SLAC NATIONAL ACCELERATOR LABORATORY

GEOMETRY AND ENTROPY

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B.CZECH, X.DONG, JS arXiv:1406.4889

Motivation

What does a theory of quantum gravity look like?

Gravity is often described in different situations as:

1. A coarse-grained/effective theory
2. A microscopic unitary quantum theory

Gravity is Entropic:

From this perspective, gravity is best understood as a thermodynamic theory. Entropy generates spacetime, but seems *agnostic* about what it is purified by.

- Black hole thermodynamics *[Bekenstein and Hawking]*
- Einstein equations as equation of state *[Jacobson]*
- Gravity as entropic force *[Verlinde]*
- Extremal surfaces from entanglement entropy *[Ryu-Takayangi; Hubeny, Rangamani, Takayangi]*
- AdS geometry as a MERA entanglement network *[Swingle]*
- *AdS Rindler horizons and ER =EPR [Van Raamsdonk; Maldacena, Susskind]*
- *Linearized Einstein equations from EE [Lashkari, McDermott, Faulkner, Hartman; Myers, Van Raamsdonk]*

Gravity is Entropic:

From this perspective, gravity is thermodynamic and entropy generates spacetime, but seems *agnostic* about what it is purified by.

Gravity is Pure:

From the other perspective, the microscopic structure of entanglement purification is important.

- Through AdS/CFT we have confirmed that gravity can be described by a microscopic unitary theory
- EFT in curved space-time: vacuum state is a particular entangled state
- Eternal AdS black hole described by particular TFD state

Which of these perspectives is correct? Is there a middle ground between the two perspectives?

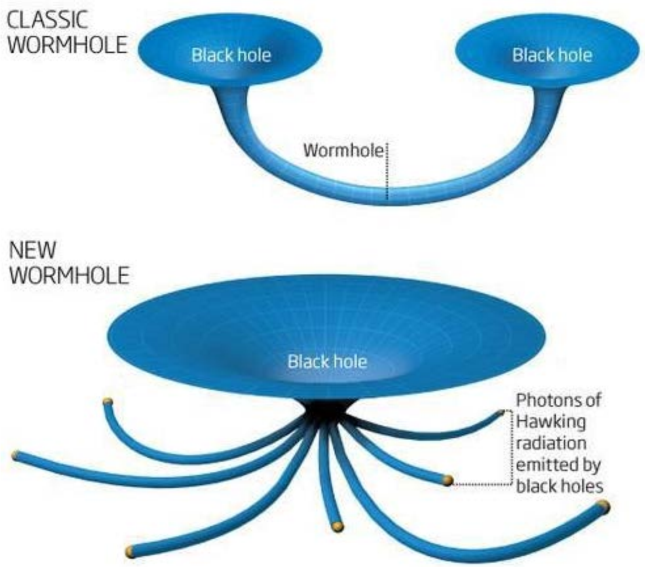
Firewalls

Resolving the tension between these two perspectives isn’t simply a question about quantum gravity at the Planck scale:

The black hole information paradox and the question of firewalls hinges on which of these two perspectives we believe:

- The reliance on EFT and the belief that the vacuum has a fixed structure seems to lead inevitably to firewalls
  - A smooth vacuum state at the horizon requires fields to be in the local Rindler state at the horizon, with entanglement between the outgoing and ingoing Hawking partners.
  - On the other hand, unitarity requires that the outgoing Hawking radiation is generically entangled with other degrees of freedom in the early radiation.

- The incompatibility of mutual entanglements that leads to the firewall can be resolved if one tracks entanglement, but not its purification
- This entropic approach builds a smooth geometry by constructing the interior Hawking modes from whatever the exterior Hawking mode happens to be entangled with.
  - This leads to constructions like the proposal of Papadodimas and Raju for building non-linear (state dependent) interior operators and the EPR=ER proposal of Maldacena and Susskind



While black holes are an invaluable pressure test for our ideas about quantum gravity, they also add to the confusion about what we are doing.

- Life is confusing enough without immediately confronting, for example, whether quantum gravity can accommodate violations of quantum mechanics
- While this may drive straight to the heart of the issue, perhaps something can be learned by less invasive surgery of what we think we know.

It seems valuable to explore the tension between entropy and purity in the absence of black holes.

We might want to ask:

- Does some measure of entropy determine the geometry of spacetime?
- If so, what is the precise entropy measure in the boundary field theory?
- What does this entropy tell us about holographic RG and the emergence of the radial direction?

I will assuredly answer none of these questions today.

Outline

1. Hole-ography and Differential Entropy
2. Differential Entropy for General Surfaces
3. 8 Open Questions & Some Wild Speculation

HOLE-OGRAPHY

BALASUBRAMANIAN, CHOWDHURY, CZECH, DE BOER, HELLER

The classic results of black hole thermodynamics suggest we should associate an entropy to black hole horizons (or any killing horizons):

$$S_{BH} = \frac{A}{4G_N}$$

Natural to then ask: can we associate a notion of entropy to any choice of (generalized) bulk area?

- And, if so, what is the meaning of this entropy in the boundary field theory?

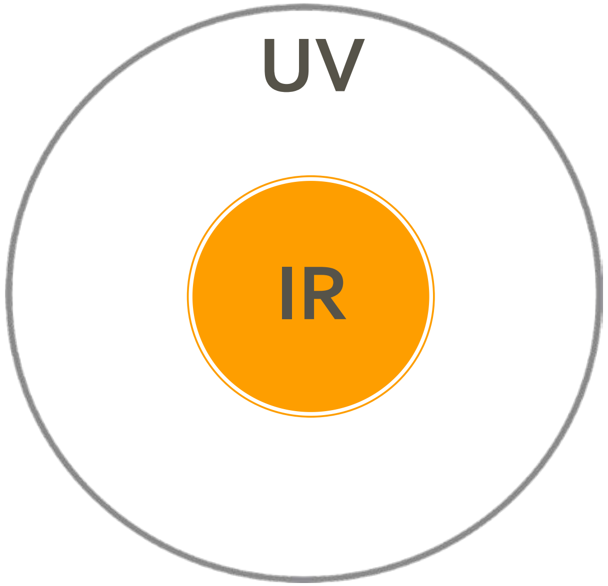
Conjecture [Bianchi, Myers]:

"In a theory of quantum gravity, for any sufficiently large region in a smooth background spacetime, one may consider the entanglement entropy between the degrees of freedom describing the given region with those describing its complement… the leading contribution from this short-range entanglement will be given precisely by the BH formula."

What is its precise manifestation in the boundary field theory?

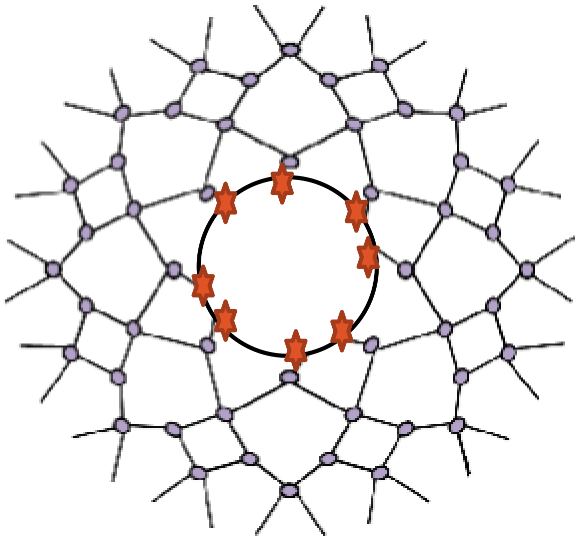
First, some broad intuition:

1) The radial direction in the bulk is associated to a holographic RG flow
   The area of some closed surface then should correspond to an entanglement between UV and IR degrees of freedom rather than some particular spatial region of the boundary.



2) The bulk geometry is perhaps associated to a MERA lattice [Swingle]
For a CFT, one can efficiently represent certain low-energy states by a lattice of unitary operators. Swingle has suggested that the structure of this lattice for the vacuum state of a CFT mimics the coarse structure of AdS.

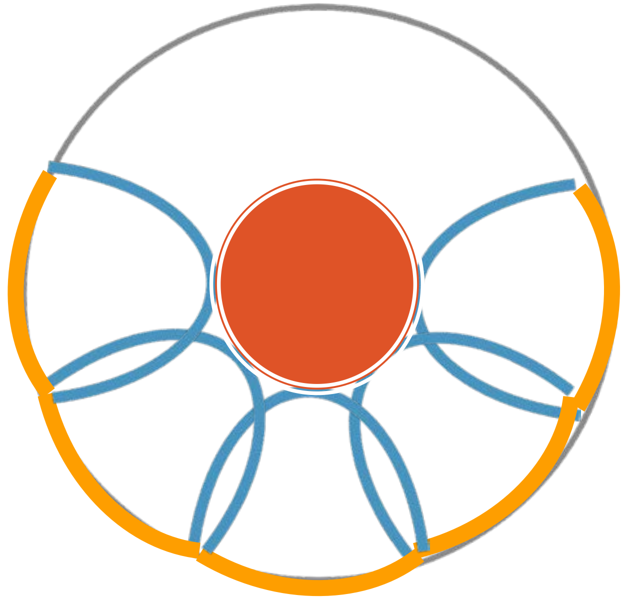
- Lattice points deeper in the bulk encode IR entanglement in the CFT



- When we cut out a region of the MERA lattice, we remove a number of unitary operators proportional to the area of the cut.
- The number of possible states that fills in the lattice is proportional to the area of the cut
- However, the subspace spanned by these states does not necessarily form a tensor factor of the Hilbert space
  - Suggests we don't think of this entropy as an entanglement entropy, but some other entropy measure

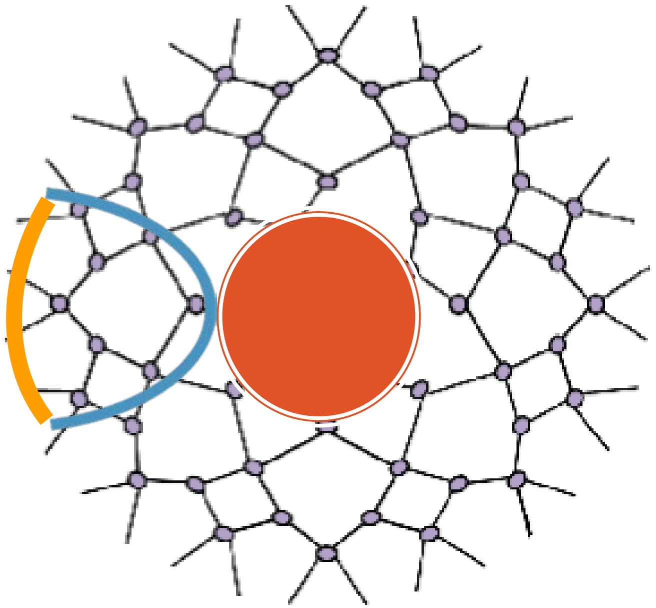
3) RT: While the surface area describes the EE, the density matrix describes the interior region

The exterior of some closed bulk surface should then be described by a collection of density matrices whose minimal surfaces are—at most—tangent to the bulk surface



This intuition is equally supported by the MERA picture:

- The RT surfaces are given by cuts of the exterior region of the MERA lattice, which contains enough information to reconstruct the density matrices for the boundary regions

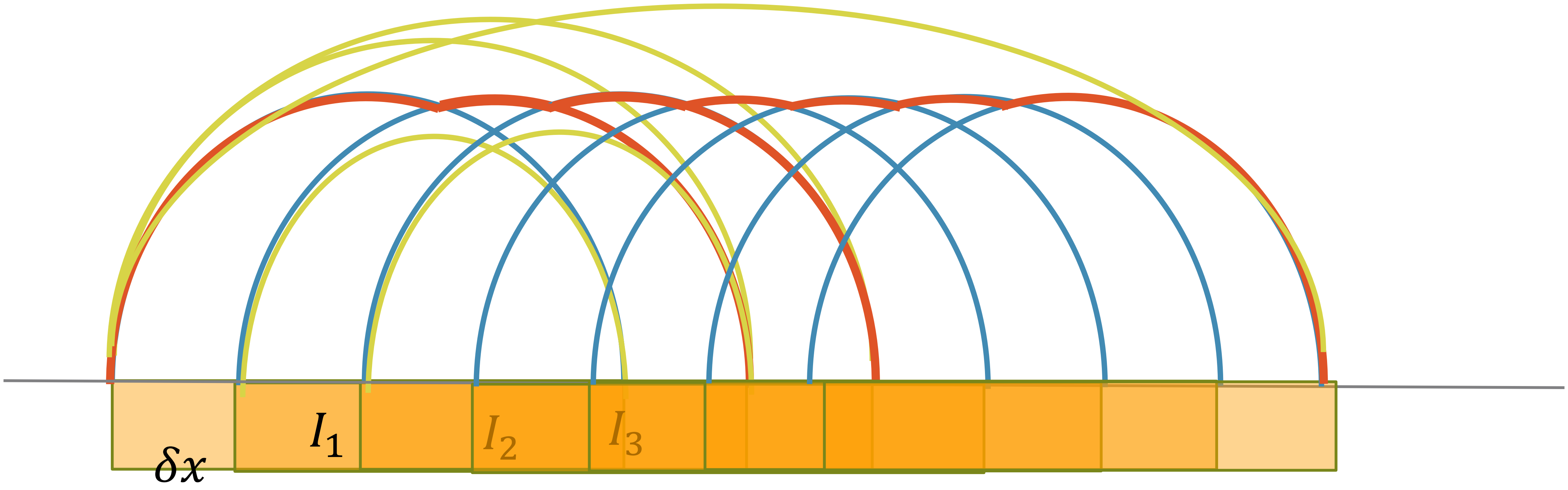


How do we assign an entropy to this collection of density matrices?

We will make use of Strong Subadditivity of Entropy: $S_A + S_B \geq S_{A\cap B} + S_{A\cup B}$

$$S_{I_1\cup I_2\cup I_3} \leq S_{I_1} + S_{I_2} + S_{I_3} - S_{I_1\cap I_2} - S_{I_2\cap I_3}$$

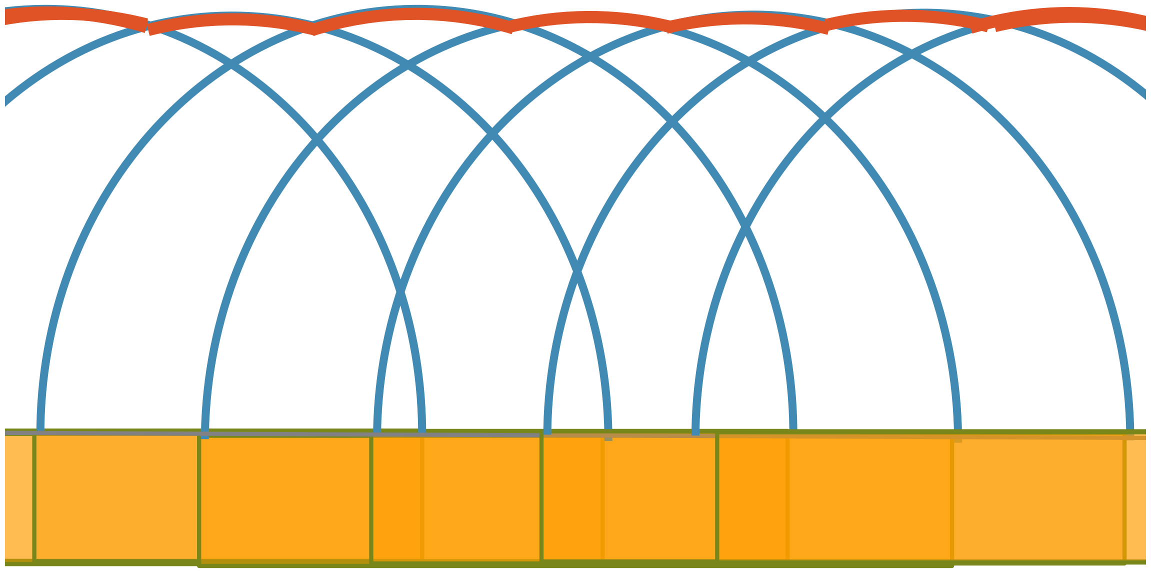
$$S_{\cup I_k} \leq \sum_{k}^{n} S_{I_k} - \sum_{k}^{n-1} S_{I_k\cap I_{k+1}}$$



The RHS of the bound we have established generally has boundary terms that give a UV divergence. When we periodically identify the boundary, we can subtract the intersection term between the first and last interval.

The RHS becomes UV finite and is approximately equal to the area of the 'outer envelope'. We define this sum of entropies to be the differential entropy:

$$S_\rho \leq \sum_k^n S_{I_k} - \sum_k^n S_{I_k \cap I_{k+1}} = S_{\text{diff}}$$



- The discrete formula for differential entropy we have written on the previous slide has a beautiful continuum limit:

$$S_{\text{diff}} = \int \mathrm{d}\lambda \frac{dx_R(\lambda)}{d\lambda} \frac{\delta S[x_L(\lambda), x_R(\lambda)]}{\delta x_R(\lambda)} = \int \mathrm{d}\lambda \frac{dx_L(\lambda)}{d\lambda} \frac{\delta S[x_L(\lambda), x_R(\lambda)]}{\delta x_L(\lambda)}$$

$$\Rightarrow \quad \boxed{S_{\text{diff}} = \int \mathrm{d}x_c \frac{\delta S[R(x_c)]}{\delta R(x_c)}}$$

- By choosing $R(x)$ so that the minimal surface is just tangent to S for every x, it is then a simple proof to show that:

$$S_{\text{diff}} = \text{Area}(S)$$

GENERAL SURFACES

CZECH, DONG, JS

We build on the work of Headrick, Myers, and Wien and Myers, Rao, and Sugishita to extend the definition of differential entropy to arbitrary* closed surfaces in generalized gravitational theories in higher dimensions.

First, some definitions:

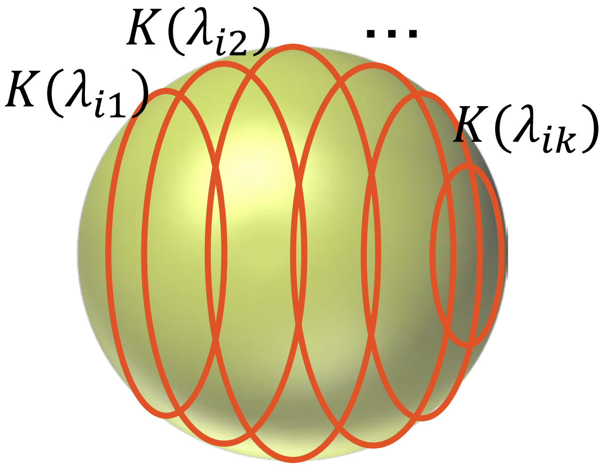
Consider:

1. A gravitational theory with a generalized entropy functional which is an integrated local density

$$S_A = \int_A \mathcal{L}$$

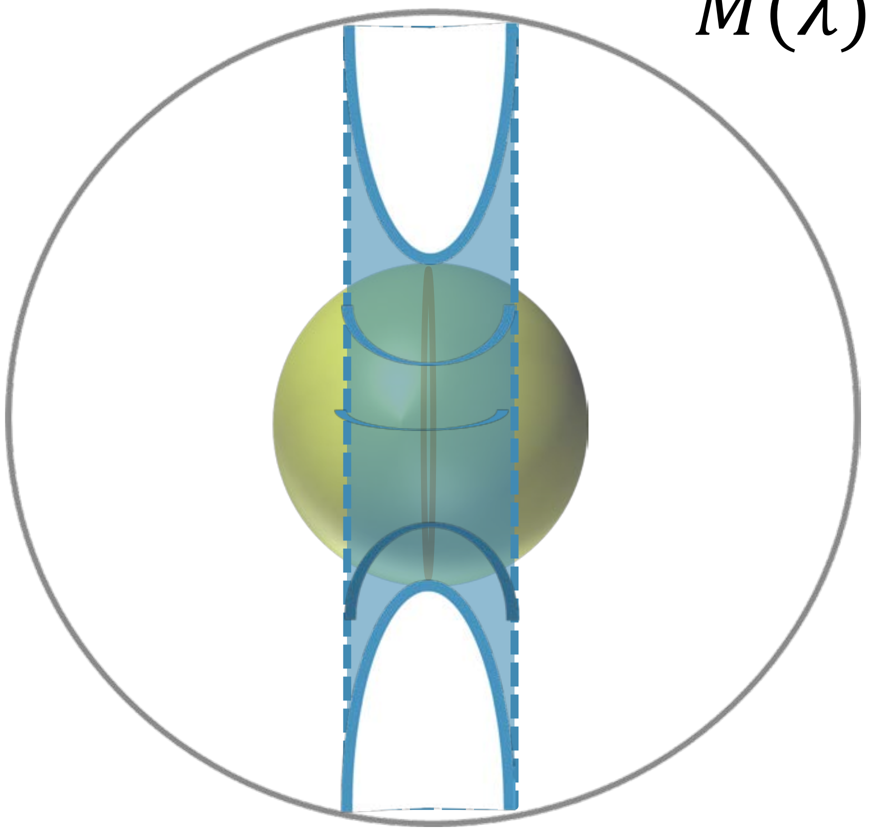
2. A spacelike slice Σ of a static background
3. A closed, codimension 1 (dimension d-1), smooth hypersurface A on Σ
4. A 1-dimensional foliation of A by closed, smooth "loops" $K(\lambda)$ of codimension 1 on A (ie. of dimension d-2)

A

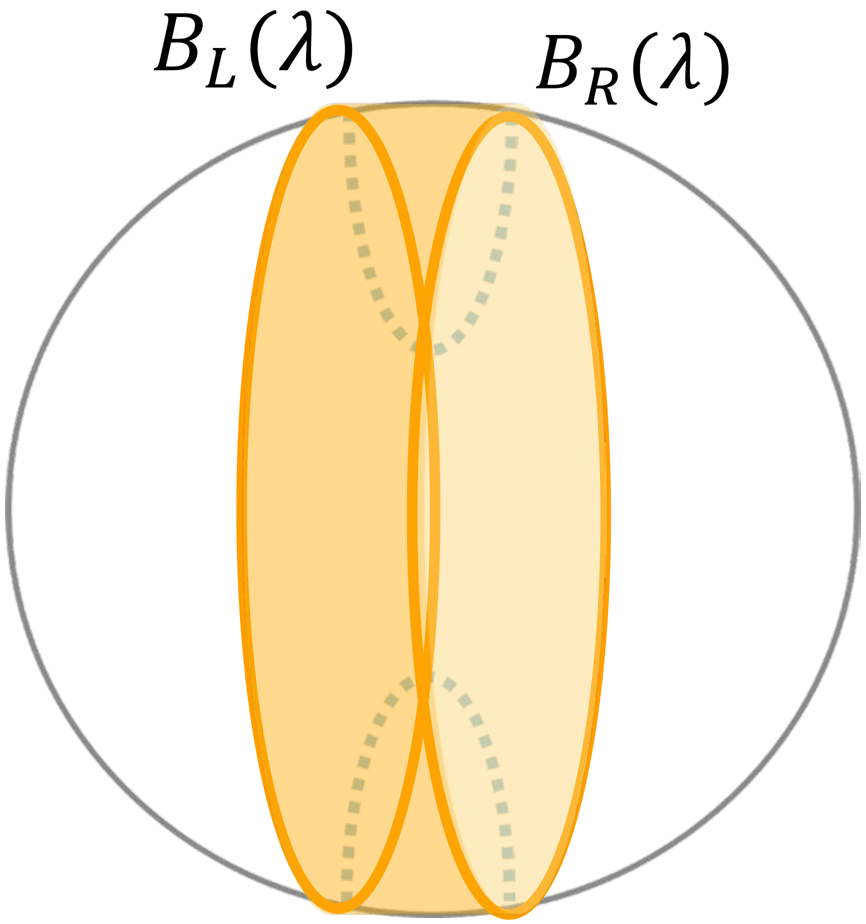


$K(\lambda)$

To each loop $K(\lambda)$ we associate the boundary anchored *extremal* surface $M(\lambda)$ that is tangent to A at $K(\lambda)$:



The boundary anchored extremal surface $M(\lambda)$ meets the boundary of the bulk geometry along two loops $B_L(\lambda)$ and $B_R(\lambda)$ (generically) enclosing a ring-shaped region:



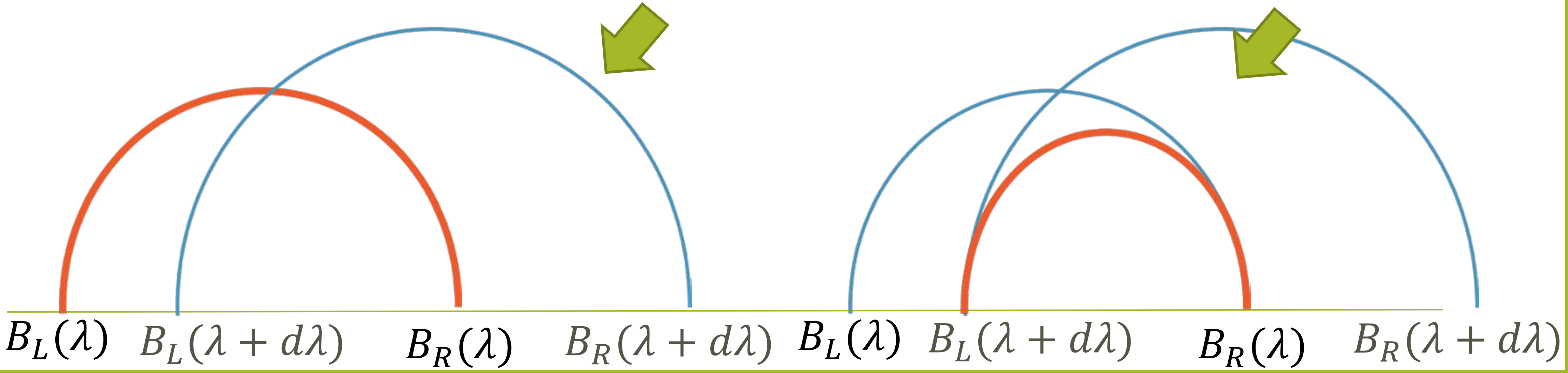
- We denote the generalized entropy associated with $M(\lambda)$ by

$$S_{M(\lambda)} = S[B_L(\lambda), B_R(\lambda)]$$

where it is labeled instead by its boundary conditions.

- Given a one parameter collection of boundary intervals $\{B_L(\lambda), B_R(\lambda)\}$, define:

$$S_{\text{diff}}[\{B_L(\lambda), B_R(\lambda)\}_\lambda] = \int \mathrm{d}\lambda \{S[B_L(\lambda), B_R(\lambda)] - S[B_L(\lambda + \mathrm{d}\lambda), B_R(\lambda)]\}$$



Differential Entropy

We will show that the above defined generalization of differential entropy reconstructs the generalized area/entropy of A:

$$S_{\text{diff}}[\{B_L(\lambda), B_R(\lambda)\}_\lambda] = \int_A \mathcal{L} = S_A$$

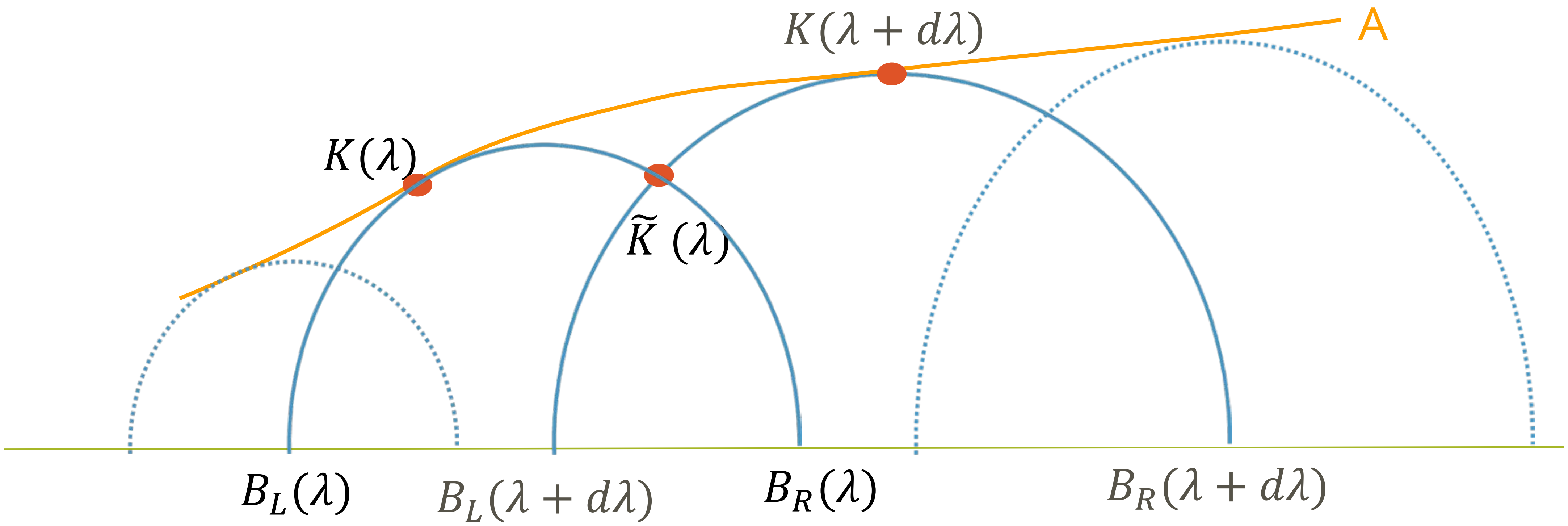
for an arbitrary* smooth surface in a generalized theory of gravity.

Proof

First note that up to second order in $d\lambda$ we can rewrite

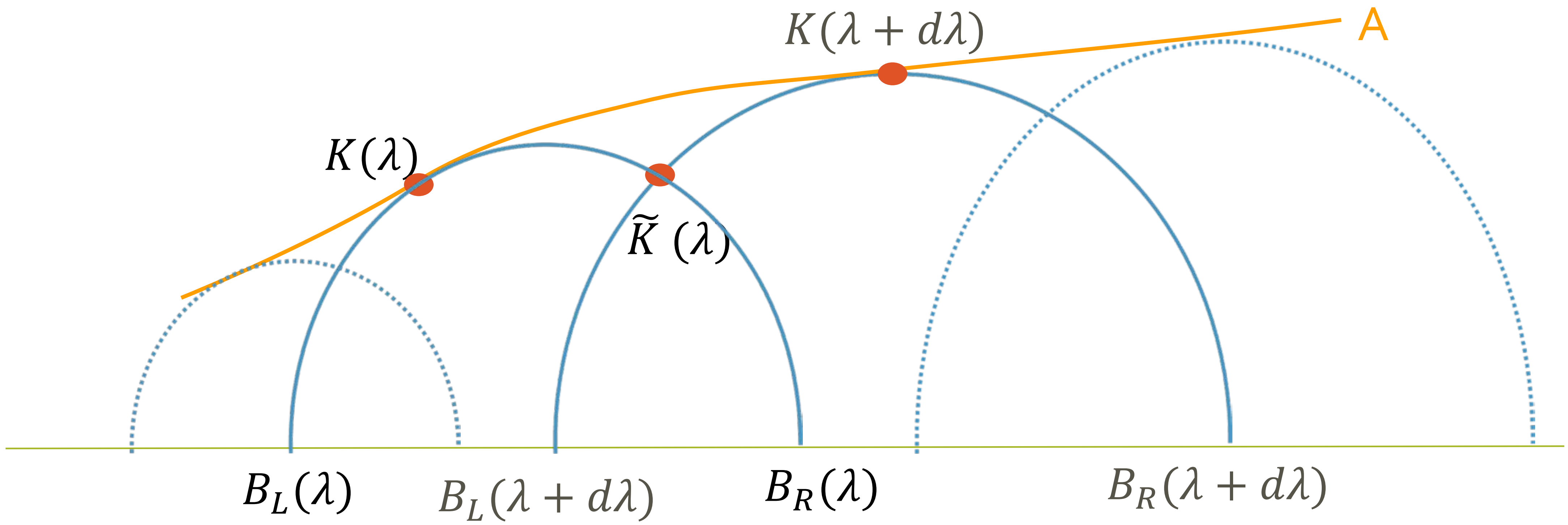
$$S[B_L(\lambda + \mathrm{d}\lambda), B_R(\lambda)] = S[B_L(\lambda + \mathrm{d}\lambda), \tilde{K}(\lambda)] + S[\tilde{K}(\lambda), B_R(\lambda)]$$

because we have just infinitesimally perturbed about an extremal solution



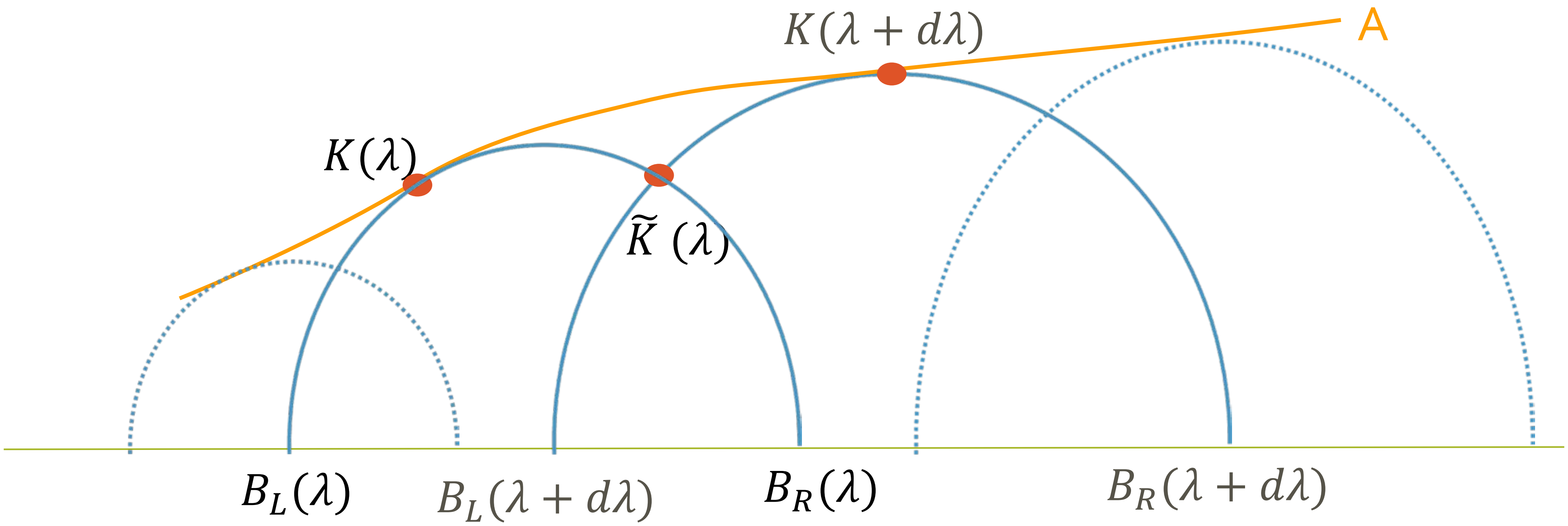
Thus we can rewrite the differential entropy as

$$
\begin{aligned}
S_{\text{diff}} &= \int \mathrm{d}\lambda \{ S[B_L(\lambda), \tilde{K}(\lambda)] - S[B_L(\lambda + \mathrm{d}\lambda), \tilde{K}(\lambda)] \} \\
&= \int \mathrm{d}\lambda \{ (S[B_L(\lambda), K(\lambda)] - S[B_L(\lambda + \mathrm{d}\lambda), K(\lambda + \mathrm{d}\lambda)]) \\
&\quad + \Big(S[K(\lambda), \tilde{K}(\lambda)] + S[\tilde{K}(\lambda), K(\lambda + \mathrm{d}\lambda)] \Big) \}
\end{aligned}
$$



$$S_{\text{diff}} = \int d\lambda \{ (S[B_L(\lambda), K(\lambda)] - S[B_L(\lambda + d\lambda), K(\lambda + d\lambda)])$$
$$+ \Big(S[K(\lambda), \tilde{K}(\lambda)] + S[\tilde{K}(\lambda), K(\lambda + d\lambda)] \Big) \}$$

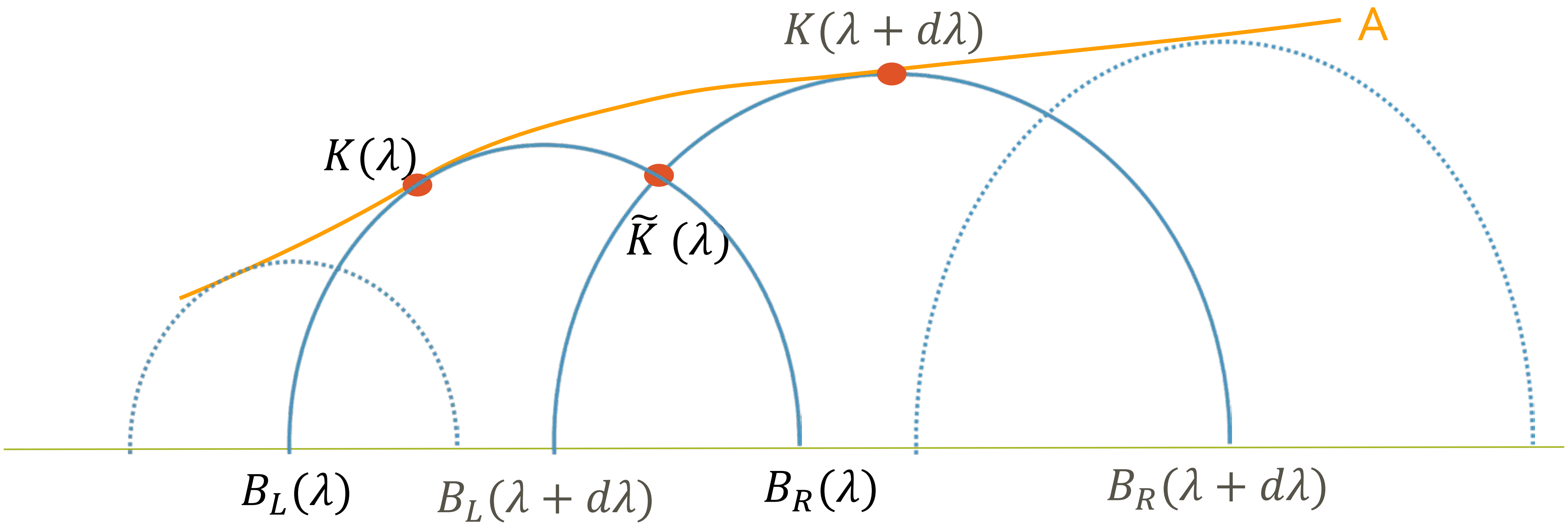
These two terms are just the differential generalized area between $K(\lambda)$ and $K(\lambda + d\lambda)$



$$S_{\text{diff}} = \int \mathrm{d}\lambda \{ (S[B_L(\lambda), K(\lambda)] - S[B_L(\lambda + \mathrm{d}\lambda), K(\lambda + \mathrm{d}\lambda)]) + \left(S[K(\lambda), \tilde{K}(\lambda)] + S[\tilde{K}(\lambda), K(\lambda + \mathrm{d}\lambda)] \right) \}$$

These two terms are just a total derivative:

$$S[B_L(\lambda), K(\lambda)] - S[B_L(\lambda + \mathrm{d}\lambda), K(\lambda + \mathrm{d}\lambda)] = -\frac{\mathrm{d}}{\mathrm{d}\lambda} S[B_L(\lambda), K(\lambda)]\mathrm{d}\lambda$$



So long as the total derivative does not produce boundary terms, the integrated quantity is exactly the generalized area:

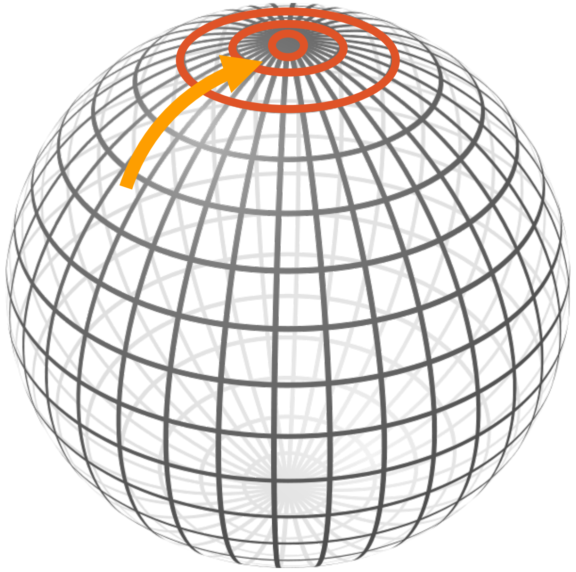
$$S_{\text{diff}}[\{B_L(\lambda), B_R(\lambda)\}_\lambda] = \int_A \mathcal{L} = S_A$$

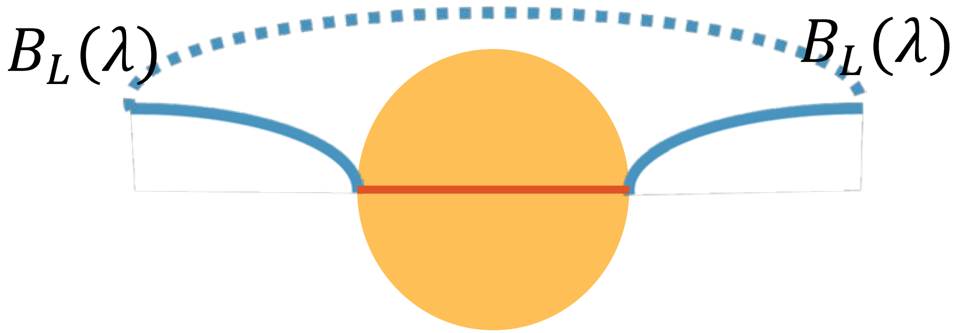
Issues and subtleties

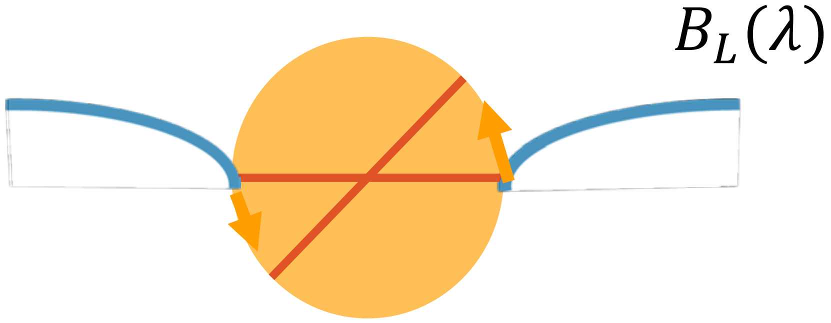
1) Degenerate foliations

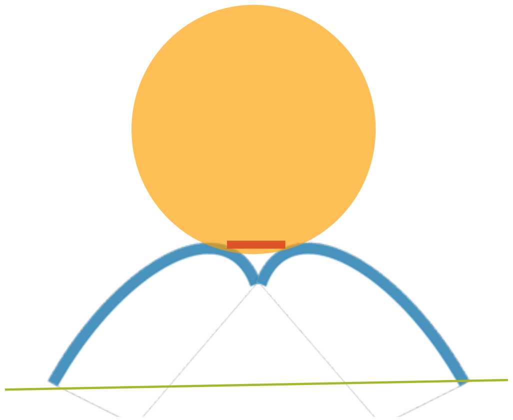
Foliating surfaces by ‘loops’ often necessitates points where this foliation degenerates (ie. even for the simple case of a fully rotationally invariant sphere in empty AdS).

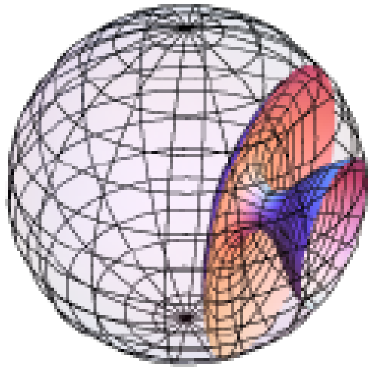
As long as the foliation only degenerates at finitely many λ, the differential entropy still gives an integral of the infinitesimal generalized area + boundary term.

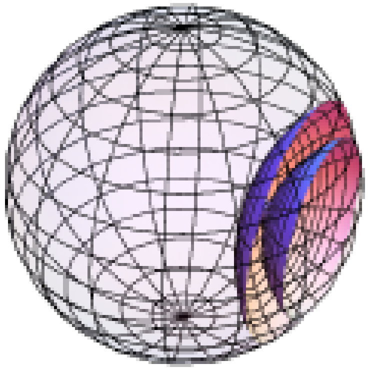










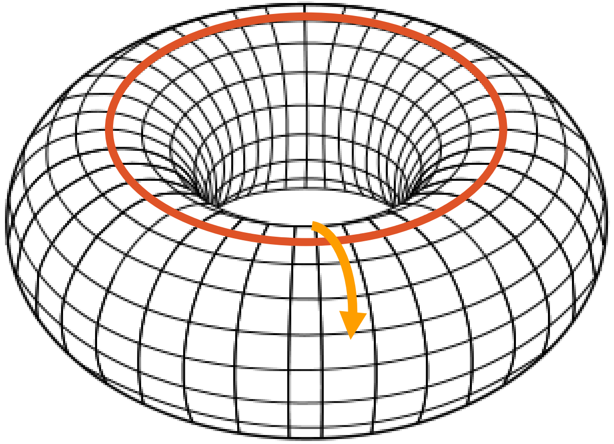


At the degeneration points, it may appear that a caustic will develop as the radius of the loop goes to zero size.

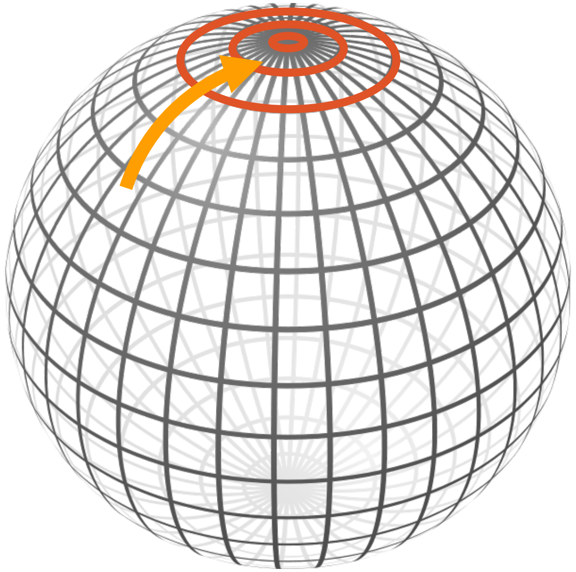
However, the extremal surfaces have vanishing extrinsic curvature. As the sectional curvature in one direction grows large, the surface is driven away in the other directions.

At the point where the extremal surface is tangent at a point, it approaches a double-cover of the minimal surface of the extremal surface with only one boundary.

2) Boundary terms



- If the parametization is non-degenerate (for example a torus), then the boundary terms will necessarily cancel.
- If the parametization degenerates, cancellation is no longer automatic, although the procedure can still work.
  - Simplest where cancellation protected by symmetry, ie. choose a foliation that respects the Z2 symmetry for the case of the sphere
  - Otherwise, must choose degeneration points such that boundary terms cancel. For two degeneration points, this is always possible as the boundary term at a degeneration point is a continuous function
  - NB: The boundary term is then generically sensitive to the choice of UV cutoff



3) Non-minimal Surfaces

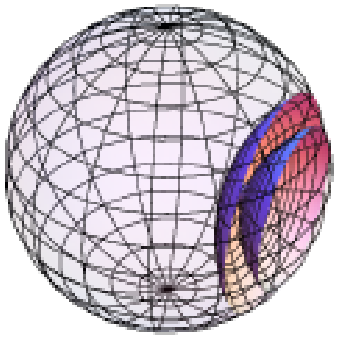
- Our proof so far has only been a geometric construction and we have not given a boundary interpretation for $S_{M(\lambda)} = S[B_L(\lambda), B_R(\lambda)]$.
- From RT, we know that for a boundary region B

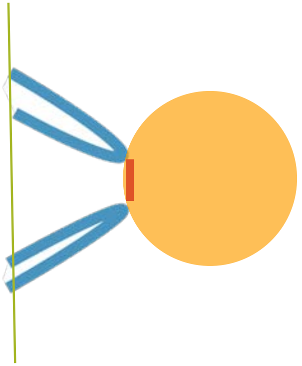
$$S_{EE}[B_L(\lambda), B_R(\lambda)] = \min_{\partial M = B} S_{M(\lambda)}$$

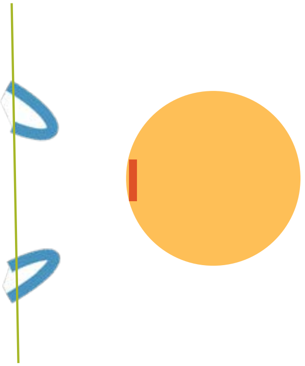
But, in our case we have no guarantee that

1. $M(\lambda)$ is the minimal surface
2. $B(\lambda)$ forms a well-defined boundary region

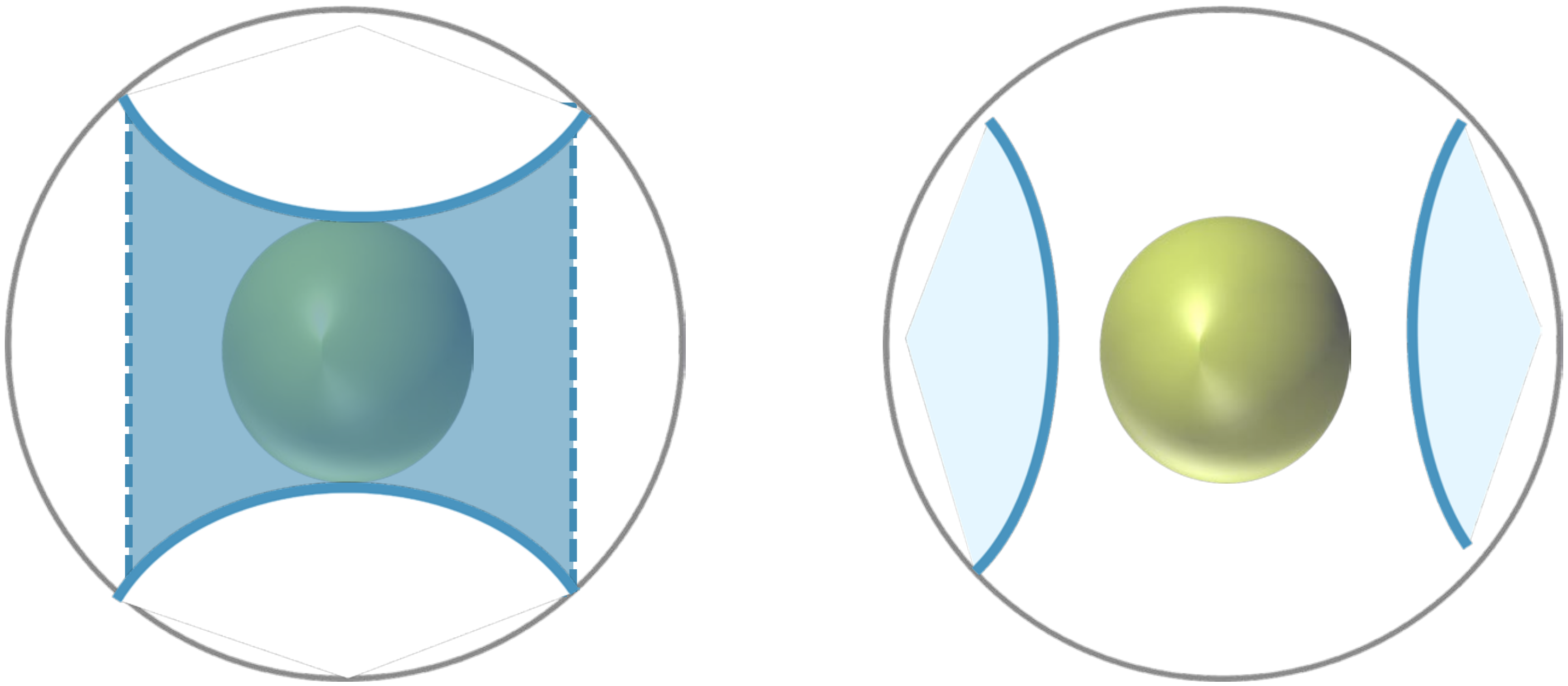
- (1) is certainly problematic near degeneration points:







- There will also be a phase transition between the minimal surfaces we use and the disconnected surface that encloses the complementary region on the boundary



- This phase transition happens when the loop on the surface is roughly the size of the AdS radius.

OPEN QUESTIONS

(AND WILD SPECULATION)

1) Is there an understanding of non-minimal surfaces in the boundary?

We are forced to conclude that *spatial* entanglement seems insufficient to understand differential entropy as a boundary quantity.

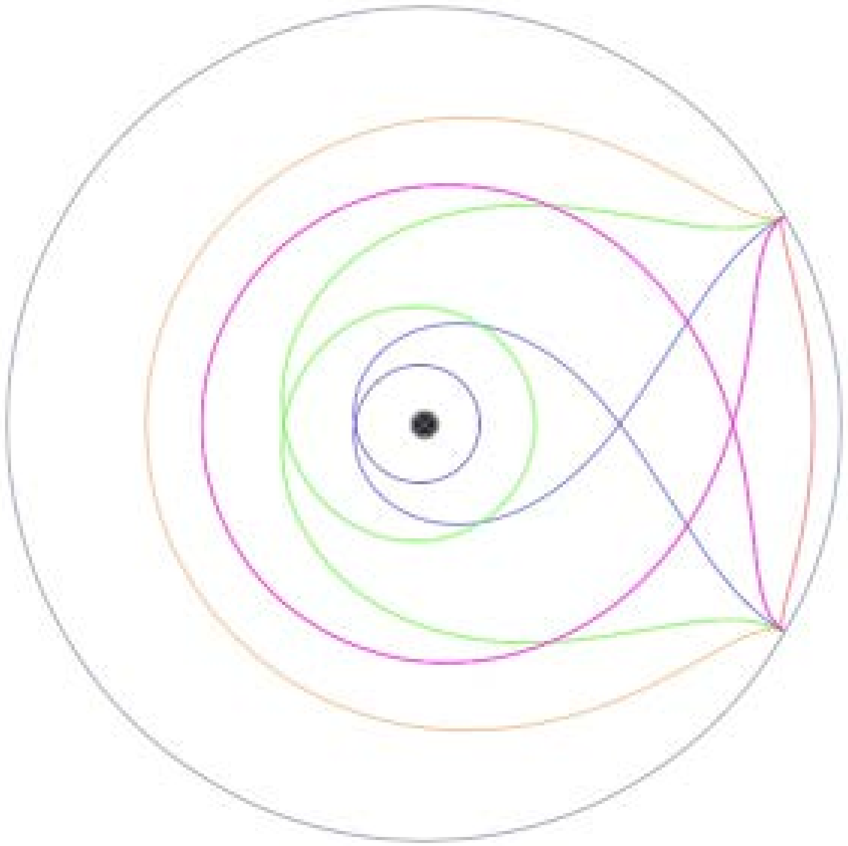
This leaves two possible choices:

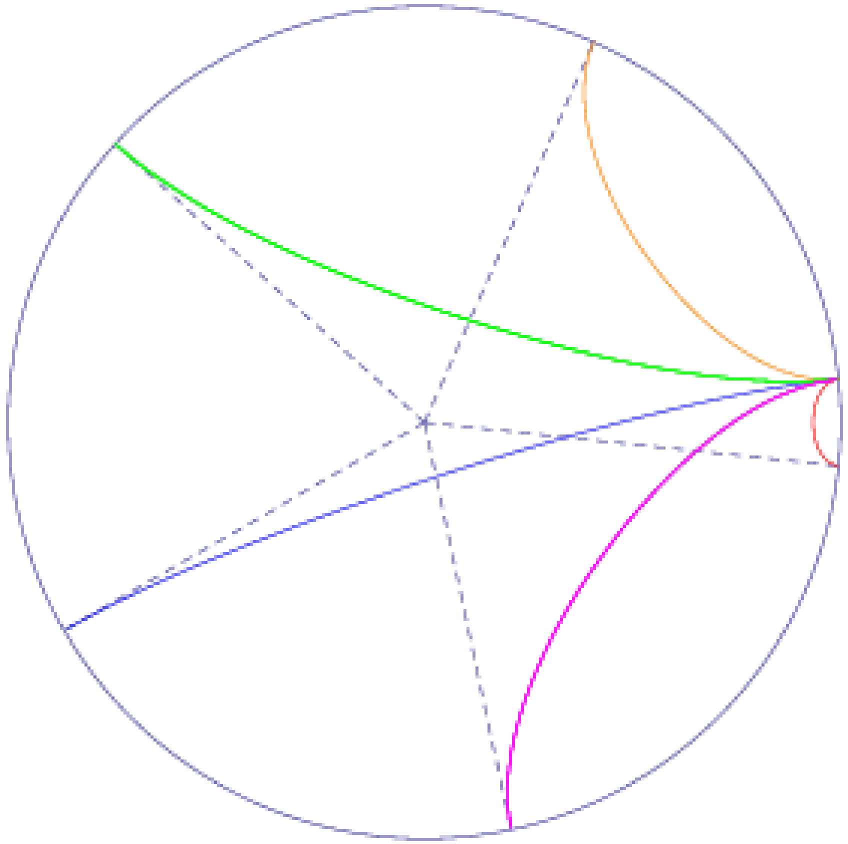
1. **Conservative and less interesting:**
   Differential entropy is only useful for a *coarse* reconstruction of bulk geometry. In general, we can only calculate an approximate area for surfaces and only measure features larger than the AdS scale.

2. **Interesting, but more speculative:**
   There is a (not yet known) field theory entropy we can associate to non-minmal extremal surfaces.

- Being more speculative for now, there are two possibilities worth pursuing:

1. They are also EE, but not purely spatial EE
    1. There has already been evidence given that there is a geometric interpretation to the entropy of factorizing CFT degrees of freedom in internal space, as opposed to spatially:
        - Entanglement between non-interacting CFTs in thermofield double is described by black hole geometry
        - More generally, entanglement between two interacting CFTs: [Mollabashi, Shiba, Takayanagi]
    2. This perspective has support from work on Entwinement [Balasubramanian, Czech, Chowdhury, de Boer]
        - Quotient of AdS3 by $\mathbb{Z}_n$ described by state in $\mathbb{Z}_n$ twisted-sector of orbifold T^{4N}/S_N CFT.
        - Long geodesics (not globally minimal curves) can be understood as descending from minimal geodesics in the covering space





- Can think of encoding the entanglement of degrees of freedom in a spatial region as well as a factorization of the internal space (although this picture is slightly complicated by the gauge constraint)

- Even if non-spatial entanglement doesn't play a role in understanding non-minimal surfaces, does it nevertheless play a role in understanding the sub-AdS-scale residual entropy?
  - Sub-AdS-scale locality is subtly encoded in matrix degrees of freedom. It would be very surprising to be able to faithfully describe bulk geometry without probing this entanglement.

2) What is the role of non-spatially organized entanglement?

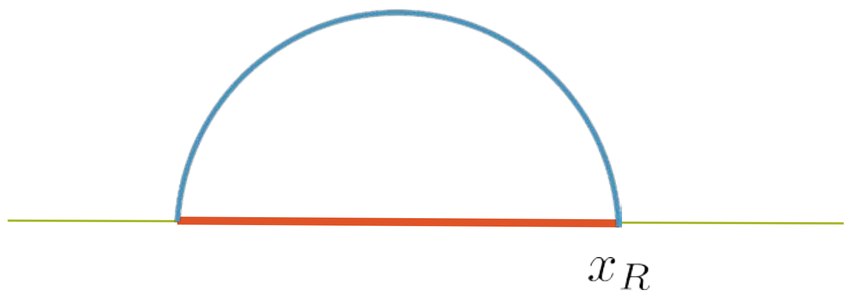
2. Non-minimal surfaces require a more general entropy measure
Recall the conjectured correspondence between Causal Holographic Information and boundary one-point entropy.
Is there a different entropy measure that is suitable for our surfaces?

- If non-minimal surfaces require a different entropy measure, should we be basing the construction on a different measure altogether? [cf. Hubeny]
- Differential entropy constructs surfaces differentially. It's important that entropies are described locally in the bulk by an action priniciple.
  - It's not at all clear how other measures that aren't determined locally in the bulk could fit into this paradigm.

3) What about other entropy measures?

4) Is there a clean UV/IR separation in gauge theories?

- Consider AdS3:
  Instead of considering the RT surface for fixed dirichlet boundary conditions in the UV, we can consider fixing two conditions at one end:



$$x(z) = (x_R - R) + \sqrt{R^2 - z^2}$$

$$x(z) = x_R - z^2 \frac{1}{2R} + \dots$$

- This type of boundary condition differentiates the minimal/nonminimal surfaces on the same region.
- Is there a corresponding specification of UV boundary conditions in the CFT that identifies the different surfaces?

- A natural identification of boundary conditions would be the boundary choice in assigning a local algebra to a region in gauge theory.
  [Casini, Huerta, Rosabal; Donnelly; Tachikawa, Ohmori]

- A reasonable expectation is that this choice should only affect UV divergent contributions and should be irrelevant for determining the IR behaviour. Is this in fact the case?

5) Is there a more covariant formulation?

The necessity of loops is not clear:

- Might seem more natural to tile surface by tangent RT surfaces for spherical regions, but this doesn't seem to work.
  - Our understanding of how differential entropy works in higher dimensions still reduces to the one-dimensional picture with a one-parameter foliation with a well-defined ordering principle
- A covariant formulation should allow us to foliate by more general classes of RT surfaces. Does this framework exist?

6) Is there a relation to integral geometry?

Czech, Lamprou, JS

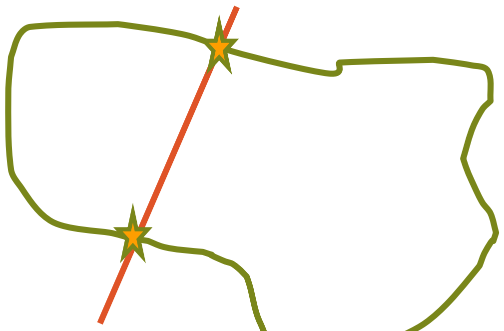
- Integral geometry studies measures on geometric spaces that are invaraint/equivariant under the action of the symmetry group
  - Concerned with integral transforms
  - Interesting connections with probability theory and stochastic geometry

- Two classic results of integral geometry:
  1. Radon Transform (cf. Penrose Transform and twistors):
     Let $f: \quad \mathbb{R}^2 \to \mathbb{R}$ be a compactly-supported continuous function. The Radon Transform of f is then a function from the space of straight lines in $\mathbb{R}^2$:

$$Rf(l) = \int_l f\,ds$$

  2. Crofton Formula:
     The area of a plane curve can be written as an integral of the intersection number over the space of lines:

$$S(C) = \frac{1}{4}\int_L \#(l \cap C)\, d^2l$$



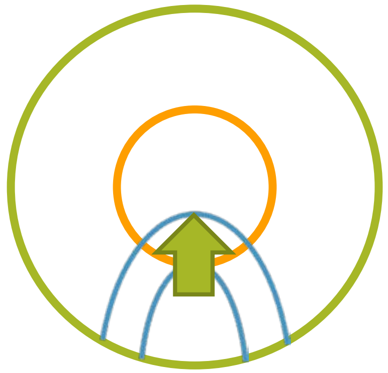
- The Radon transform and the Crofton formula can be naturally extended to hyperbolic space:
  - Let Γ be the space of planes in $\mathbb{H}^2$, with the unique invariant measure. Then:

$$S(C) = \frac{1}{4} \int_\Gamma \#(\gamma \cap C) \, d^2\gamma$$

- This appears distinct from the differential entropy formula, but they are in fact equivalent: Differential Entropy = Crofton Formula
  - We can write the invariant measure on the space of planes as

$$ds^2 = \frac{\partial^2 S}{\partial \alpha^2} \left(d\alpha^2 - d\theta^2 \right)$$

  (This is just the metric on Lorentzian de Siter space.)
  - Then we can first integrate over the interval size coordinates at each point (with support starting at $\alpha(\theta)$) to give the differential entropy formula:



$$\int_\Gamma \#(\gamma \cap C) \, d^2\gamma \quad \rightarrow \quad \int \frac{\partial S}{\partial \alpha}(\alpha(\theta)) d\theta$$

- Does this connection generalize to higher dimensions?
  - Differential entropy is a construction that generates area/entropy from area/entropy (spatial co-dimension 1 to spatial co-dimension 1)
  - BUT, the Crofton formula generalizes to higher dimensions with complementary dimension surfaces.
    We can write a formula for the area of surfaces in terms of geodesics:

$$A(S) \propto \int_L \#(l \cap S)\, \Phi_L$$

    We can write a formula for the length of curves in terms of co-dimension 1 surfaces:

$$l(C) \propto \int_S \#(C \cap s)\, \Phi_s$$

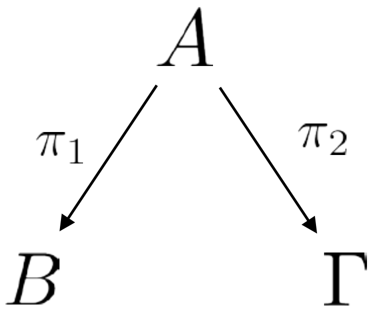
  - Neither of these has as nice an interpretation in terms of entropy alone.

- Does this connection generalize to less symmetric geometries?
  - The formula for differential entropy requires no symmetry
  - A. Weil: Integral geometry belongs "within the framework of E. Cartan's theory of homogenous spaces." Should we be worried?

Perhaps not.

  - There is a more general framework [Paiva, Fernandes]: Gelfand Transform: Consider some manifold B, dim(B)=n, with a family of submanifolds B_γ, dim(B_γ)=n-k, parametized by points γ on the manifold Γ, Φ a smooth measure on Γ.
  - Let A be the double-fibration over Γ,B of incidence relations:

$$A = \{(b,\gamma) \in B \times \Gamma \,|\, b \in B_\gamma\}$$



For some immersed submanifold N of dimension k, we have:

$$S_\Phi(N) = \int_\gamma \#(N \cap B_\gamma)\Phi = \int_N \pi_{1!}\pi_2^*\Phi$$

  - This seems to be a generalization of some of the ideas in differential entropy, but not necessarily the one we want

- Our construction in higher dimensions is clearly incomplete.
  a) Is there a generalized notion of Crofton formula that matches differential entropy?

  b) Is differential entropy the wrong generalization of the idea from the 2D Crofton formula?

7) What are we counting?

We have defined the differential entropy in terms of a sum of entanglement entropies, but what does this entropy actually measure? Does it correspond to the von Neumann entropy of some different density matrix?

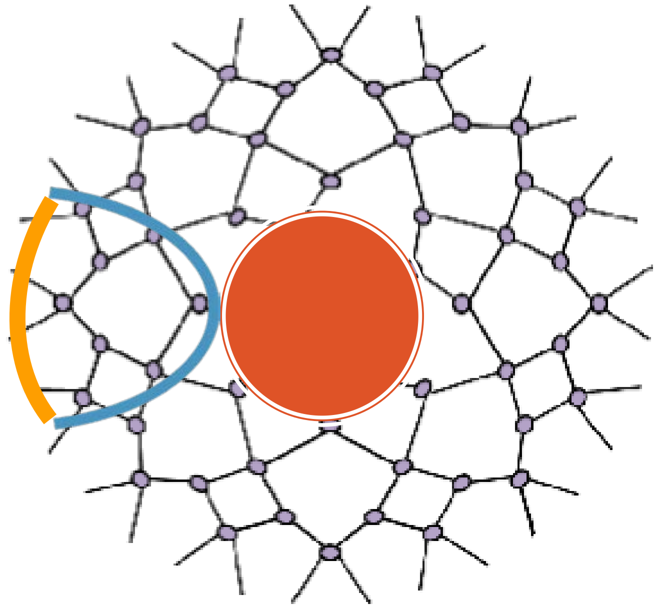
$$S_{\tilde{\rho}} = \sum_{k}^{n} S_{I_k} - \sum_{k}^{n} S_{I_k \cap I_{k+1}} = S_{\text{diff}} \quad ?$$

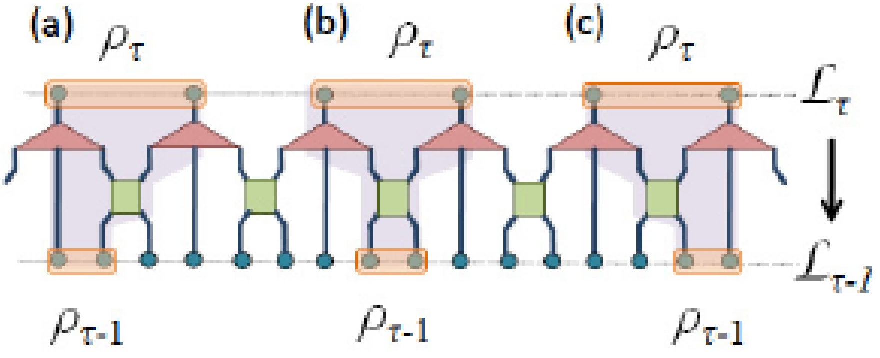
The answer is NO.

- This is easiest to see when we consider the entropy of a hole in empty AdS: [Kim, Swingle]
  Assume the ground state is unique.
  The collection of local density matrices $\{\rho_x\}_{x\in\partial B}$ is sufficient to calculate the expectation value of the local Hamiltonian.
  Thus, there is a unique state ρ_{vac} that is consistent with our collection of density matrices. It's impossible to "fill in" the hole in the spacetime without propagating energy out the boundary.

To see where we went wrong, it's helpful to return to the MERA picture:

- Cutting out a hole in the interior of the MERA lattice does NOT actually uniquely determine the collection of density matrices.
  - The density matrix depends on the complete causal cone of the boundary region.
  - The exterior region cuts the causal cone at its shortest width.
- The information encoded in the exterior of the MERA lattice is really the Descending Superoperator that maps the IR density matrix specified at the hole to UV density matrix at the boundary.
- The Descending Superoperator can be thought to propagate the conserved charges of interior excititations to the UV boundary; it enforces gravitational Gauss' Law.





(a') $\rho_{\tau\text{-}1} = \mathcal{D}_{\mathcal{L}}(\rho_\tau)$

[Vidal, Evenbly]

- So while for QFT we may associate an entropy with a fixed region of spacetime, the differential entropy should be associated with a region of fixed boundary area mapped into different exterior spacetimes.

- Perhaps we should associate differential entropy not with $\{\rho_x\}_{x\in\partial B}$, but with $\{\mathcal{D}_x\}_{x\in\partial B}$

$$S_{\text{diff}}: \quad \{\{\mathcal{D}_x\}_{x\in\partial B}\} \to \mathbb{R}$$

- While it is clear how to determine D from MERA lattice, it is not clear how to extract from continuum density matrix.

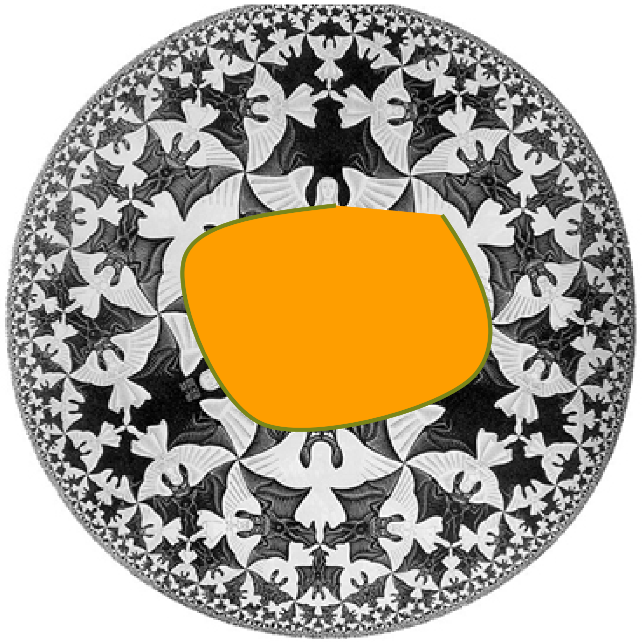
8) Differential Entropy and the RG?

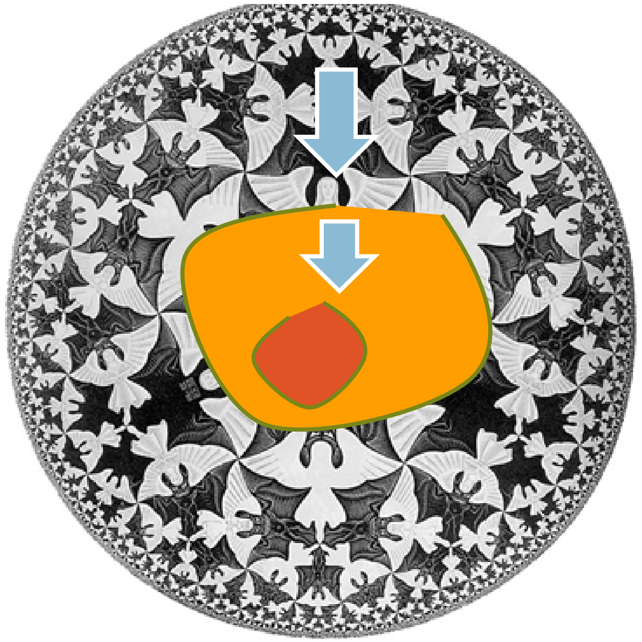
- Differential entropy is a functional that takes a collection of density matrices $\{\rho(\lambda)\}_\lambda$ and outputs one number, the generalized area.
- But the question of reconstructing the bulk is one that is much stronger than just computing areas
- And, from the boundary perspective, $\{\rho(\lambda)\}_\lambda$ contains much, much more information than just S_{diff}. How should we understand what this collection of density matrices is?

~~Conjecture~~ **Guess:**

The map $\rho \rightarrow \{\rho(\lambda)\}_\lambda$ should be understood as an information theoretic RG (coarse-graining) on the state. To $\{\rho(\lambda)\}_\lambda$ we should *associate* an IR field theory that lives on the holographic screen whose area is computed by $S_{diff}[\{\rho(\lambda)\}_\lambda]$.

- $\{\rho(\lambda)\}_\lambda$ explicitly seems to describe the exterior of the holographic screen. The CFT which lives on the screen then should describe the structure of IR entanglement which stitches them back together into ρ.
- A precise boundary formulation of holographic RG has been difficult. This suggests a framework, although we lack a user's manual. It's much more clear in the MERA setting.
- Czech and Lamprou have developed a nice picture for understanding the topology of a spatial slice in terms of differential entropy. This RG structure extends the picture nicely:
  - An IR CFT defines an open set on the spatial slice
  - We define inclusion of sets if one CFT can be obtained from another by RG flow
  - Points are the infinite limits of these RG flows



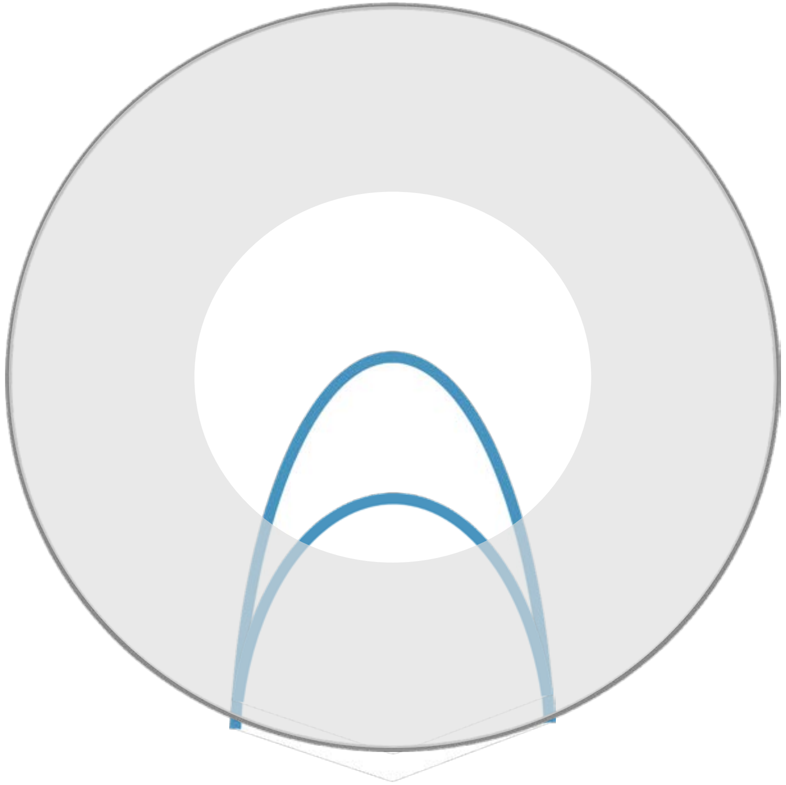


- There are also interesting connections between the RG and our understanding of non-minimal extremal surfaces

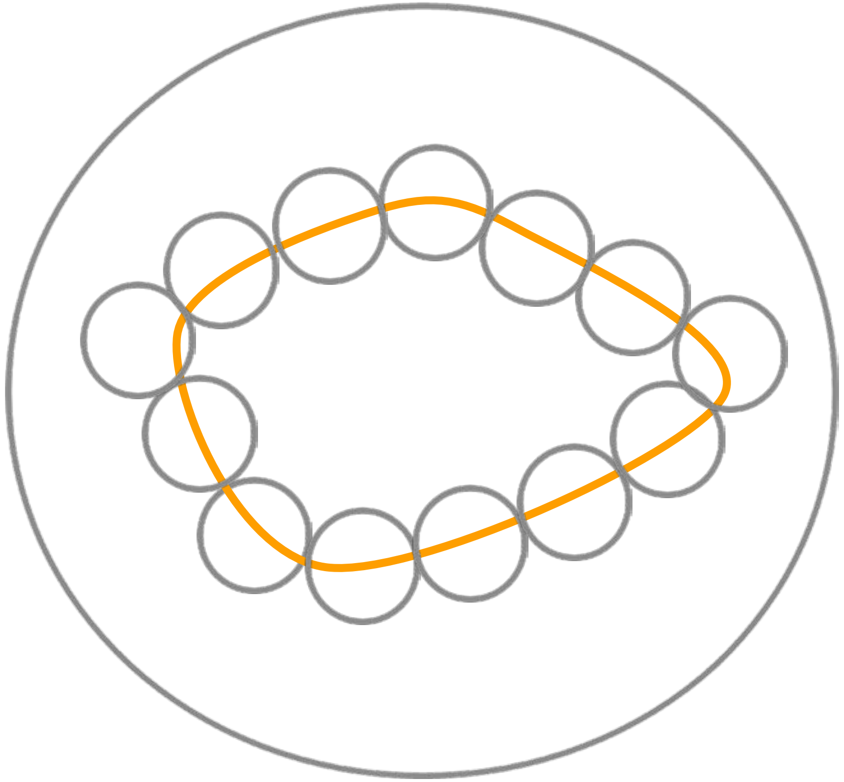
- So there is an RG flow between non-minimal and minimal extremal surfaces

- With respect to a particular choice of holographic RG, the non-minimal surface encodes IR spatial entanglement entropy (+ a non-spatial UV entropy that we still don’t understand)

- The non-minimal surface is also specified at the UV boundary by an non-standard set of boundary conditions—can we understand the flow of boundary conditions from the UV theory?



- This RG perspective also gives an interesting interpretation for what we are calculating with differential entropy:



- The differential entropy can then be understood as summing the differential contribution of IR entanglement entropy with respect to mutually incompatible RG schemes.

Conclusions

- We have given a geometric definition of differential entropy that generates the generalized area of arbitrary surfaces in higher-derivative theories of gravity.
- This definition of differential entropy has an incomplete boundary description:
  - Using only spatial entanglement entropy, we can only associate entropies to coarse-grained bulk surfaces
  - With a (not currently understood) boundary prescription for all boundary anchored extremal surfaces, we could assign an entropy to all bulk surfaces
- Differential entropy is only one piece of data that we can assign to a collection of boundary density matrices. A much richer structure is to assign to this coarse-graining an information theoretic RG scheme
  - Is this the right boundary interpretation of holographic RG?
  - There seems to be an interesting connection between this RG flow, the flow of non-spatial EE, and differential entropy.