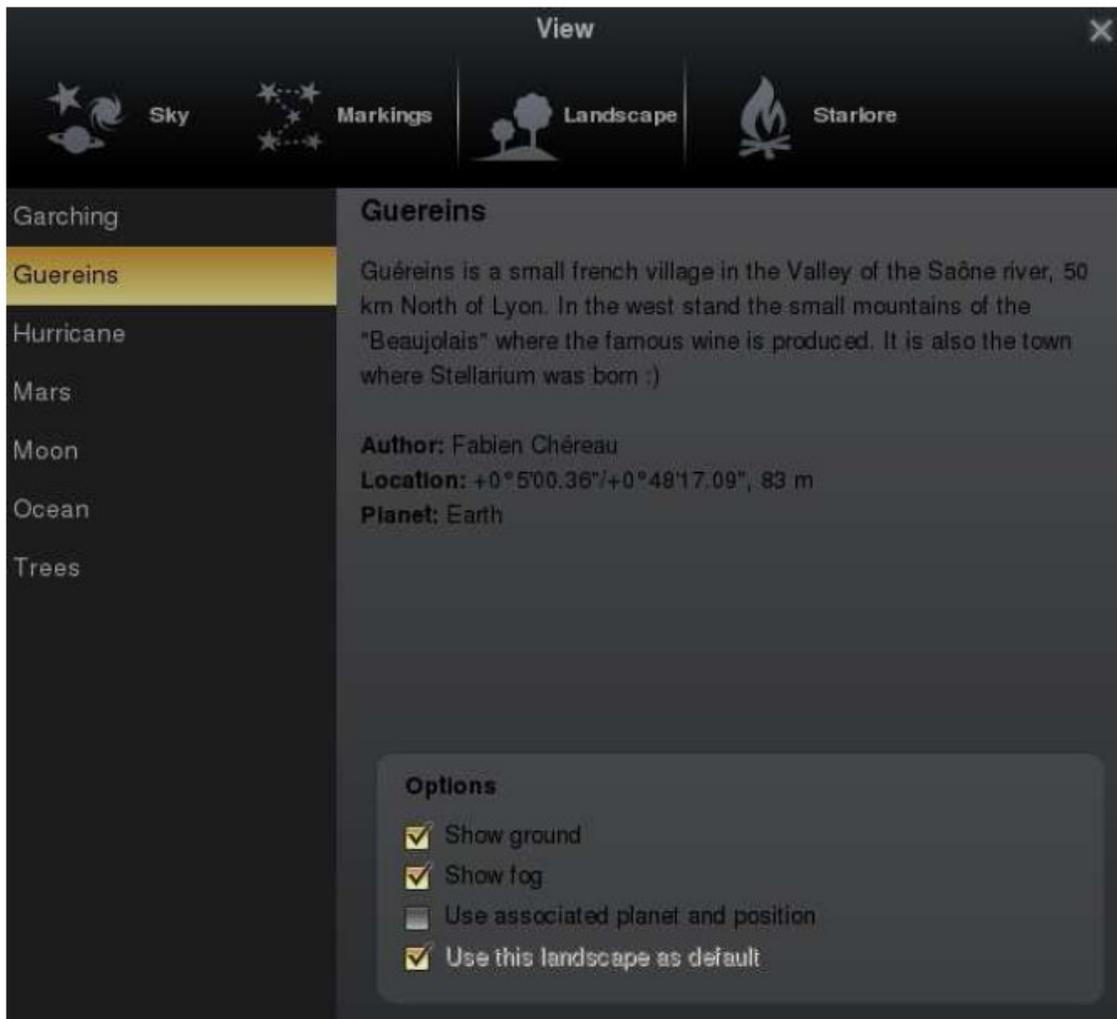


Figure 6 The “Landscape” tab of the “View” window

The controls at the bottom right of the window operate as follows:

Show ground This turns on and off landscape rendering (same as the button in the main tool-bar).

Show fog This turns on and off rendering of a band of fog/haze along the horizon.

Use associated planet and position When enabled, selecting a new landscape will automatically update the observer location.

Starlore Tab

The Starlore tab of the View window controls what culture's constellations and bright star names will be used in the main display. Some cultures have constellation art (Western and Inuit), and the rest do not.

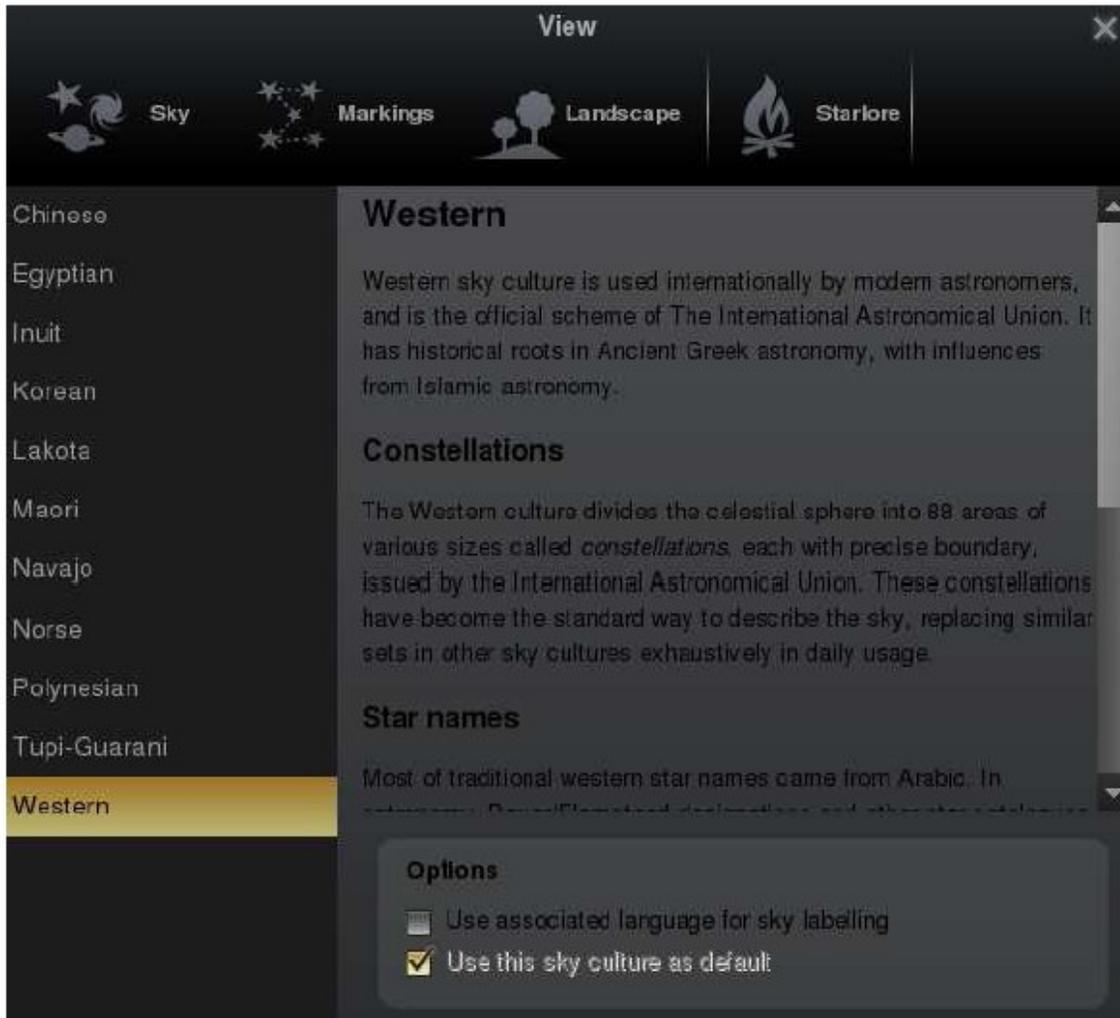


Figure 7 The “Starlore” tab of the “View” window

Dance of the Planets

Appendix B

Introduction

Dance of the Planets is the modern-day version of an "orrery" - a complete Solar System simulator- It will track the orbits of the planets, most of their known satellites and an extensive catalogue of asteroids and comets through hundreds of years. This is quite a complex task, and it can take a while to become completely familiar with the operation of the program- To save time, the projects involving Dance of the Planets have been set up as preconfigured files which you can select directly from a command menu, saving you the effort of becoming a computer expert in two hours.

Using Dance of the Planets

After Dance of the Planets starts, you will have to press the Space bar twice to clear the two information screens at the beginning. For most purposes, all you need to know is how to load pre-configured setups into the program. Press **M** and a menu will appear - Press **M** again to select "More Options". Now press **E** to select "Resumes" and to load the file. You will be prompted for the file name- Type the name of the file (which will be given in the appropriate lab instructions; see, e.g., Chapter 3) and press RETURN. The view will change to the new configuration. Then follow the instructions in the lab manual.

When you have completed the lab tasks involving SkyGlobe, you may wish to try other things with it - from checking past and future passages of Halley's Comet to following the shepherd satellites which keep Saturn's rings in place.

Additional commands

Dance of the Planets operates in several modes and the commands available to you will depend somewhat on the mode. The modes are:

1. the solar system as seen from 270 AU,
2. the sky from the surface of the Earth,
3. planets as the central object, as seen from space,
4. view from Earth but with a planet, the Sun or the Moon locked at the centre of the screen, and
5. close-up view of the Earth from inside the Moon's orbit.

Most of the time, modes 1 to 3 will be used in the lab exercises in this course.

- F1** Help! Displays a list of commands with a brief description-
- M** Displays a menu of options. This list will be different in different modes. The option you will need most is **M** for "More Options" followed by **R** to load a new con-figuration. Just type the name of the configuration file that you want to load and press **RETURN**.
- L** Label the objects on the screen. This will pause the simulation until you press **L** again to remove the labels.
- D** Change the date- To enter a new date move the cursor using the arrow keys and type in the new date. The format is MM/DD-HH/YYYY. The hour range is 00 - 23 (midnight is 00). The year can range from -4680 to 10000. Pressing **C** will set the time to the current time and date.
- V** Changes the viewing direction. When the view is from Earth use the arrow keys to change the RA and Dec that will be in the centre of the screen. When viewing from space, the coordinates are in ecliptic latitude and longitude. An object's ecliptic latitude is its angle above or below the plane of the ecliptic - analogous to terrestrial latitude, which is an angle above or below the Earth's equator. A ecliptic latitude of 90° S means that you are looking vertically down onto the Solar System; one of 0° means you are looking at it "edge on". Ecliptic longitude is the object's angle along the ecliptic. After changing the view direction, press **RETURN** to continue the simulation from this new view.
- P** Changes the pace of the simulation. Use the ↑ and ↓ keys to cycle through Available speeds. Negative speeds run the simulation backwards. A speed of "true" is real time.
- Z** Zoom in on the center of the screen. A zoom of "1x" is a direct view - again, use the ↑ and ↓ keys to change this value.
- F** Find an object, constellation or star. The objects that Find will look for change depending on the viewing mode. Generally just type as much of the objects name as makes it unarnbiguous.
- S** An on/off toggle for sky lines (i.e., traces of orbits, constellation lines, grid lines and star labels).
- R** Refresh the screen.
- I** Invert the display to correspond to a telescope view. Changing the view direction whilst in this mode may be confusing.
- A** Displays a text screen showing detailed information and configuration options. You will not normally need to use this feature.
- ESC** Quit Dance of the Planets. The program will ask "Quit? (Answer **Y** or **N**) **ESC** can also be used to cancel the menus started by **M**.

1. Types of error

Any physical measurement obtained from the reading of an instrument or graph can never be exactly correct. Its accuracy is always limited by the particular measuring device used, by independent random effects which introduce small deviations, and sometimes by systematic biases of the observer herself. To give a recorded result physical meaning, it is essential to know the reliability of the measurement; i.e., how likely is it to be close to the “true” value.

For instance, if we are measuring the separation of two points with a ruler scaled in mm, the reading obtained can hardly be more accurate than 1/10th of a division. Even this depends on your confidence that you can subdivide the mm divisions into ten equal and distinguishable parts by eye. This type of uncertainty is called an *instrumental error*, or a *reading error*. In many cases the estimation of this error is subjective and based on common sense – into how many distinct parts can the smallest scale division of this instrument be sub-divided? If we are convinced that the mm graduations on the ruler can be divided into ten parts, a reading of 7.2 mm would be recorded as (7.2 ± 0.1) mm or $7.2 \text{ mm} \pm 0.1 \text{ mm}$.

Consider a voltmeter with an analogue (rather than a digital) readout whose markings on the dial are even closer together. If we decide that we can only subdivide each interval into only five distinguishable parts, a measurement might be recorded as (6.4 ± 0.2) V.

Keep in mind that, if a reading x has an estimated error Δx , then the error must have the same dimensions or units as the quantity itself. One can remove the dependence on units by calculating the ratio $\Delta x / x$, known as the *relative error*, or $\Delta x / x \times 100$, the *percentage error*. Since these quantities are dimensionless, relative errors from many different types of measurements (with different units) can be readily combined (see Section 2 below), whereas the errors in their original units cannot.

1.1 Random errors

By repeating a measurement and making many independent readings of the same quantity, you can improve the accuracy of the result. How the accuracy improves with increasing number of measurements n depends on the nature of errors, but it generally improves as $1/n^{1/2}$ (see Section 1.2).

Say you are measuring the brightness of a star of constant brightness with a telescope and electronic detector like those used in these lab projects. Our

telescope collects photons of starlight (as well as the background sky) which are focused onto the detector and recorded. In ten seconds, we count a certain number. In a second 10-sec exposure, we count a similar but slightly different number. If we continue, we will eventually accumulate a large number of individual photon counts which tend to scatter about some average, presumably the true value. The variations in readings may be due in part to the detector, but for a fainter star, much of the scatter will be due to genuine fluctuations in the number of arriving photons. (Astronomers refer to this as photon noise or photon counting statistics.) The larger the flux of photons, the lower is this source of noise.}

Errors of this type are dubbed random errors and can be treated by statistical methods. If n measurements are made of a physical quantity x , the average value is $\langle x \rangle$, where

$$\langle x \rangle = (1/n) (x_1 + x_2 + \dots + x_i + \dots + x_n) = (1/n) \sum_{(i=1 \rightarrow n)} x_i \quad (1)$$

Such errors can be treated statistically, leading to the standard deviation or the standard error associated with a single observation within a data set and with the average of the entire set itself.

1.2 Standard deviation

In our example above, we were accumulating 10-second integrations of photons from a star. After obtaining 25 readings, say we plotted a histogram or bar graph of the number of measurements which fell within certain ranges of photon count (in bins 1000 counts wide). An example is shown in Figure 1. In the idealised case, where one could make an infinite (or at least extremely large) number of readings, your histogram would approach a continuous distribution resembling the curve in Figure 2. This is known as a Gaussian distribution with the mean given by the Greek letter μ (“mu”).

The value μ will be the accurate mean for the “population” of measurements (i.e., the average of a very large number of readings (where n goes to ∞), whereas $\langle x \rangle$ is the experimental average value of a relatively small number of discrete readings. For the Gaussian curve of Figure 2, the population’s standard deviation σ (“sigma”), or its root-mean-square deviation, is

$$\sigma_{true} = [(1/n) \sum_{(i=1 \rightarrow n)} (x_i - \mu)^2]^{1/2} \quad (2)$$

where n is very large. This quantity σ_{true} is a measure of the “sharpness” of the peak of the curve. Any single measurement of this population has a 68% chance of falling in the range from $\mu - \sigma$ to $\mu + \sigma$, and a 95% chance of being in the range $\mu \pm 2\sigma$. Thus, the standard deviation is a very practical way to quantify the accuracy of a measurement.

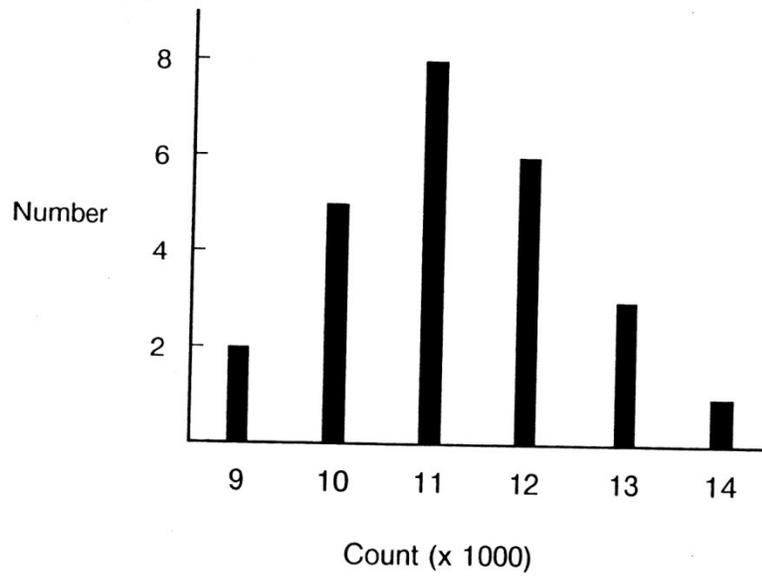


Figure 1 A distribution of photon counts with a mean of 11,000.

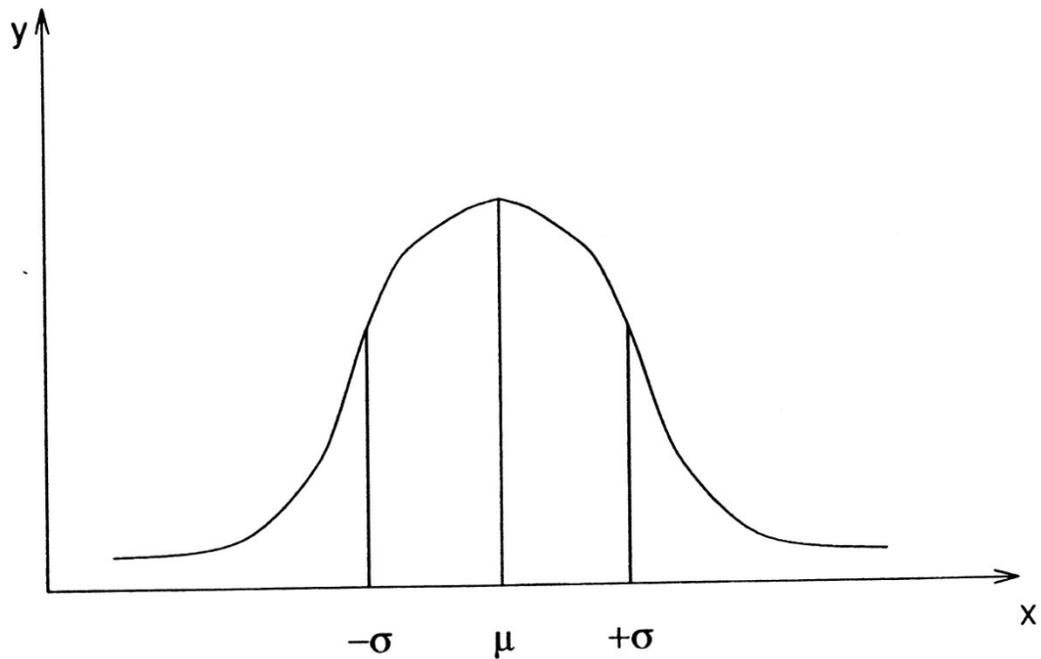


Figure 2 A Gaussian distribution with mean μ and standard deviation σ .

Unfortunately, in real life, we cannot calculate a true σ_{true} since we do not know the population value of $\langle x \rangle = \mu$. (If we did, we wouldn't have to make the measurement in the first place!) However, we do know the experimental mean $\langle x \rangle$ of our sample of data. The best estimate of the standard deviation of the population from which the sample is taken is given by

$$\sigma = [(1/(n - 1)) \sum_{(i=1 \rightarrow n)} (x_i - \langle x \rangle)^2]^{1/2} \quad (3)$$

This is the formula you will use in the labs when you are asked to calculate the standard deviation σ of a single measurement. For reasonably large numbers of data, this equation approaches the ideal case. Even with only 10 readings, the ratio $(n/(n-1))^{1/2} = (10/9)^{1/2} = 1.05$, so equation (3) is a good approximation to the actual uncertainty.

Often, however, you are asked to determine the uncertainty of the mean itself, known as the standard error. As you might expect, this will be smaller than the error of any single measurement in the set. It can be shown the standard deviation of any sample of measurements is

$$\sigma(\langle x \rangle) = \sigma / n^{1/2} \quad (4)$$

To calculate the standard error in a result based on an average of several measurements, you will use equations (3) and (4) together. Notice that equation (4) means, to halve the error in the mean, you must quadruple the number of measurements. Improving experimental accuracy takes a great deal of effort. Because the standard error drops relatively slowly with sample number, in general, a medium-sized sample of accurate readings is better than a very large set of data of poor quality.

The treatment above assumes there is a spread of readings. If repeated readings all give the same value – for instance, if we read temperature from a thermometer only to the nearest degree and the deviations are less than that – then the error quoted must be the reading error; in this case, ± 1 degree.

Many scientific calculators are designed to find the standard deviation σ directly.

Example 1

Take the 25 readings plotted in Figure 1.

The mean value of x is

$$\langle x \rangle = (1/25) [(2 \times 9000) + (5 \times 10000) + (8 \times 11000) + (6 \times 12000) + (3 \times 13000) + (1 \times 14000)] = 11,240$$

The best estimate of standard deviation (from equation (3)) is $\sigma = [(1/24) \sum_{(i=1 \rightarrow n)} (x_i - \langle x \rangle)^2]^{1/2} \approx 1270$. This means that any given reading taken has a 68% chance of being between $11240 - 1270 = 9970$ and $11240 + 1270 = 12510$.

The best estimate of the standard error of the mean (equation (4)) is $\sigma(\langle x \rangle) = 1270 / (25)^{1/2} \approx 250$.

Therefore, the average photon count should be recorded as **11240 ± 250**.

1.3 Systematic errors

There is another type of error which is not random. Such systematic errors are constant factors present throughout the measurements. Examples of systematic errors include: measuring a meter reading from one edge of the dial needle rather than the middle; failing to take into account a constant in the readings (e.g., the orbital velocity of the Earth in measuring the radial velocity of a star); applying the wrong constant in a calculation; etc.

Note that simple counting of integral quantities (such as the number of stars in a certain portion of the sky) is in a different category as far as errors are concerned. The resulting number is exact, assuming that the counting has been done accurately. However, such measurements may be subject to observational selection effects. For example, you can only count the number of stars in an image which are bright enough to be detected. Many of your experiments in these lab projects (such as sorting stars according to spectral type or galaxies according to appearance on a survey image) may suffer from such selection effects.

2. Combining errors

2.1 One variable

The relative error of a function can usually be calculated from the derivative of that function. For example, consider a quantity x which is measured with an uncertainty Δx . What would the error be in a quantity $Z(x) = x^n$? Differentiating $Z(x)$ gives $dZ/dx = n x^{(n-1)} = n (Z/x)$, or $\Delta Z/Z \approx n \Delta x/x$. The relative error in Z is therefore n times the relative error in x . If $Z = x^2$, then a 3% error in x means that there will be 6% error in Z . Note that this relation is only usable for *small* relative errors in x .

The same approach can be applied to other expressions (e.g., $Z(x) = \log x$).

2.2 Two or more variables

Sums and Differences

Take a quantity Q which is the sum or difference of two independent measurements x and y , each of which has estimated errors Δx and Δy . It would be very pessimistic to say that the error in Q should be given by $\Delta Q = \Delta x + \Delta y$ since this means that both x and y have their maximum error at the same time. This is the worst-case scenario. It is more realistic to consider the standard error in Q , which can be derived from equation (3) by taking $n = 2$:

$$\Delta Q = [(\Delta x)^2 + (\Delta y)^2]^{1/2} \quad (4)$$

Example 2

Consider the separation of two lines on a spectrum, as you observe in some of these labs.

Position of line 1 = (14.7 ± 0.2) mm

Position of line 2 = (12.3 ± 0.2) mm

Combined uncertainty = $[(0.2)^2 + (0.2)^2]^{1/2} \approx 0.28$

Reported separation = (2.4 ± 0.3) mm

Note that we have only worked this example to one decimal place (see Sect. 3).

Products and quotients

If a quantity Q is the product or quotient of several variables x, y, \dots which have errors $\Delta x, \Delta y, \dots$ then the *relative* error in Q is the square root of the sum of the squares of the relative errors in x, y, \dots such that

$$\Delta Q/Q = [(\Delta x/x)^2 + (\Delta y/y)^2 + \dots]^{1/2} \quad (5)$$

Example 3

What is the resistance of a resistor from measurements of current and voltage?

Voltage across resistor = (15.1 ± 0.1) V

Current through resistor = (0.91 ± 0.01) A

The resistance $R = V / I = 16.59$ ohms.

The relative error is $[(0.1/15.1)^2 + (0.01/0.91)^2]^{1/2} = 0.013 = \mathbf{1.3\%}$

The actual error is $0.013 \times 16.6 = 0.21$ and the value is recorded as (16.6 ± 0.2) ohms.

Note that relative (or percentage) errors must be used here since we are combining errors of quantities which have different dimensions. It will often be found that, because the fractional errors are squared, one of them often dominates the calculation.

3. Significant figures

The number of significant figures given in a final result is obviously dependent on the accuracy to which the physical quantities used in calculating the result were measured. The data and results in an elementary laboratory experiment are rarely significant to more than three figures and any result quoted to more than three significant figures usually implies a false accuracy. Writing $g = 9.81034 \text{ m/s}^2$ implies that g lies between 9.81033 and 9.81035 m/s^2 and that its value has been measured to 1 part in a million.

Example 4

Consider the following calculation:

Mass of object = $(21.2 \pm 0.1) \text{ g}$; Volume = $(4.1 \pm 0.1) \text{ cm}^3$

Density = $M / V = 5.170731707 \text{ gm/cm}^3$ (Result from calculator)

% error in density = $[(0.5/100)^2 + (2.4/1001)^2]^{1/2} = 2.5\%$

Actual error in density = $0.025 \times 5.170731707 \approx 0.13$ gm/cm³.

Therefore, the density should be quoted as **$(5.17 \pm 0.13) \text{ gm/cm}^3$** .

In general, the error is rounded off to two significant figures (or in some cases, only one) and the calculated result is rounded off to the same number of decimal places. Error calculations can often be done mentally without using calculators. If you are in doubt about the last figure, give it in brackets. For example, in the density calculation above, the volume is given to only two significant figures although the mass is given to three. The density might better be recorded as $5.1(7) \pm 0.13 \text{ gm/cm}^3$.

Zeroes which are not used merely to locate a decimal point must be considered as significant figures. For instance, if we know the result is only accurate to three significant figures, then \$410,000\$ should be written as 4.10×10^5 . Sometimes an approximation symbol like “ \sim ” will ensure there is no misunderstanding of the accuracy if you include zeroes.

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The above notes were adapted from a similar set issued by the Department of Astronomy, University of St. Andrews.

NOTE: The symbols used for various errors in these notes may differ slightly from those used by authors in the above references. In particular, the symbol s is used by some authors for the unbiased estimate of the standard deviation of the population whereas it is designated σ in these notes and most texts.

4. Graphs

4.1 Fitting a straight line to a graph

In handling data you collect in some of these lab projects, you may be asked to plot x and y data and fit a straight line to them, such that

$$y = m x + c$$

where m is the slope of the line and c , the intercept on the y -axis where $x = 0$. Note that x and y might not be the actual measurements themselves, but functions which enable a straight line graph to be drawn; e.g., $\log y$ vs. $\log x$. In a graph, it is conventional to plot the x quantity on the horizontal axis.

To find the best values for the slope m and intercept c of a set of points which have random scatter, one usually applies a statistical criterion: The 'best' straight line is the one for which the sum of the squares of the perpendicular distances from all the points to the line is a minimum. The equation of this least-squares fit can be computed for any data set and will give m and c together with their standard errors. However the calculation can be cumbersome without an appropriate computer program or calculator. The special (but common) case where y is a measured quantity and x is known exactly can be handled more easily. Here, only the vertical deviations of the points from the line are important and you can follow the procedure below:

- 1) Choose the axes and scales of the graph to utilise as much of the graph paper as possible, but avoid using scales of 3 or thirds. Including the (0,0) origin is not usually essential unless the measurements pass near this point – a good spread of points is generally far more desirable.
- 2) Plot the data points using symbols to enclose the points. Circles, squares, crosses, triangles and so on, generally help to distinguish different data points where necessary. Error bars can help to evaluate the reality of any trends in the graph.
- 3) If the points look as if they can be reasonably fitted by a straight line, draw what you consider to be the 'best straight line' by eye using a fine pencil. A transparent plastic ruler is handy for this. Another useful tip is to calculate the mean values of x and y and put this point ($\langle x \rangle, \langle y \rangle$) on the graph. (Do not confuse it with a real data point, however.) The best straight line should normally pass through this point; the mathematically calculated least-squares line always passes through this point.
- 4) Determine the slope of this line and (if required) the intercept, which is the value of y when $x = 0$. If the line $x = 0$ is not on your scale, then the

intercept can be found by simple proportion – imagine the straight line to be extrapolated to cut an imaginary line at $x = 0$.

- 5) Two faint lines can be drawn parallel to your straight line so that they just touch the most extreme data points above and below it. Two vertical short lines will complete a parallelogram which encloses all the points. Connect the corners of this parallelogram of uncertainty with diagonal lines. Calculate the slopes m_1 and m_2 of these two diagonals, and extrapolate them to obtain the intercepts c_1 and c_2 respectively.
- 6) The difference in slope $m_1 - m$ (which is the same as $m - m_2$) leads to the uncertainty in the best slope of $\Delta m = |(m_1 - m)| / (n - 2)^{1/2}$, where n is the number of data points. The intercept differences $(c_2 - c)$ and $(c - c_1)$ are not in general equal, so we take the greater of these as Δc and quote the error in c as $\Delta c = |\Delta c| / (n - 2)^{1/2}$. The reason for the factor $(n - 2)$ in the denominator (rather than n) is that there are only $(n - 2)$ degrees of freedom since two have already been used to draw the best line and therefore do not contribute towards the error estimate.
- 7) *In practice*, all these construction lines need not be drawn, since they clutter the graph. All you need are the four corners of the parallelogram, and these can be indicated by short lines. The slopes can be calculated from the coordinates of opposite corners of the parallelogram, and the intercepts obtained (provided the line $x = 0$ is on the graph) using a ruler placed between these corners.

4.2 Formulae for the least-squares line

When y is the observed quantity and x is given precisely, the slope and intercept of the least-squares line $y = mx + c$ is defined as

$$m = [(\sum x)(\sum y) - n(\sum xy)] / [(\sum x)^2 - n(\sum x)^2] \quad (6a)$$

$$c = [(\sum x)(\sum xy) - (\sum x)^2(\sum y)] / [(\sum x)^2 - n(\sum x)^2] \quad (6b)$$

This line is referred to as the linear regression of y on x . Note that this will produce a slope and intercept for *any* set of data, even if they are not close to a straight line. So you must use equations (6a) and (6b) discriminately.

