On Spacetime Entanglement

with M. Headrick & J. Wien; J. Rao & S. Sugishita

Black Hole Entropy:

• Bekenstein and Hawking: “black holes carry entropy!”

thermodynamics relativity geometry

$$S_{BH} = \frac{k_B\, c^3}{\hbar}\, \frac{\mathcal{A}}{4G}$$

quantum gravity

• “horizons carry entropy!”: de Sitter space and Rindler wedge

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- “horizons carry entropy!”: de Sitter space and Rindler wedge

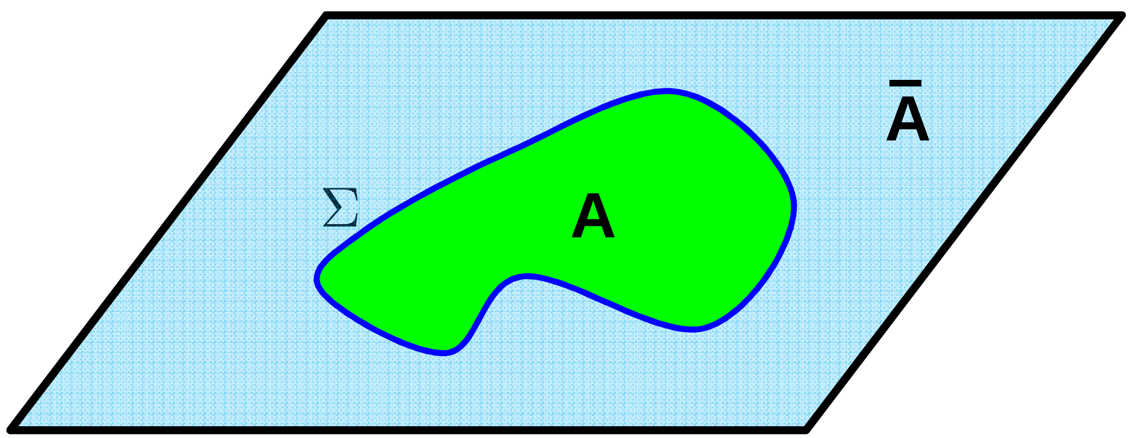
- window into quantum gravity?!?

Spacetime Entanglement Conjecture

(Bianchi & RM)

- in a theory of quantum gravity, for any sufficiently large region A in a smooth background, consider entanglement entropy between dof describing A and $\bar{\text{A}}$; contribution describing short-range entanglement is finite and described in terms of geometry of entangling surface with leading term:

$$S_{\text{EE}} = \frac{\mathcal{A}_\Sigma}{4G_N} + \cdots$$



- higher order terms controlled by higher curvature gravitational couplings, similar to Wald entropy (RM, Pourhasan & Smolkin)

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- arguments:
  1. holographic $S_{\rm EE}$ in AdS/CFT correspondence
  2. QFT renormalization of G_N
  3. induced gravity, eg, Randall-Sundrum 2 model
  4. Jacobson's "thermal origin" of gravity
  5. spin-foam approach to quantum gravity

AdS/CFT Correspondence:

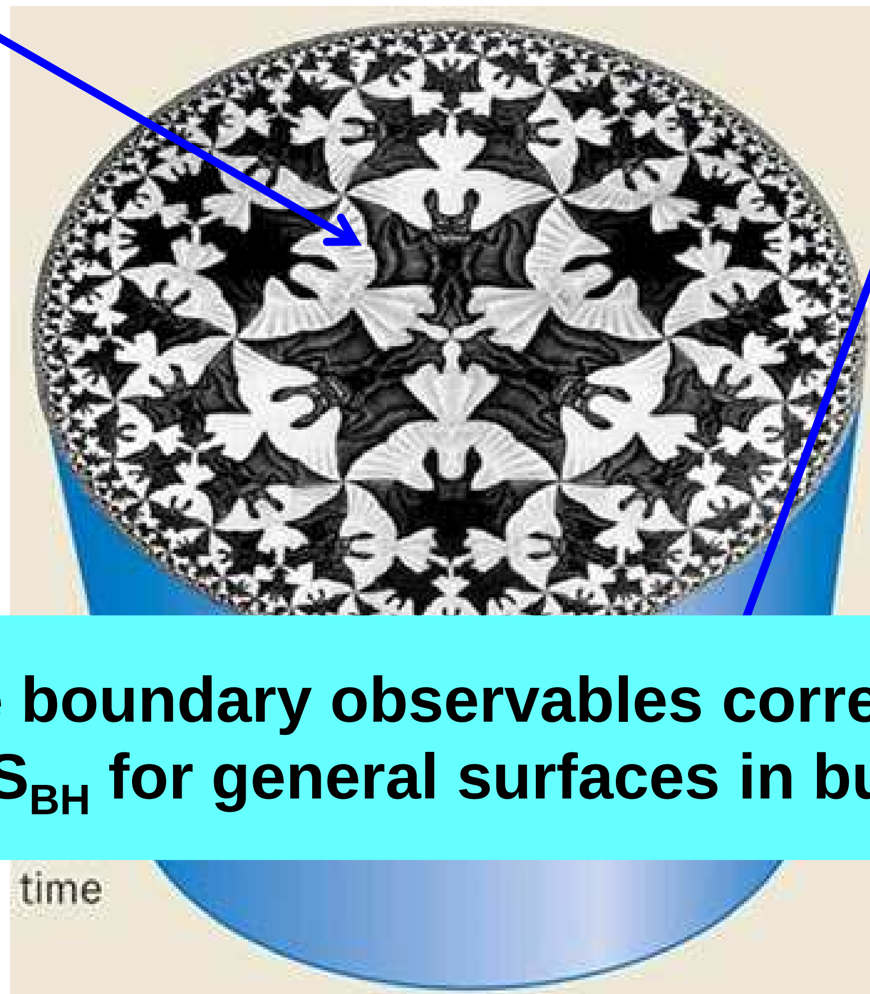
Bulk: gravity with negative Λ
in **d+1** dimensions

↔ "holography"

Boundary: quantum field theory
in **d** dimensions

anti-de Sitter space

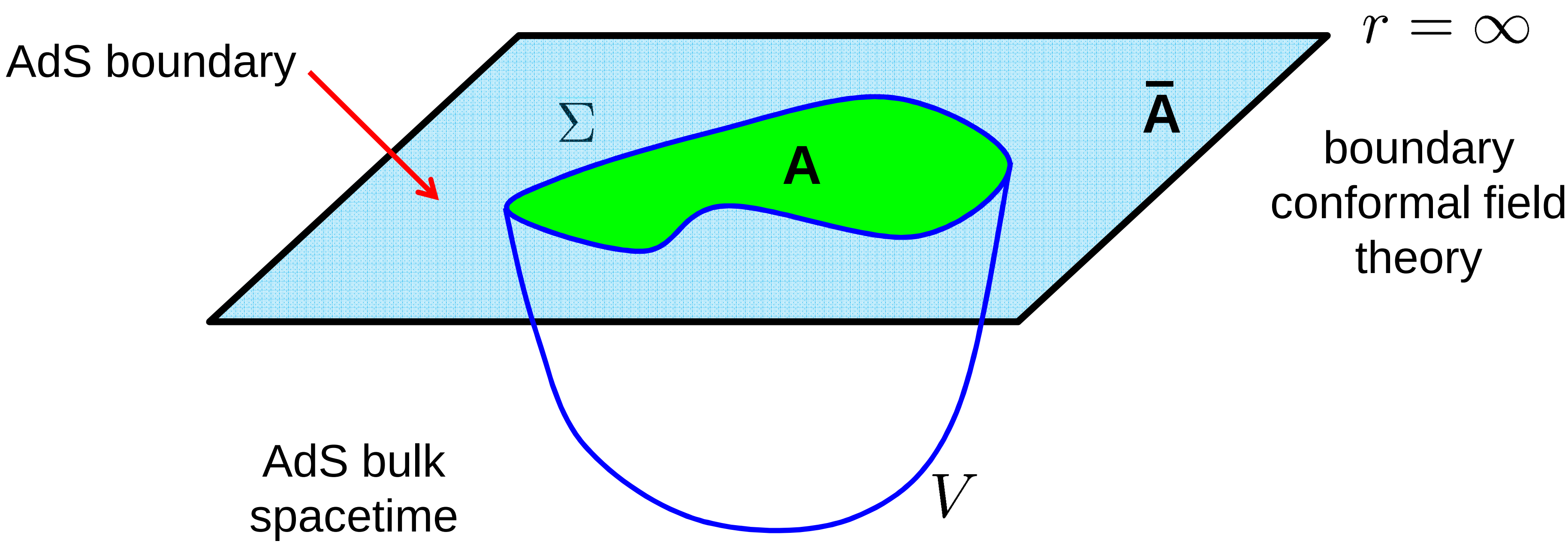
conformal field theory



Are there boundary observables corresponding to S_{BH} for general surfaces in bulk?

(Ryu & Takayanagi `06)

Holographic Entanglement Entropy:



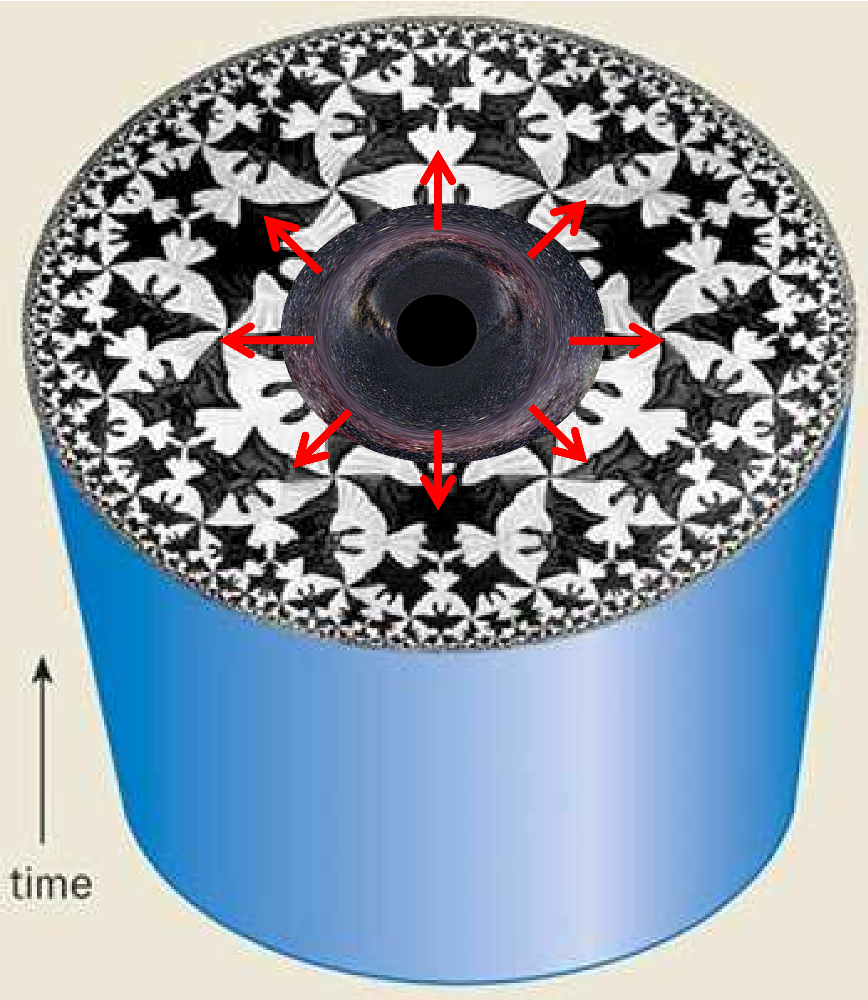
$$S(A) = \min_{\partial V = \Sigma} \frac{A_V}{4G_N}$$

S_{BH} applied in unusual circumstances

Lessons from Holographic EE:

AdS/CFT Dictionary:

Boundary: thermal plasma ⟷ Bulk: black hole



Temperature	Temperature
Energy	Energy
Entropy	Entropy

Lessons from Holographic EE:

(entanglement entropy)$_{boundary}$ = (entropy of extremal surface)$_{bulk}$

- R&T prescription assigns gravitational entropy $S_{BH} = \mathcal{A}/(4G_N)$ to "unconventional" bulk surfaces/regions:

  not black hole! not horizon! not boundary of causal domain!

- **indicates S_{BH} applies more broadly but more examples?**

- S_{BH} on other surfaces speculated to give new entropic measures of entanglement in boundary theory

  → causal holographic information
  (Hubeny & Rangamani; H, R & Tonni; Freivogel & Mosk; . . .)

  → entanglement between high and low scales
  (Balasubramanian, McDermott & van Raamsdonk)

  → hole-ographic spacetime
  (Balasubramanian, Chowdhury, Czech, de Boer & Heller)

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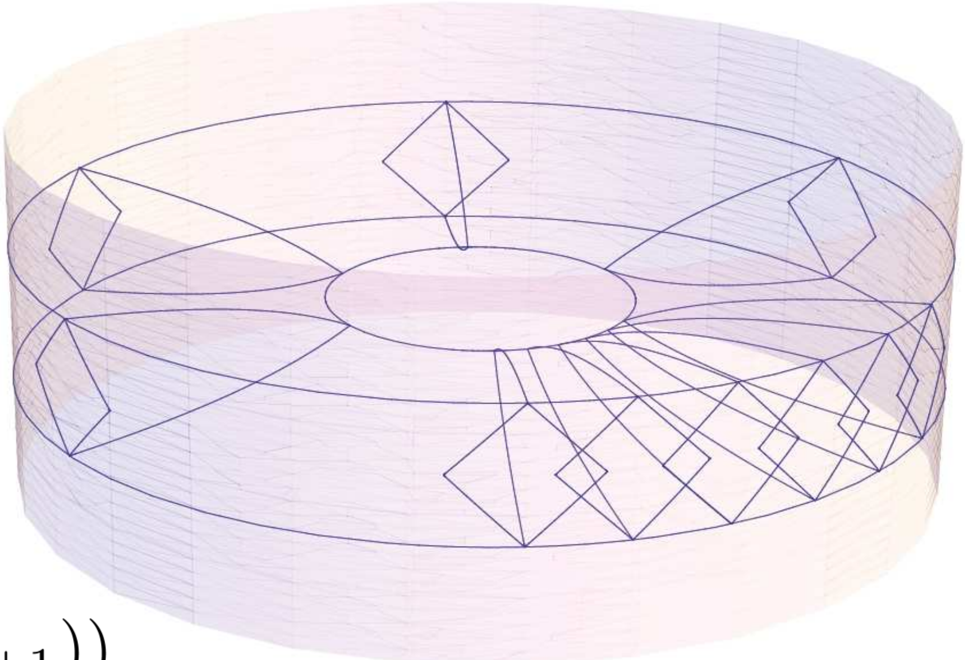
arXiv:1310.4204 [hep-th]

“hole-ographic spacetime”:

two new ideas:

- **residual entropy**: collective uncertainty associated with family of observers confined to finite time strip; maximum entropy of global density matrix consistent with density matrices of subsy’s



- **differential entropy**:

$$E = \sum \left(S(I_j) - S(I_j \cap I_{j+1})\right)$$

boundary observables which yield gravitational entropy of closed curves inside of d=3 AdS space with certain continuum limit

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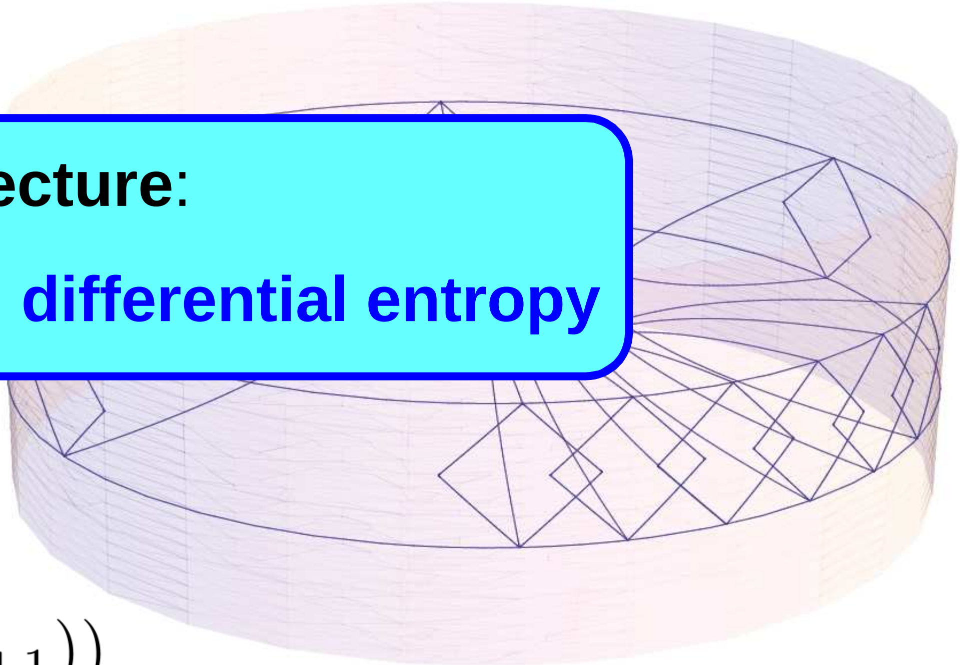

Conjecture:

residual entropy = differential entropy

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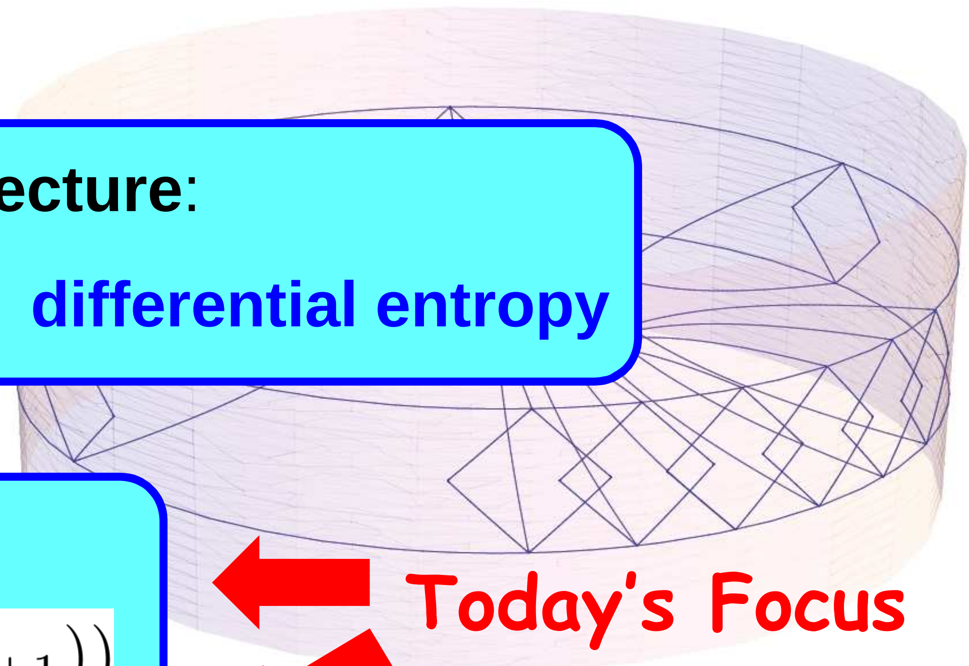

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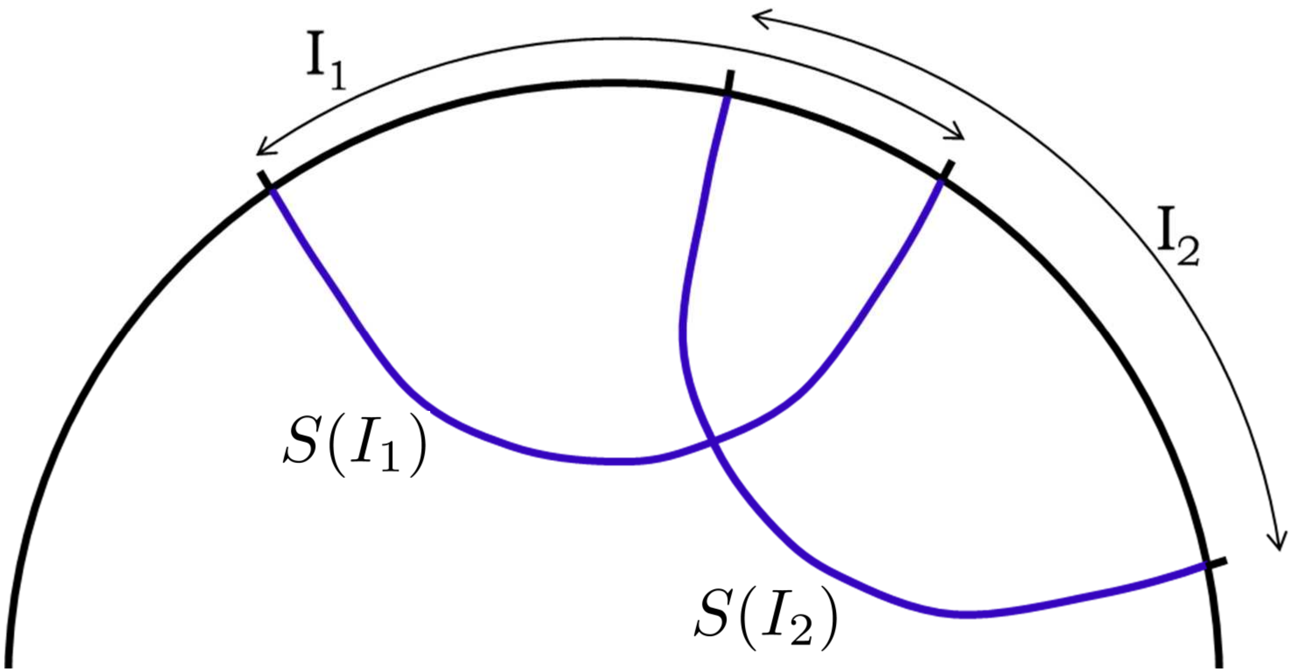
Today’s Focus

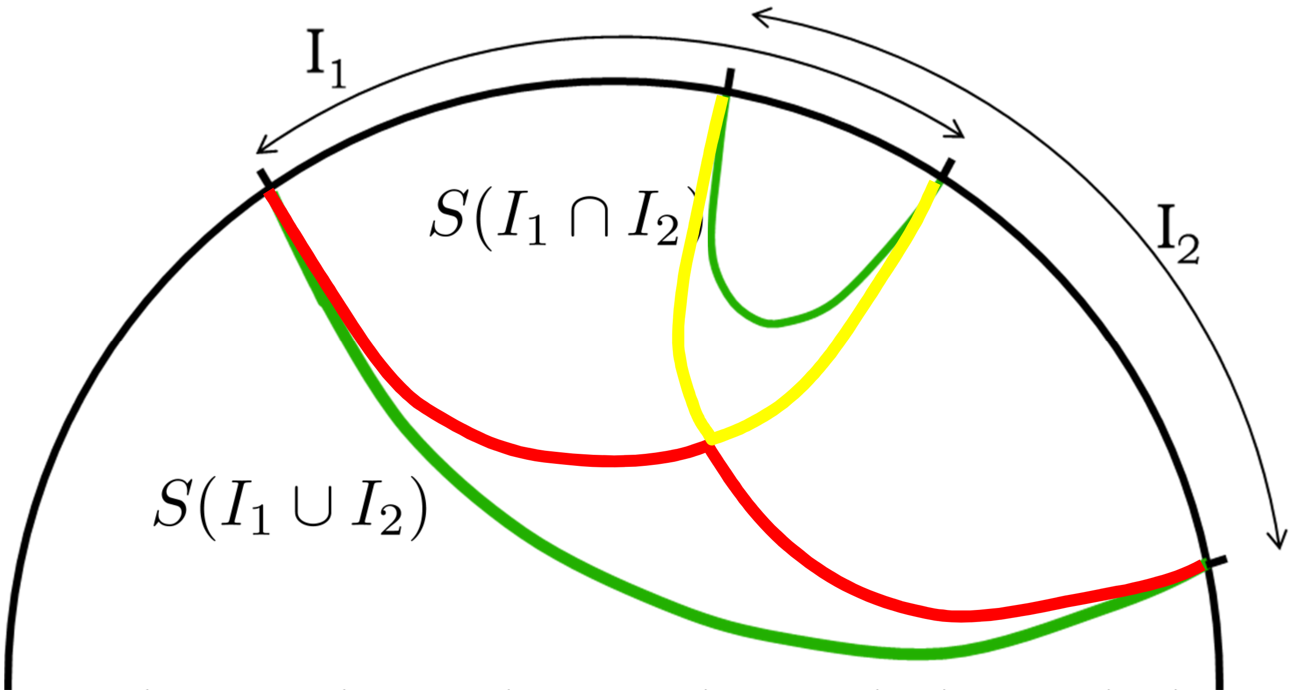
boundary observables which yield gravitational entropy of closed curves inside of d=3 AdS space with certain continuum limit

Strong Sub-Additivity: $S(I_1 \cup I_2) + S(I_1 \cap I_2) \leq S(I_1) + S(I_2)$

- recall proof that RT prescription satisfies SSA

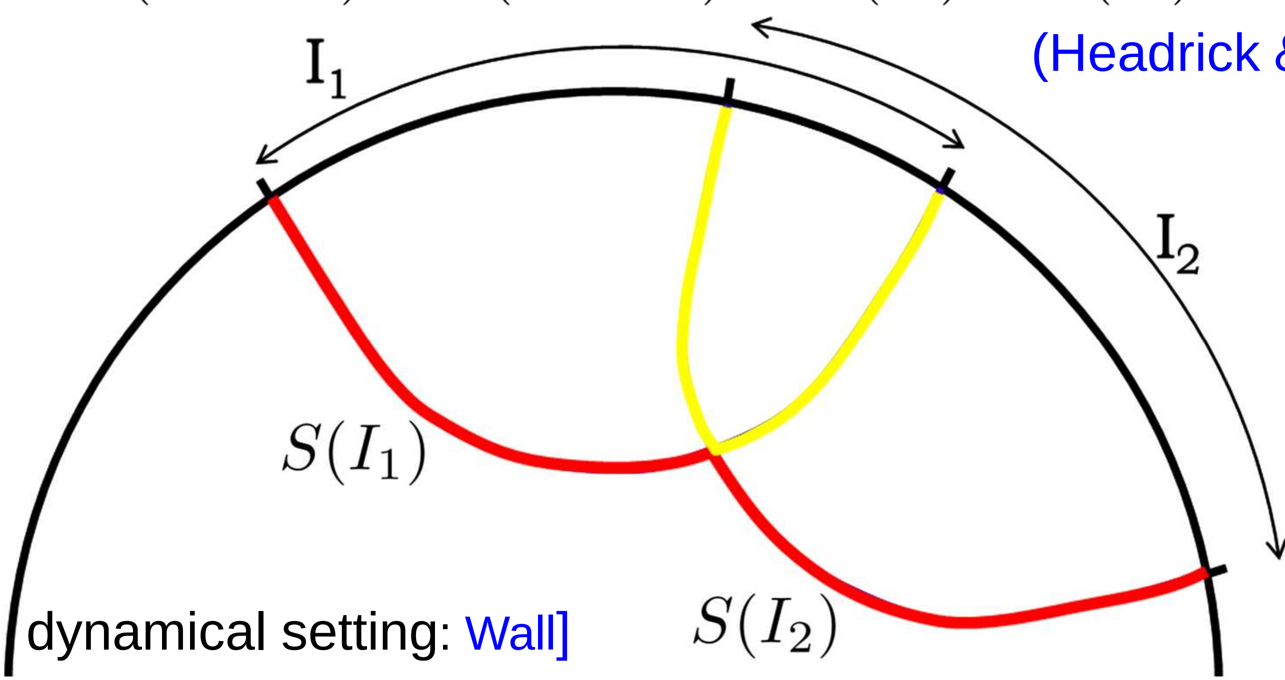
(Headrick & Takayanagi)





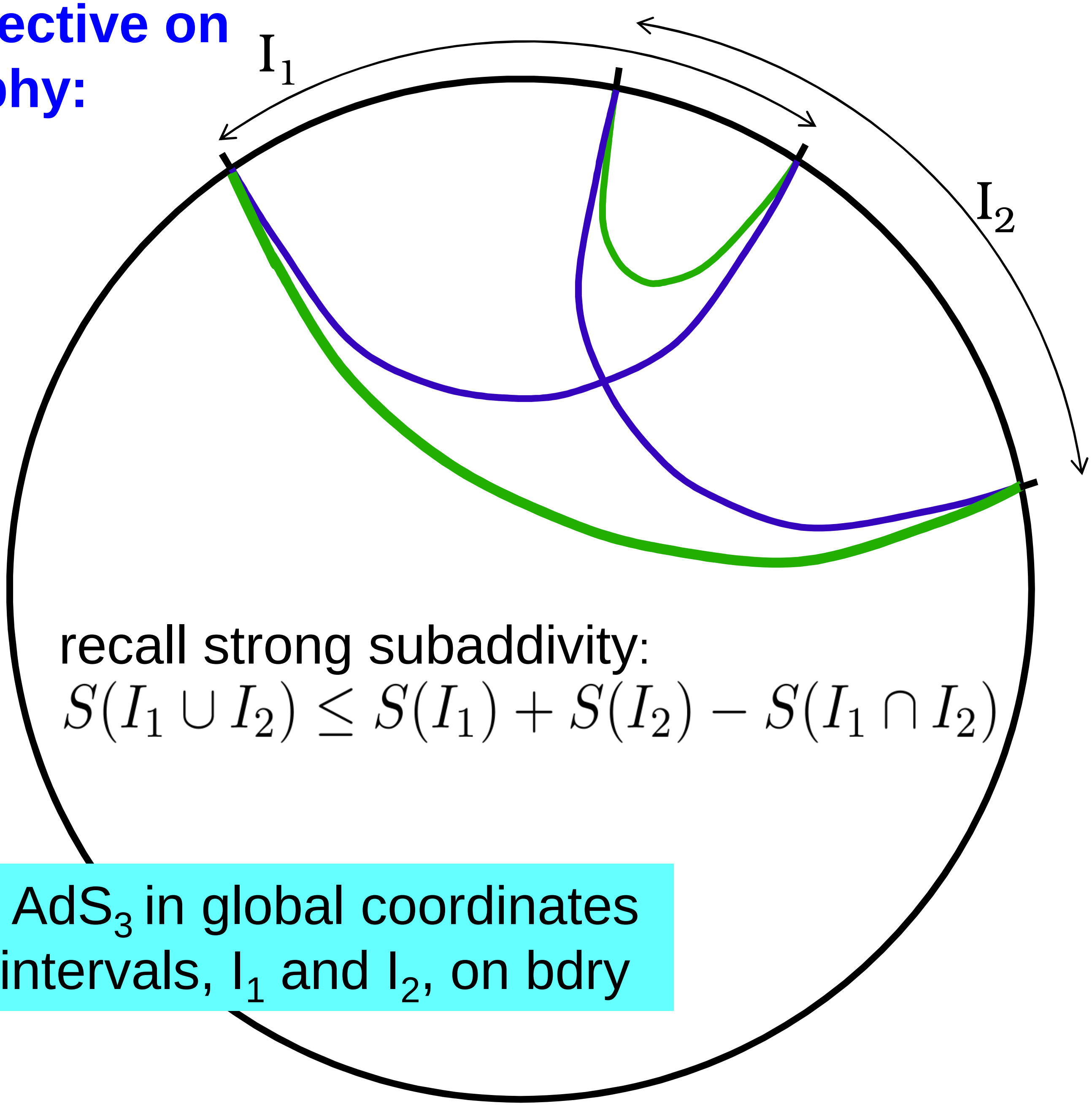
$$S(I_1 \cup I_2) + S(I_1 \cap I_2) \leq S(I_1) + S(I_2)$$

(Headrick & Takayanagi)



[extended to dynamical setting: Wall]

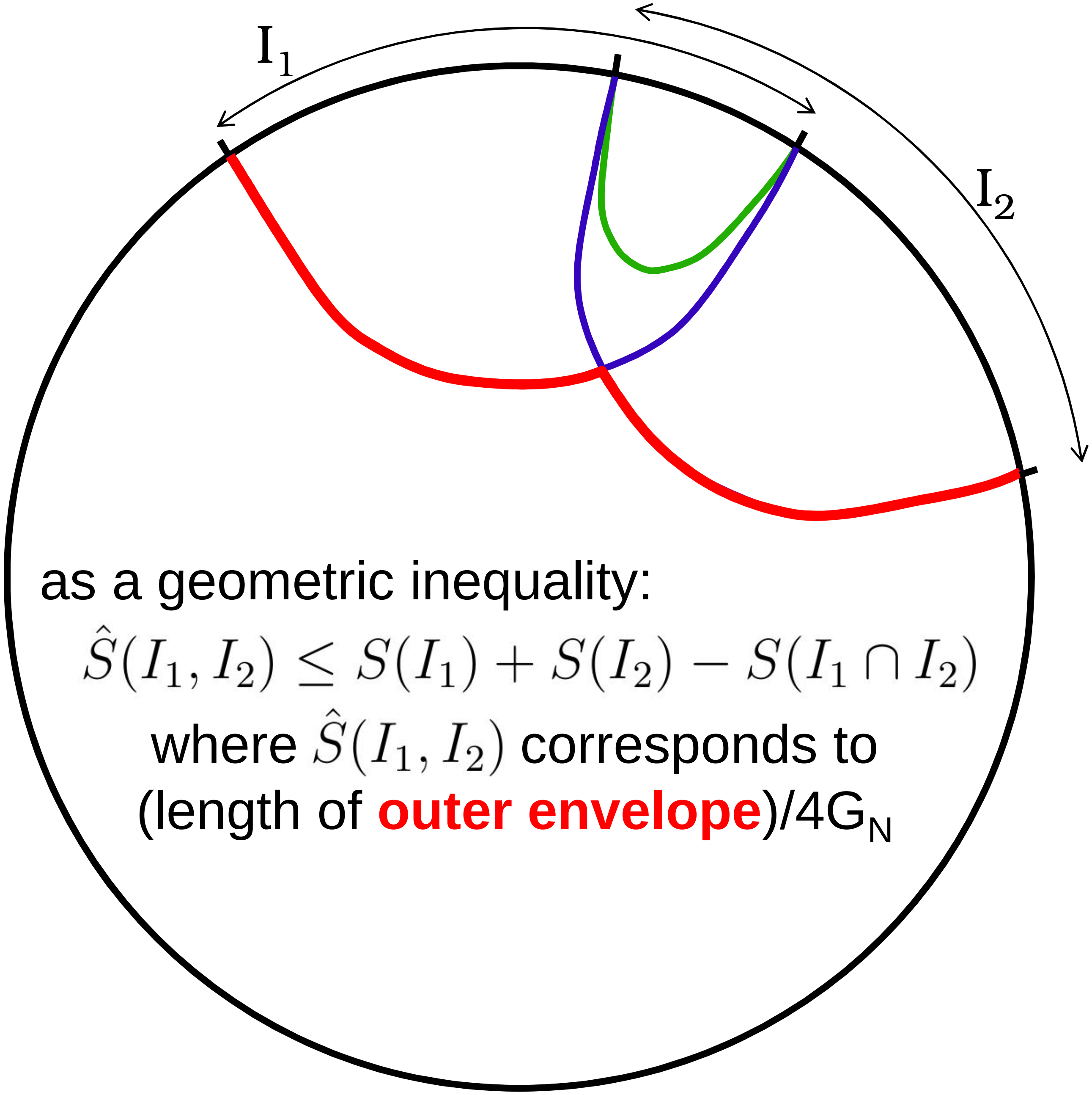
One perspective on hole-ography:



recall strong subadditivity:

$$S(I_1 \cup I_2) \leq S(I_1) + S(I_2) - S(I_1 \cap I_2)$$

consider AdS_3 in global coordinates with two intervals, I_1 and I_2, on bdry

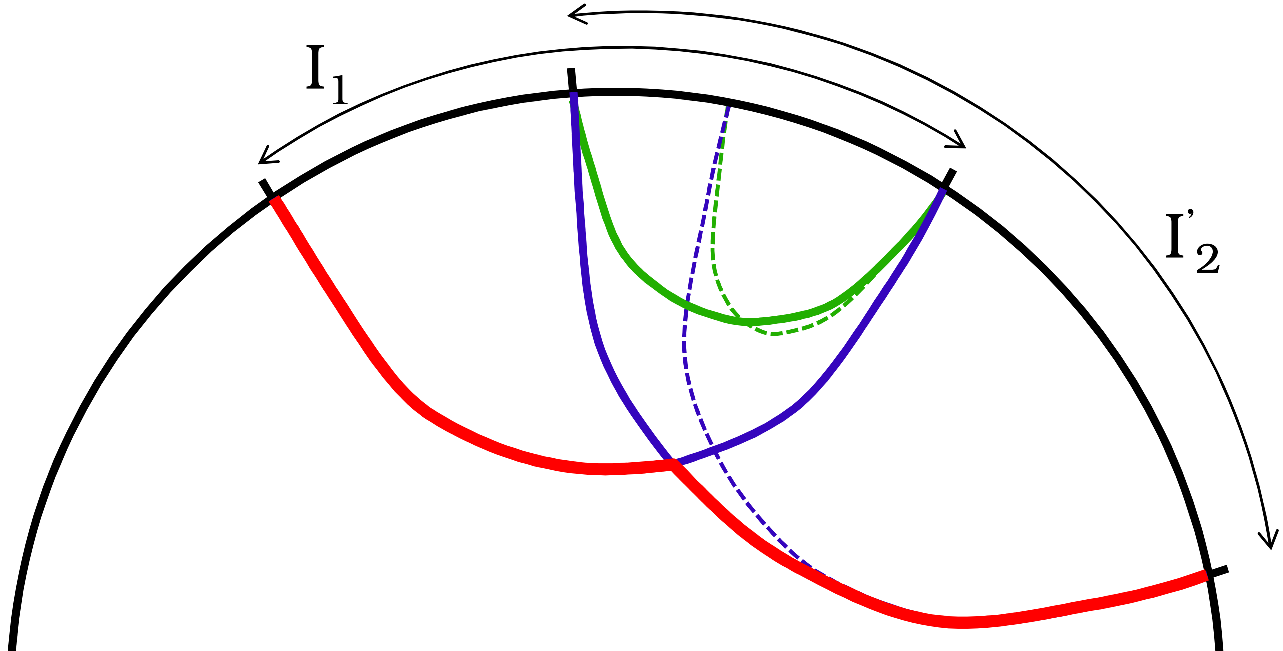


as a geometric inequality:

$$\hat{S}(I_1, I_2) \leq S(I_1) + S(I_2) - S(I_1 \cap I_2)$$

where $\hat{S}(I_1, I_2)$ corresponds to

(length of **outer envelope**)/$4G_N$



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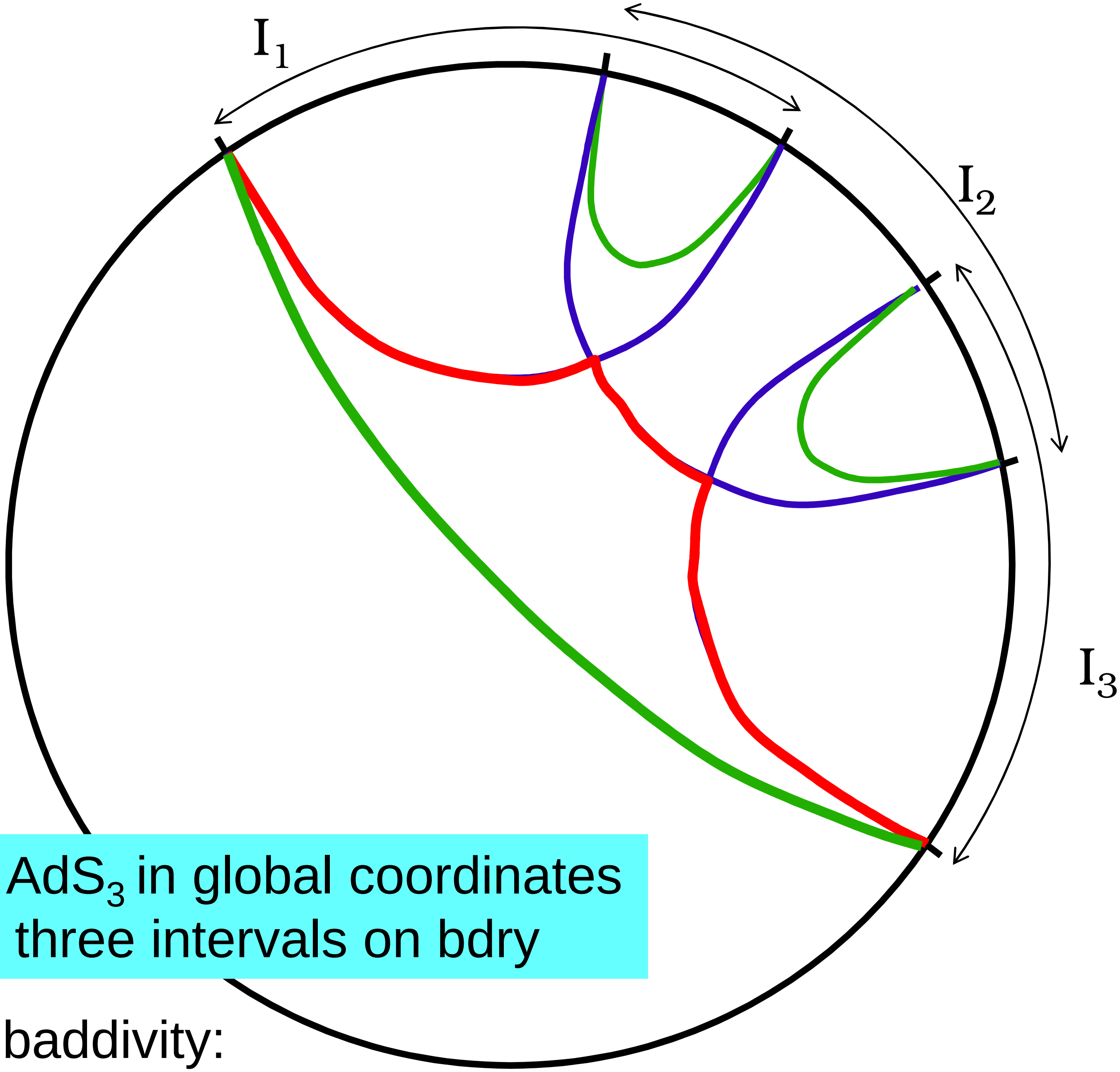
Note: $\hat{S}(I_1,I_2)$ is not a function of $I_1 \cup I_2$ alone; instead depends on details of partition

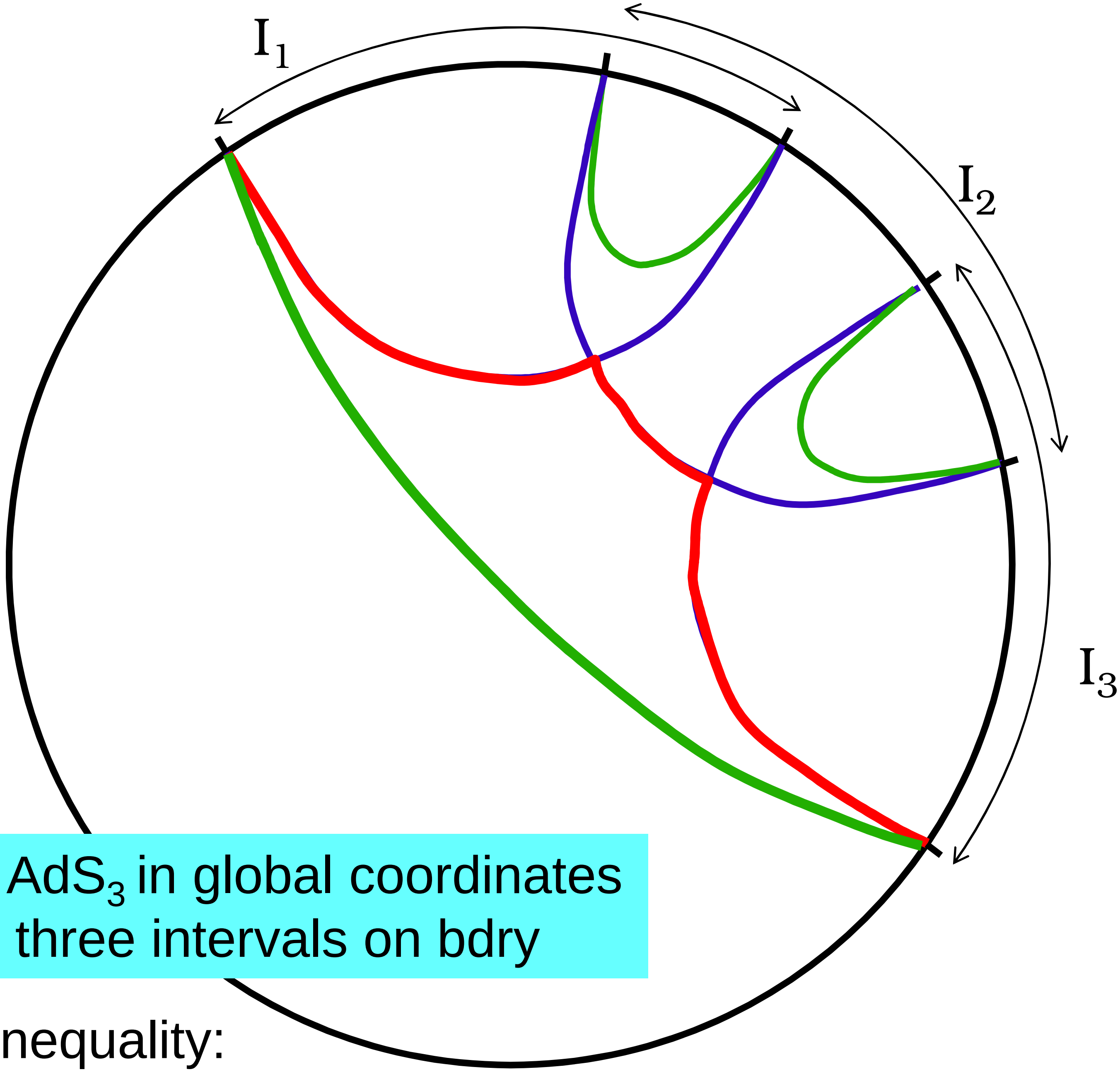
eg, $I_1 \cup I_2 = I_1 \cup I_2'$ but $\hat{S}(I_1, I_2) \neq \hat{S}(I_1, I_2')$



consider AdS_3 in global coordinates
now with three intervals on bdry

strong subaddivity:

$$S(I_1 \cup I_2 \cup I_3) \leq S(I_1) + S(I_2) + S(I_3) - S(I_1 \cap I_2) - S(I_2 \cap I_3)$$



consider AdS_3 in global coordinates
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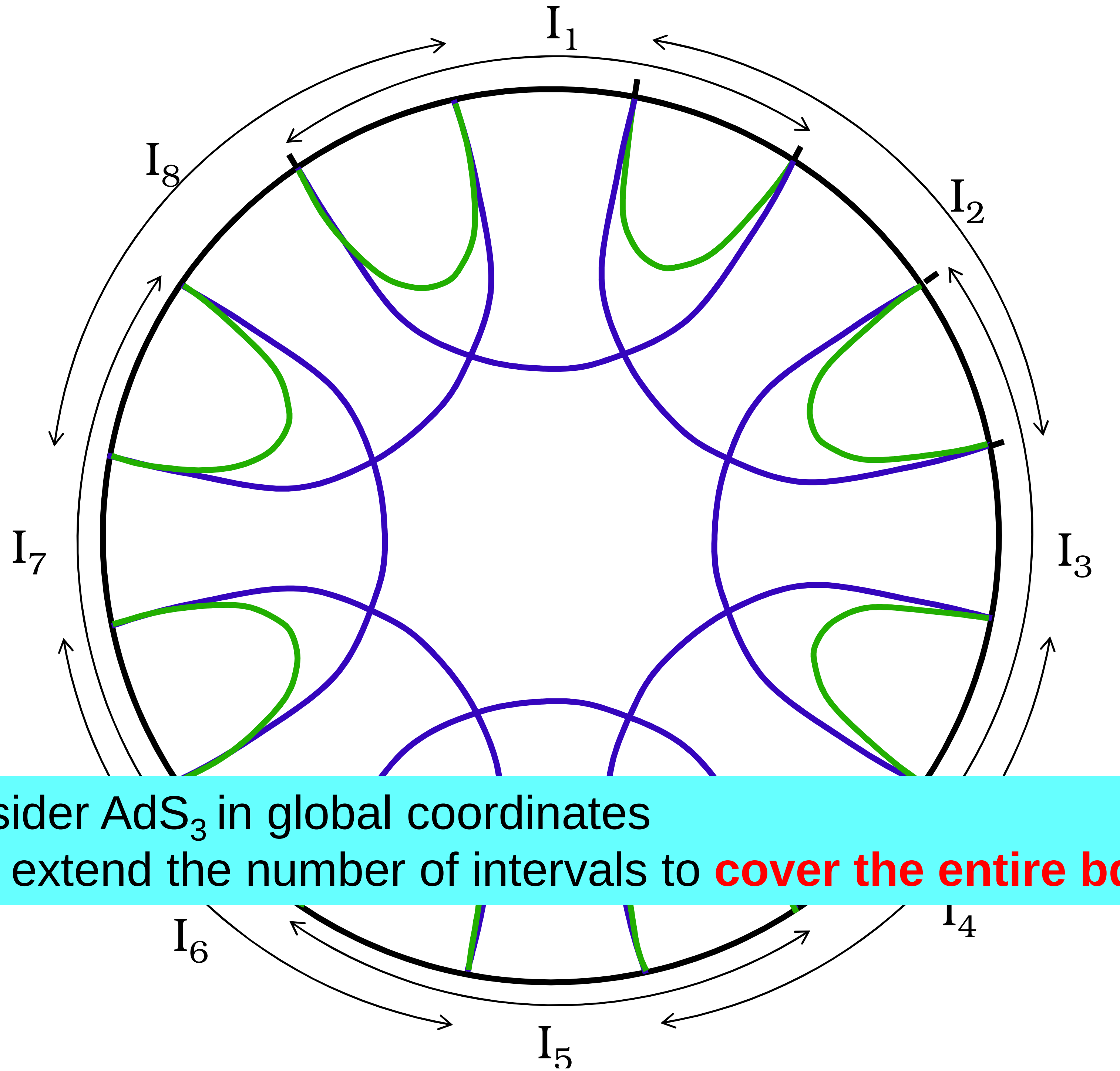
geometric inequality:

$$\hat{S}(I_1, I_2, I_3) \le S(I_1) + S(I_2) + S(I_3) - S(I_1 \cap I_2) - S(I_2 \cap I_3)$$



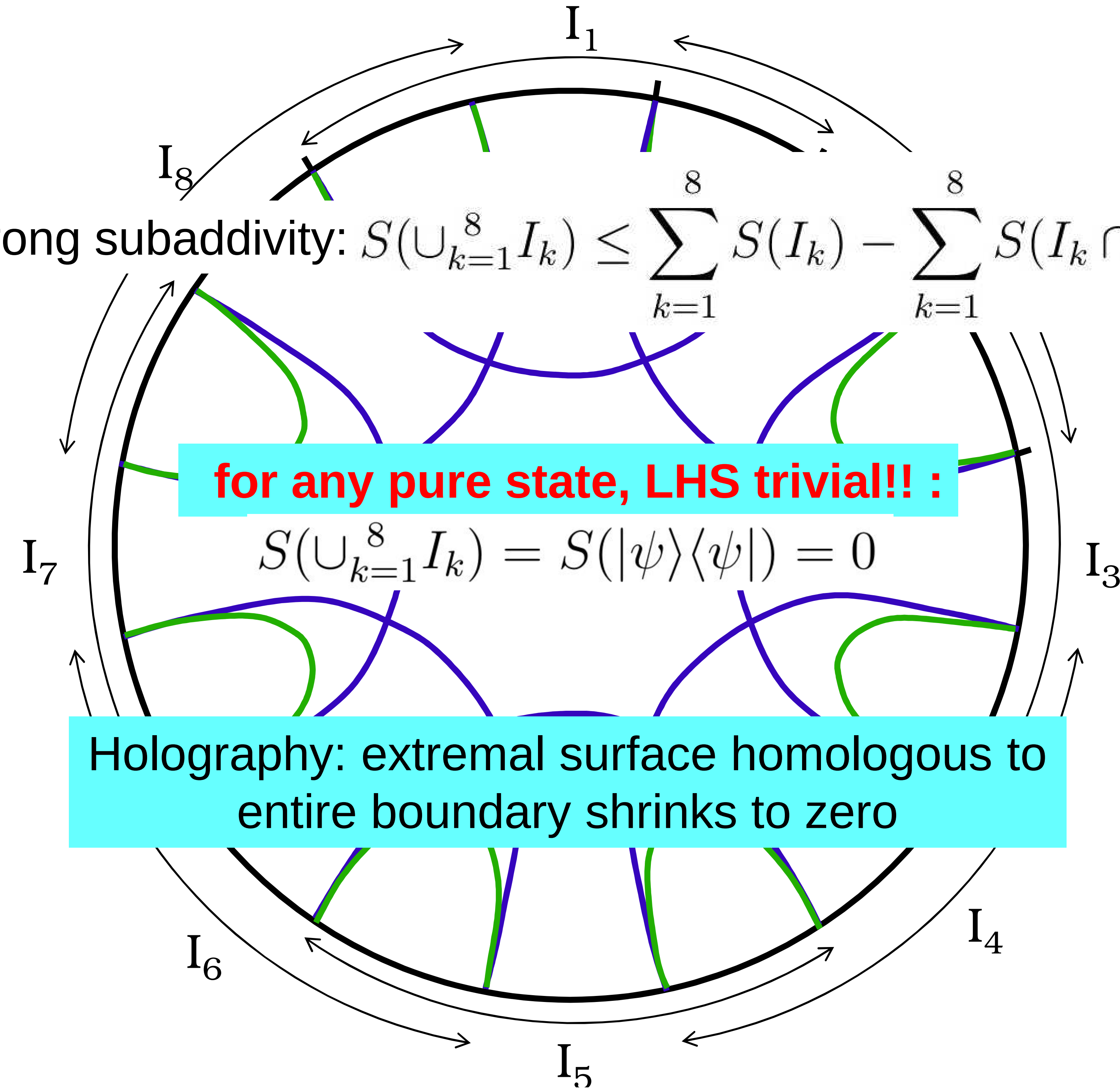
consider AdS_3 in global coordinates

now extend the number of intervals to **cover the entire bdry** strong subaddivity: $S(\cup_{k=1}^{8} I_k) \leq \sum_{k=1}^{8} S(I_k) - \sum_{k=1}^{8} S(I_k \cap I_{k+1})$

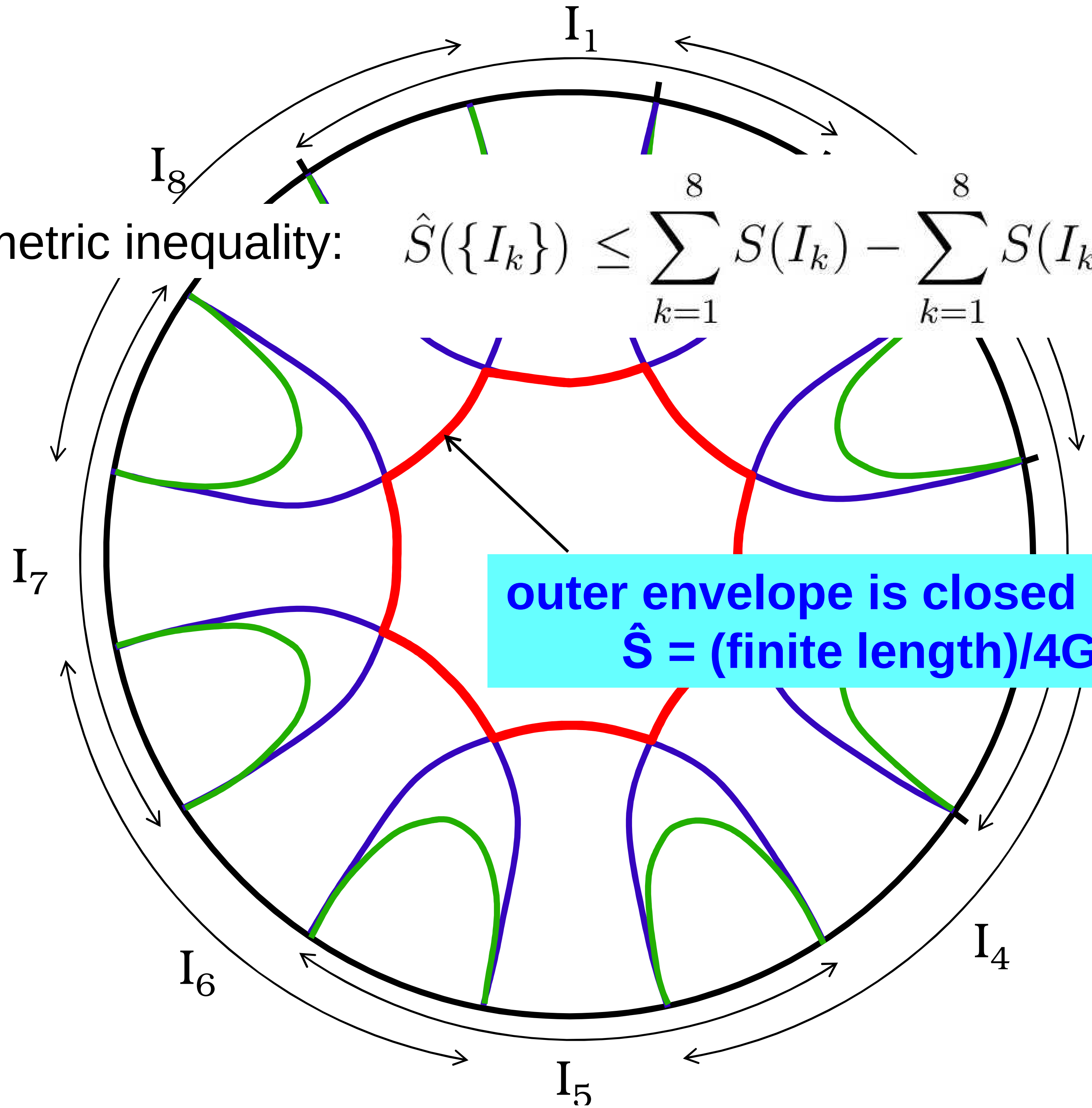
for any pure state, LHS trivial!! :

$$S(\cup_{k=1}^{8} I_k) = S(|\psi\rangle\langle\psi|) = 0$$

Holography: extremal surface homologous to entire boundary shrinks to zero



geometric inequality: $\hat{S}(\{I_k\}) \leq \sum_{k=1}^{8} S(I_k) - \sum_{k=1}^{8} S(I_k \cap I_{k+1})$

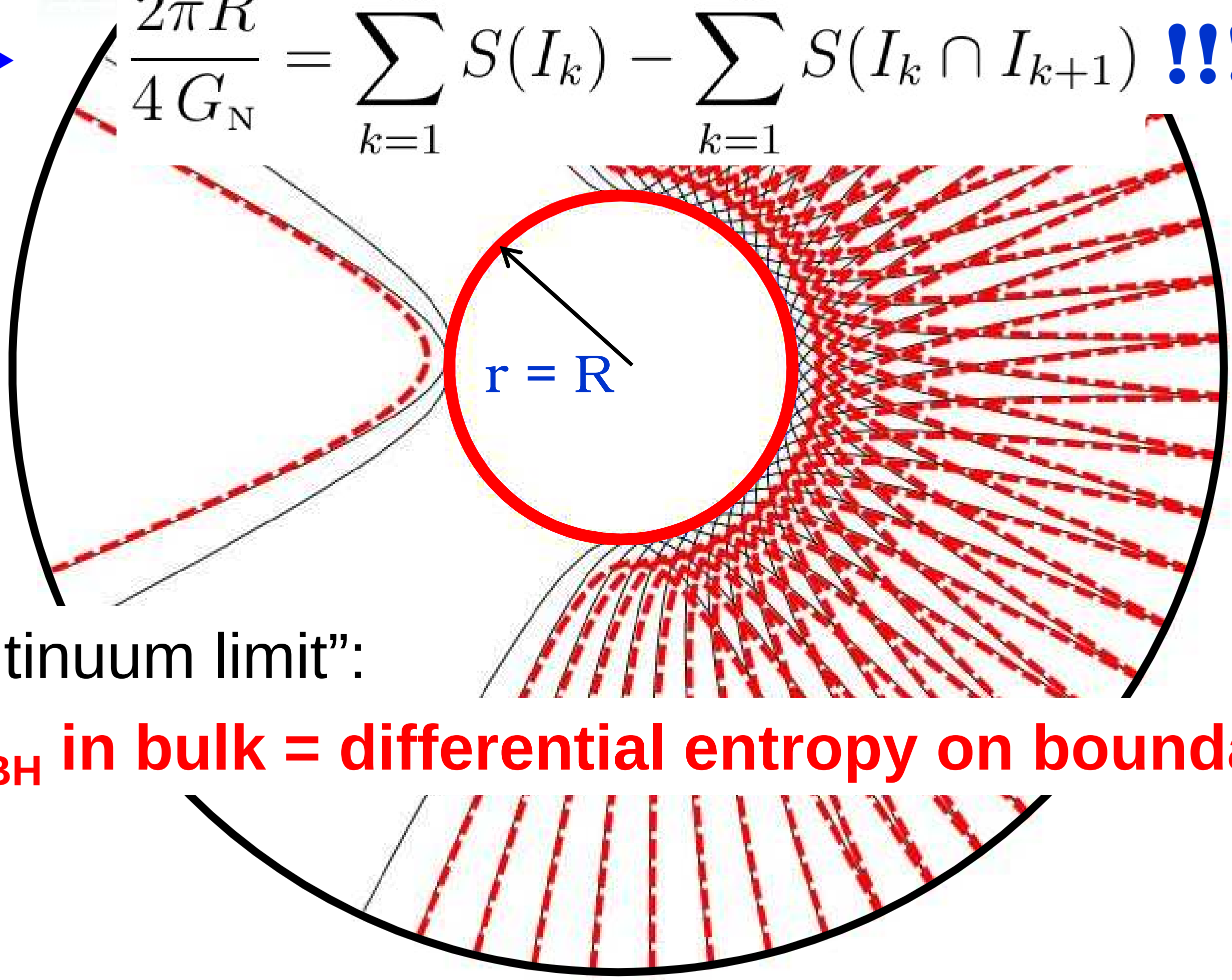


outer envelope is closed curve!
$\hat{S}$ = (finite length)/$4G_N$

(Balasubramanian, Chowdhury, Czech, de Boer & Heller)

- keep length of intervals is fixed but take number of intervals to infinity
- outer envelope becomes a smooth circle of constant radius
- **surprise is that the geometric inequality is precisely saturated!**

→ $$\frac{2\pi R}{4\,G_N} = \sum_{k=1}^{\infty} S(I_k) - \sum_{k=1}^{\infty} S(I_k \cap I_{k+1})$$ **!!!**



in "continuum limit":

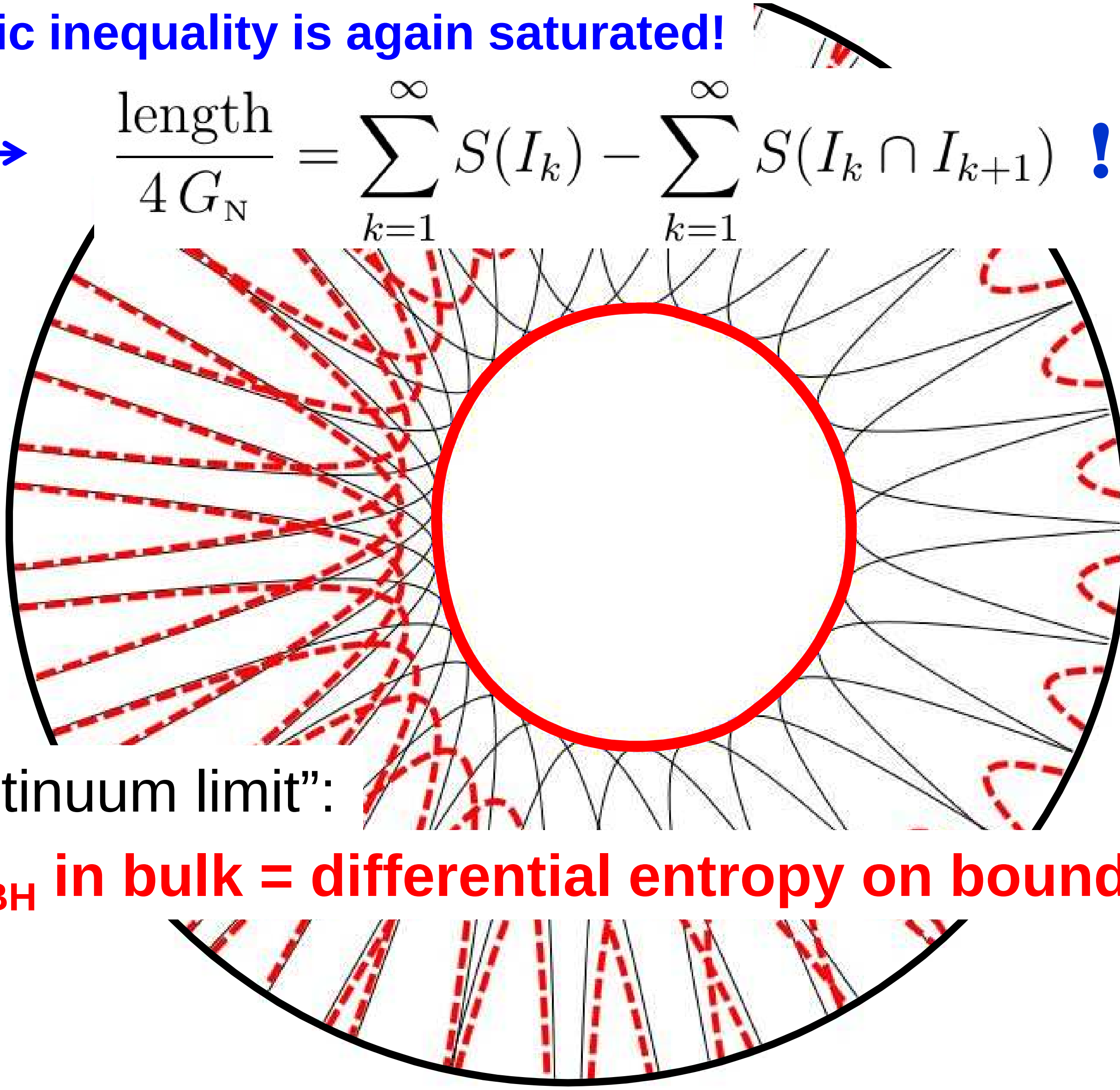
S_{BH} in bulk = differential entropy on boundary

geometric inequality: $\hat{S}(\{I_k\}) \leq \Sigma_{k=1}^{\infty} S(I_k) - \Sigma_{k=1}^{\infty} S(I_k \cap I_{k+1})$

(Balasubramanian, Chowdhury, Czech, de Boer & Heller)

- prescription extends to general curves in the bulk with Ŝ ∝ length of curve
- **geometric inequality is again saturated!**

→ $$\frac{\text{length}}{4\,G_{\rm N}} = \sum_{k=1}^{\infty} S(I_k) - \sum_{k=1}^{\infty} S(I_k \cap I_{k+1})$$ **!!!**



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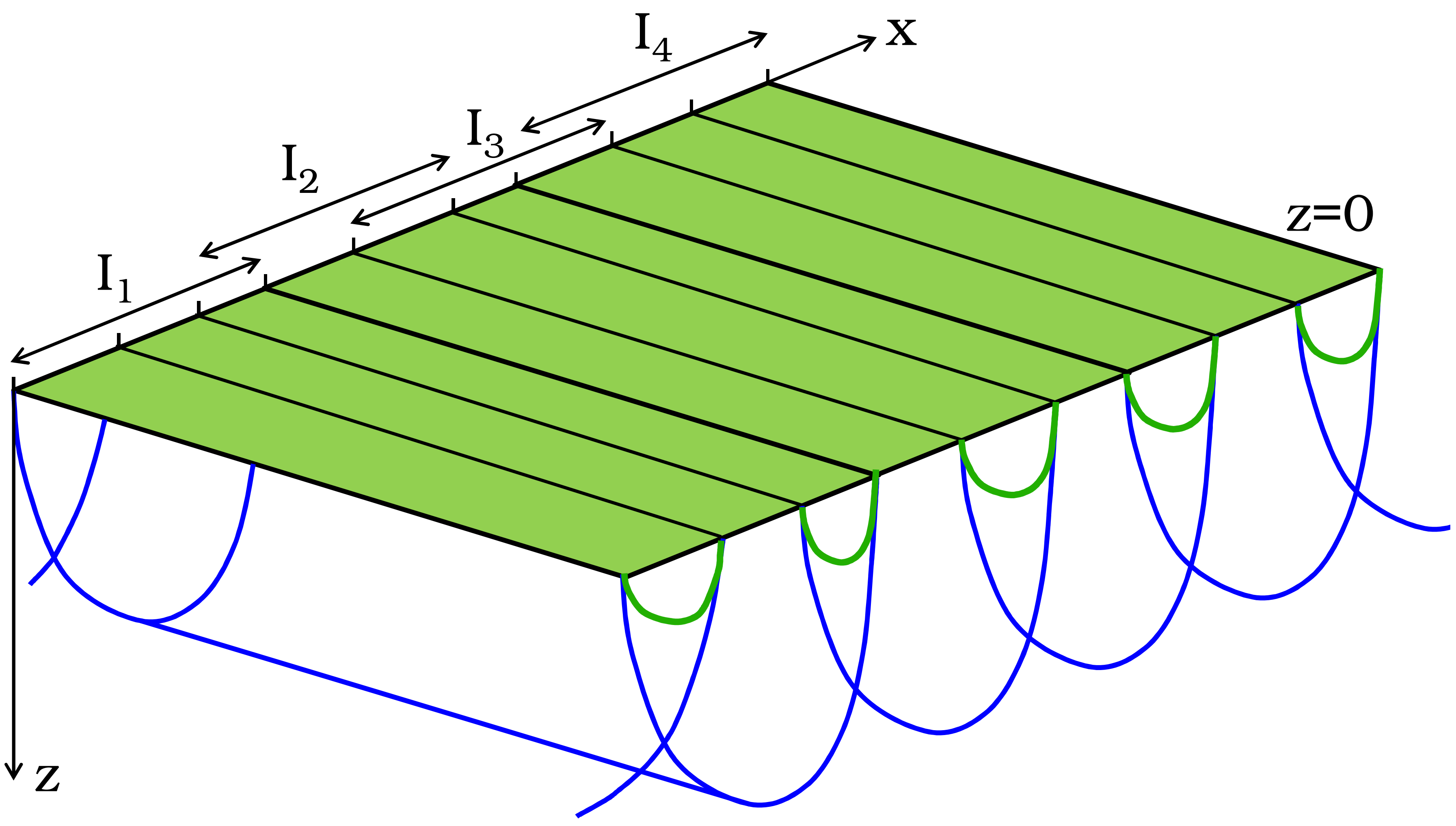
Higher dimensional "holes":

- "outer envelope" readily extends to higher dimensions
- extend to AdS_{d+1} in Poincare coordinates and tile t=0 surface with strips/slabs of constant width



Higher dimensional “holes”:

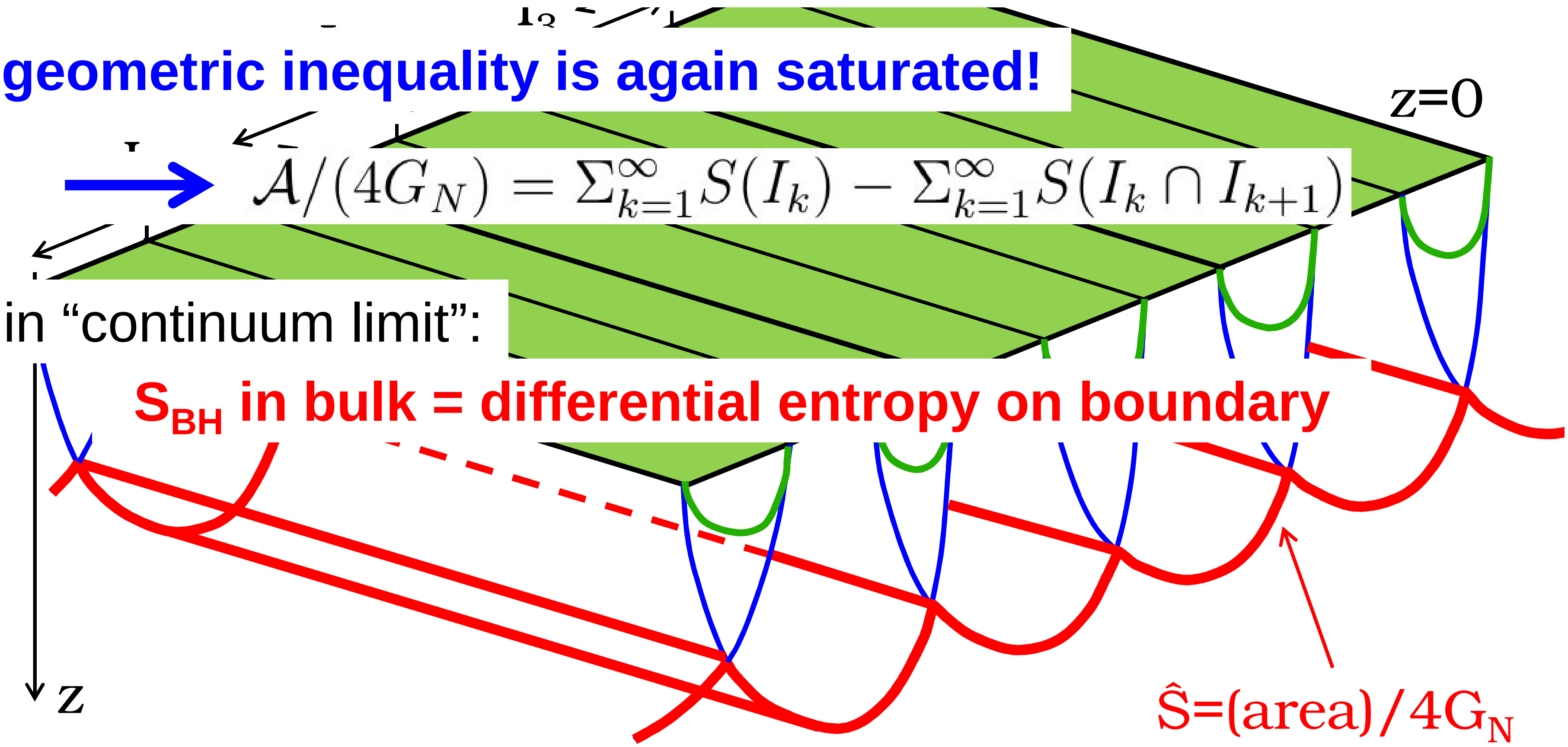
- “outer envelope” readily extends to higher dimensions
- extend to AdS_{d+1} in Poincare coordinates and tile t=0 surface with strips/slabs of constant width
- hole-ographic prescription extends to general surfaces (with planar symmetry, ie, z=z(x)) in the bulk with Ŝ ~ area of surface

- **geometric inequality is again saturated!**

→ $$\tilde{A}/(4G_N) = \Sigma_{k=1}^{\infty} S(I_k) - \Sigma_{k=1}^{\infty} S(I_k \cap I_{k+1})$$

in “continuum limit”:

S_{BH} in bulk = differential entropy on boundary

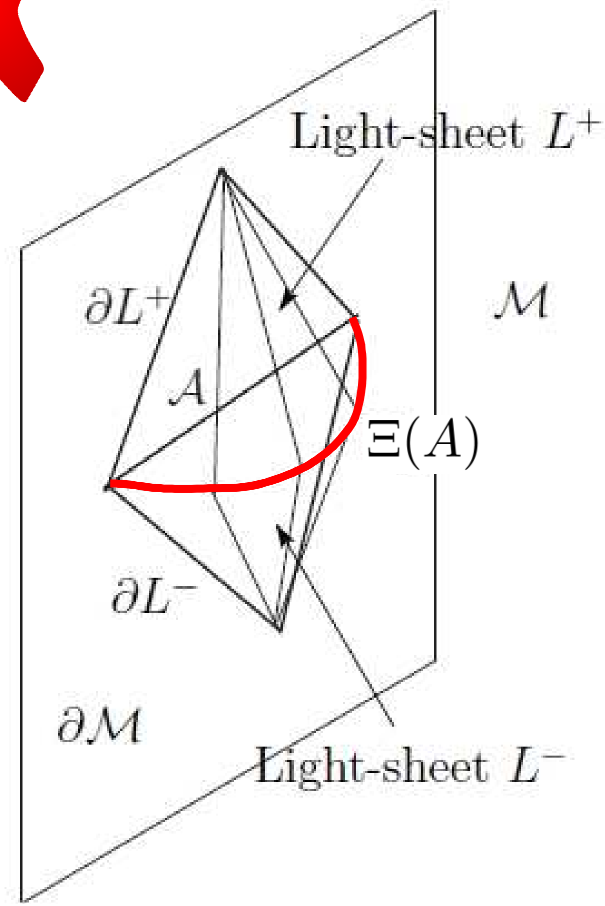


Causal Holographic Information:

(Hubeny & Rangamani)

- evaluate S_{BH} for extremal surface on boundary of bulk causal wedge

$$\chi(A) = \frac{\Xi(A)}{4\,G_N}$$



- might give natural extension of “observer story” to higher dimensions

- applying hole-graphic prescription to strip decomposition in higher dimensions, find: “geometric inequality” is **NOT** saturated!

$$\longrightarrow \sum_{k=1}^{\infty} \chi(I_k) - \sum_{k=1}^{\infty} \chi(I_k \cap I_{k+1}) \sim \left(\frac{z_{max}}{\delta}\right)^{d-4}$$

- sub-leading divergences are nonlocal!! (in contrast to S_{EE})

(Freivogel & Mosk)

→ lesson: hole-ographic construction requires extremal surfaces

General Backgrounds:

• consider more general holographic backgrounds:

$$ds^2 = -g_0(z)\,dt^2 + g_1(z)\,dx^2 + \sum_{i=2}^{d-1} g_i(z)\,(dx^i)^2 + f(z)\,dz^2$$

$$\longrightarrow \quad S_{grav} = \sum\nolimits_{k=1}^{\infty} S(I_k) - \sum\nolimits_{k=1}^{\infty} S(I_k \cap I_{k+1})$$

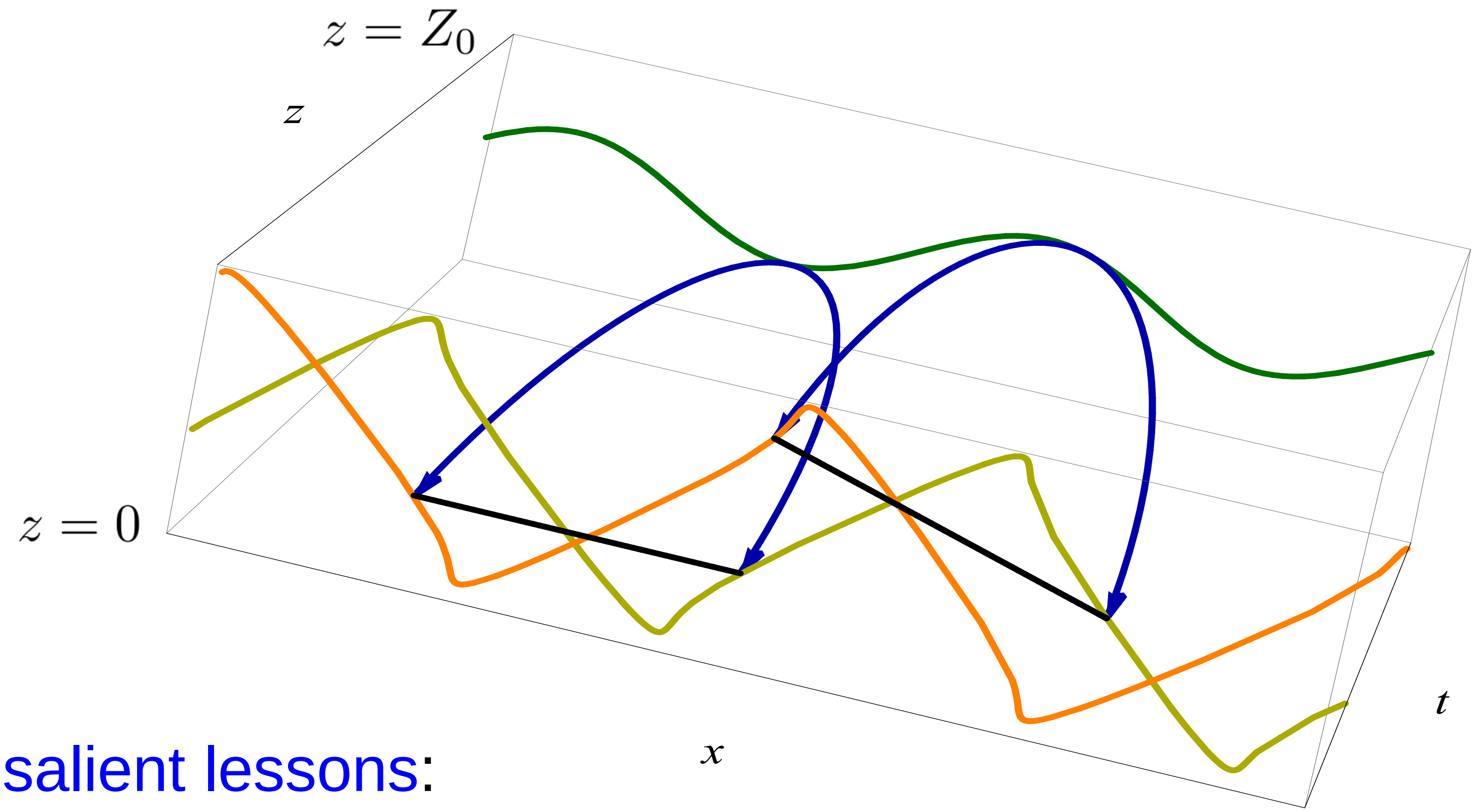
⟶ lesson: AdS vacuum (or even AdS asymptotics) not essential; extremal surfaces are again essential ingredient

Higher Curvature Gravity:

• construction extends to Lovelock gravity

⟶ lesson: essential ingredient is S_{EE} determined by extremizing appropriate entropy functional

Time-dependent bulk surfaces:



- salient lessons:
  - boundary data: two "independent" surfaces defining family of intervals: $\vec{\gamma}_L(\lambda) = \{t_L(\lambda), x_L(\lambda)\}$; $\vec{\gamma}_R(\lambda) = \{t_R(\lambda), x_R(\lambda)\}$
  - define intervals by finding extremal HEE surface which is tangent to bulk surface at each point

general “hole-ographic” construction can be packaged in terms of

classical mechanics lemma:

- consider on-shell action: $S_{on} = \int_{s_i, q_i^a}^{s_f, q_f^a} ds\ \mathcal{L}(q^a, \partial_s q^a)$
- varying boundary conditions:

$$\delta S_{on} = p_f^a\, \delta q_f^a - E_f\, \delta s_f - p_i^a\, \delta q_i^a + E_i\, \delta s_i + \int ds [\, \cancel{eom \cdot \delta q}\,]$$

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- consider family of boundary conditions: $\{s_i(\lambda), q_i^a(\lambda)\},\ \{s_f(\lambda), q_f^a(\lambda)\}$

$$\partial_\lambda S_{on} = p_f^a\, \partial_\lambda q_f^a - H_f\, \partial_\lambda s_f - p_i^a\, \partial_\lambda q_i^a + H_i\, \partial_\lambda s_i$$

general “hole-ographic” construction can be packaged in terms of **classical mechanics lemma:**

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- consider periodic family of boundary conditions: $\{s_i(\lambda), q_i^a(\lambda)\}, \; \{s_f(\lambda), q_f^a(\lambda)\}$

$$0 = \int_0^1 d\lambda \left[p_f^a \, \partial_\lambda q_f^a - H_f \, \partial_\lambda s_f - p_i^a \, \partial_\lambda q_i^a + H_i \, \partial_\lambda s_i \right]$$

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- further require reparametrization invariance: $s \;\rightarrow\; \tilde{s}(s)$

→ vanishing energy: $H = 0$

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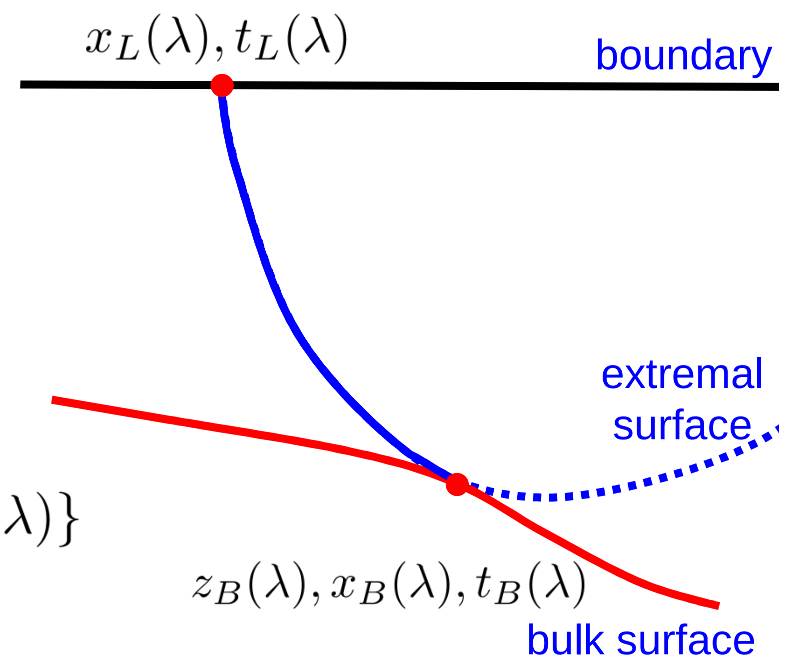
$$\int_0^1 d\lambda \; p_f^a \, \partial_\lambda q_f^a \;\; = \;\; \int_0^1 d\lambda \; p_i^a \, \partial_\lambda q_i^a$$

- further require reparametrization invariance: $s \;\; \rightarrow \;\; \tilde{s}(s)$

→ vanishing energy: $H = 0$ → operator iden: $\dot{q}^a \, \frac{\partial \mathcal{L}}{\partial \dot{q}^a} = \mathcal{L}$

→ momentum invariant: $\frac{\partial \mathcal{L}}{\partial(\partial_s q^a)} \; = \; \frac{\partial \mathcal{L}}{\partial(\partial_{\tilde{s}} q^a)}$

classical mechanics lemma:



- apply lemma to entropy problem with end-point data:

$$\{s_i(\lambda), q_i^a(\lambda)\} = \{s=0, z=0, x_L(\lambda), t_L(\lambda)\}$$

$$\{s_f(\lambda), q_f^a(\lambda)\} = \{s_{tang}(\lambda), z_B(\lambda), x_B(\lambda), t_B(\lambda)\}$$

- also use reparametrization invariance of entropy functional

$$\longrightarrow \underbrace{\int_0^1 d\lambda\, \mathcal{L}(q_B^a, \partial_\lambda q_B^a)}_{\text{Gravitational Entropy}} = \underbrace{-\int_0^1 d\lambda\, \frac{dq_L^a}{d\lambda}\frac{dS_{EE}}{dq_L^a}}_{\text{Differential Entropy}}$$

for general surfaces in general backgrounds

(with generalized planar symmetry)

$$\int_0^1 d\lambda\ \mathcal{L}(q_B^a, \partial_\lambda q_B^a) = -\int_0^1 d\lambda\ \frac{dq_L^a}{d\lambda}\ \frac{dS_{EE}}{dq_L^a}$$

Gravitational Entropy **Differential Entropy**

for general surfaces in general backgrounds (with g.p.s.)

- "hole-ography" seems a robust entry of holographic dictionary, eg, extends from AdS_3 to higher dimensions, higher curvatures, general holographic backgrounds

$$\int_0^1 d\lambda\ \mathcal{L}(q_B^a, \partial_\lambda q_B^a) = -\int_0^1 d\lambda\ \frac{dq_L^a}{d\lambda}\ \frac{dS_{EE}}{dq_L^a}$$

Gravitational Entropy **Differential Entropy**

for general surfaces in general backgrounds (with g.p.s.)

- **generalized planar symmetry:**

  one-parameter bulk profile, $\{t(\lambda), x(\lambda), z(\lambda)\}$

  → same applies for extremal surfaces, $\{t(s), x(s), z(s)\}$

- latter restricts allowed backgrounds:

includes z, t & x!!

$$ds^2 = g_{ij}(x)\, dx^i\, dx^j + g_{ab}(x,y)\, dy^a\, dy^b$$

$x^i = \{t, x, z\}$ $y^a = d-2$ "planar" coord's

along with $\det[\, g_{ab}(x,y)\,] \;=\; f(x)\, h(y)$

- ensures $y^a = \sigma^a$ is valid extremal solution

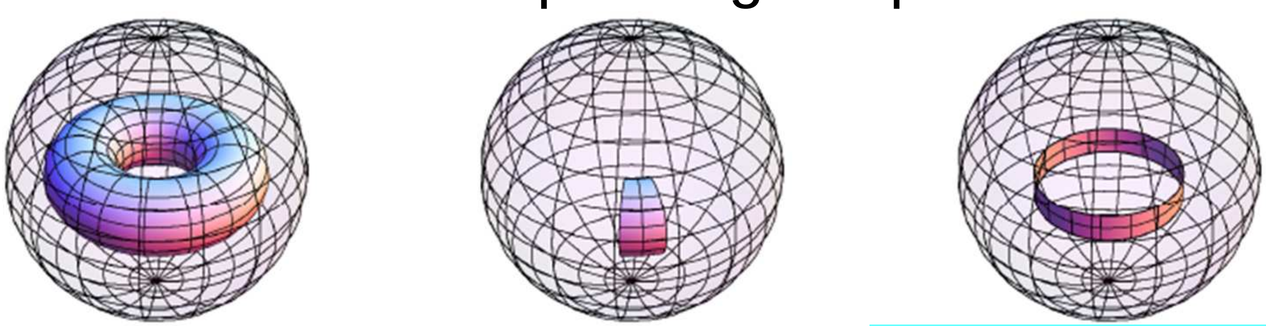
$$\underbrace{\int_0^1 d\lambda\; \mathcal{L}(q^a_B, \partial_\lambda q^a_B)}_{\text{Gravitational Entropy}} = \underbrace{-\int_0^1 d\lambda\; \frac{dq^a_L}{d\lambda}\, \frac{dS_{EE}}{dq^a_L}}_{\text{Differential Entropy}}$$

Gravitational Entropy **Differential Entropy**

for general surfaces in general backgrounds (with g.p.s.)

- **generalized planar symmetry:**
  one-parameter bulk profile, $\{t(\lambda), x(\lambda), z(\lambda)\}$
- **beyond generalized planar symmmetry**: (Czech, Dong & Sully)

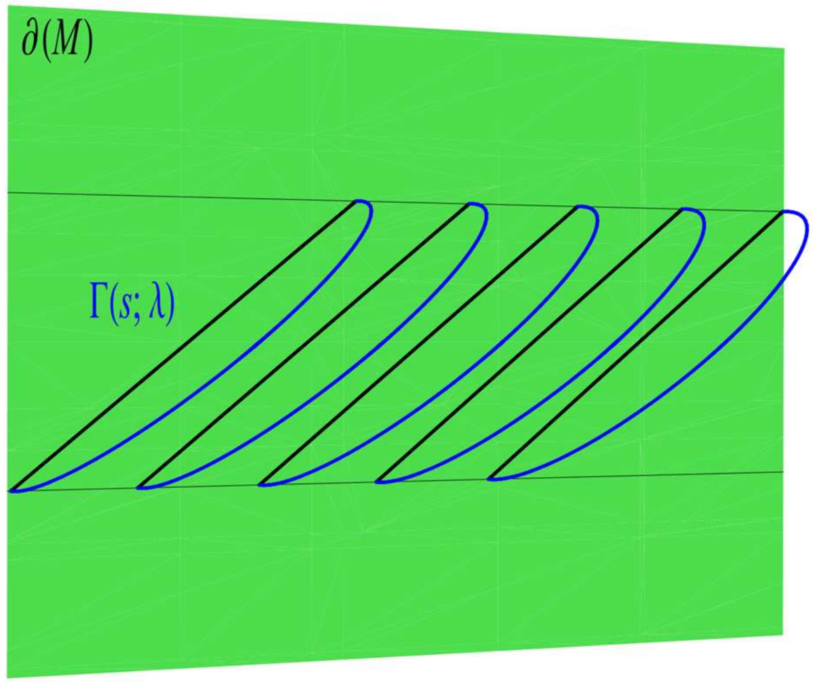
strategy is to foliate bulk surface with codimension one "loops" and use as b.c. (like alignment of tangent vectors) to construct Extremal surfaces and corresponding "loops" in boundary theory



see next talk by Sully

Question:

- "differential entropy" defined a boundary observable where input is a family of intervals in boundary geometry
- have bulk-to-boundary construction: start with bulk surface and construct corresponding boundary data, ie, $\vec{\gamma}_L(\lambda)$ and $\vec{\gamma}_R(\lambda)$
- boundary-to-bulk construction?? can we reverse engineer bulk surface from boundary data??

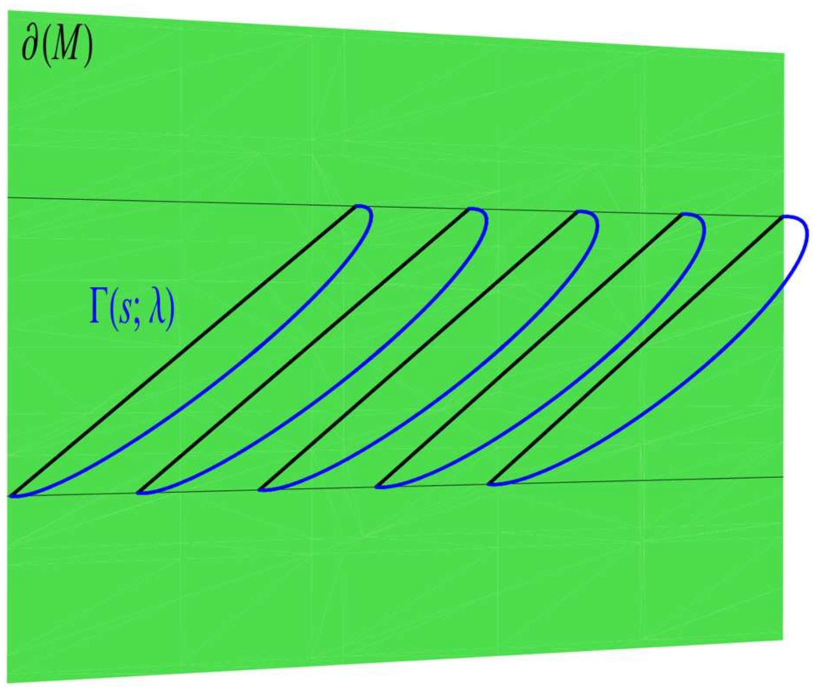


⟶ in general, seems answer may be **NO!?!?**

Question:

- boundary-to-bulk construction?? can we reverse engineer bulk surface from boundary data??

→ are these intervals still associated with bulk surface??



Question:

- boundary-to-bulk construction?? can we reverse engineer bulk surface from boundary data??

→ are these intervals still associated with bulk surface??

- look at "guts" of proof of bulk-to-boundary construction*

→ require $\frac{\partial_s x_B}{|\partial_s x_B|} \cdot \partial_\lambda x_B = \frac{\partial_\lambda x_B}{|\partial_\lambda x_B|} \cdot \partial_\lambda x_B$

Einstein gravity: $\frac{\partial \mathcal{L}}{\partial(\partial_s x^i)} = g_{ij} \frac{\partial_s x^i_B}{|\partial_s x_B|}$

proposed solution: $\frac{\partial_\lambda x^i_B}{|\partial_\lambda x_B|} = \frac{\partial_s x^i_B}{|\partial_s x_B|}$

* rest only applies for Einstein gravity in bulk and S_{BH}=A/4G

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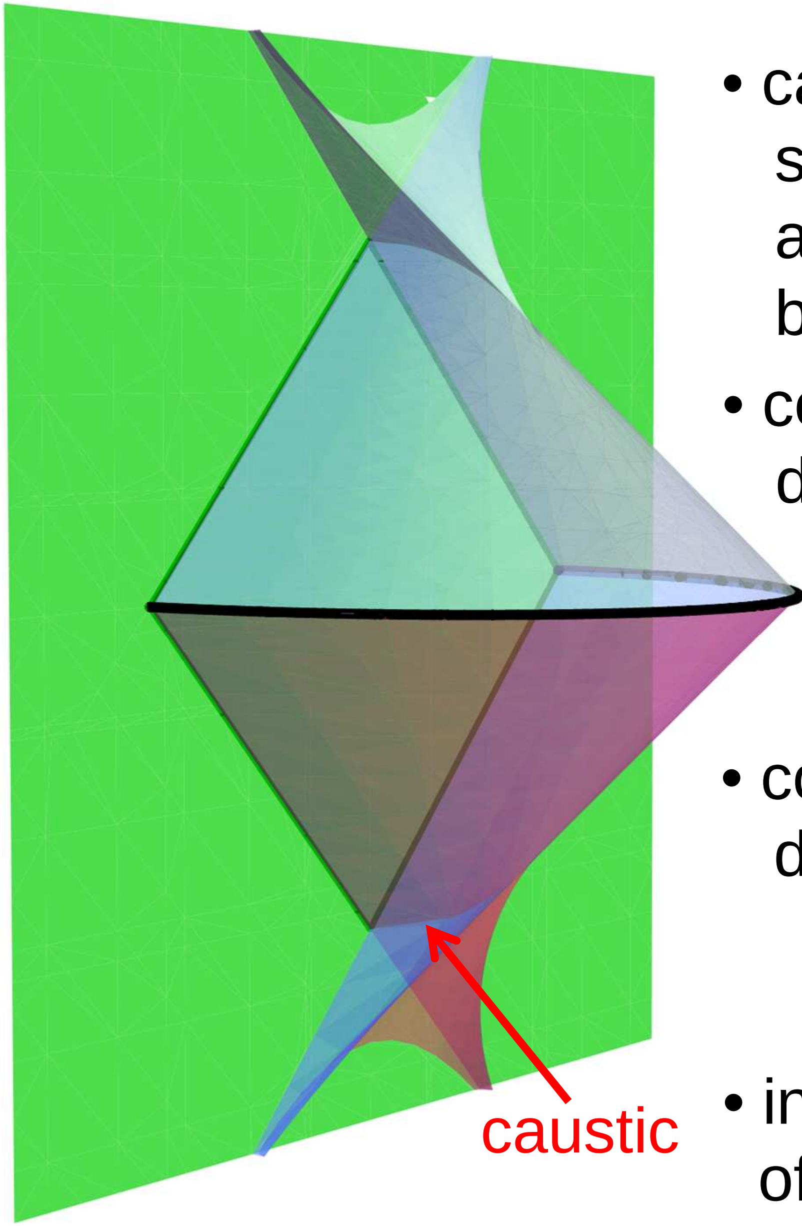
general solution: $\frac{\partial_\lambda x_B^i}{|\partial_\lambda x_B|} = \frac{\partial_s x_B^i}{|\partial_s x_B|} + k^i$

where $k \cdot k = 0$ and $k \cdot \partial_\lambda x_B = 0$

Entanglement Wedge:

(Headrick, Hubeny, Lawrence & Rangamani)



- causal development of bulk Cauchy surface bounded by extremal surface and entangling region in asymptotic boundary
- connects to boundary of causal development in asymptotic boundary
- conjectured bulk region dual to density matrix in boundary theory

(see also: Czech, Karczmarek, Nogueira & van Raamsdonk)

- in differential entropy, use intersection of extremal curves **with entanglement wedge** of neighbouring curve

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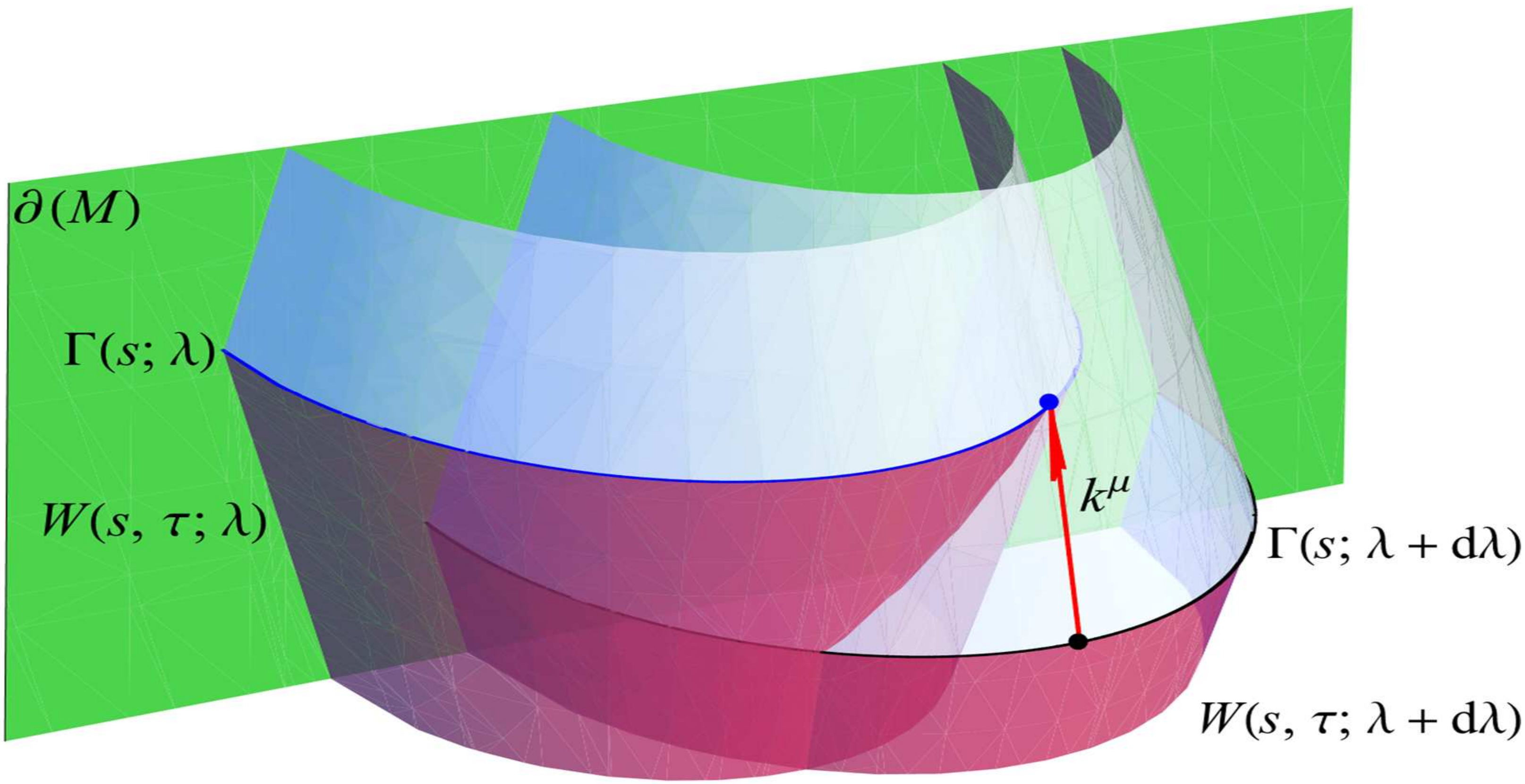

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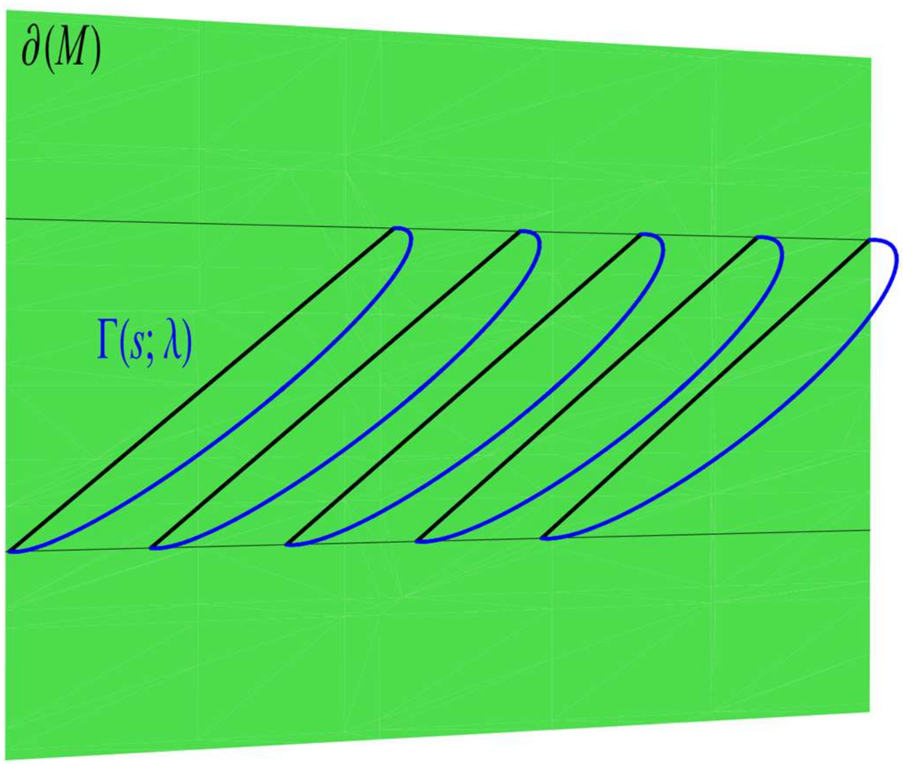

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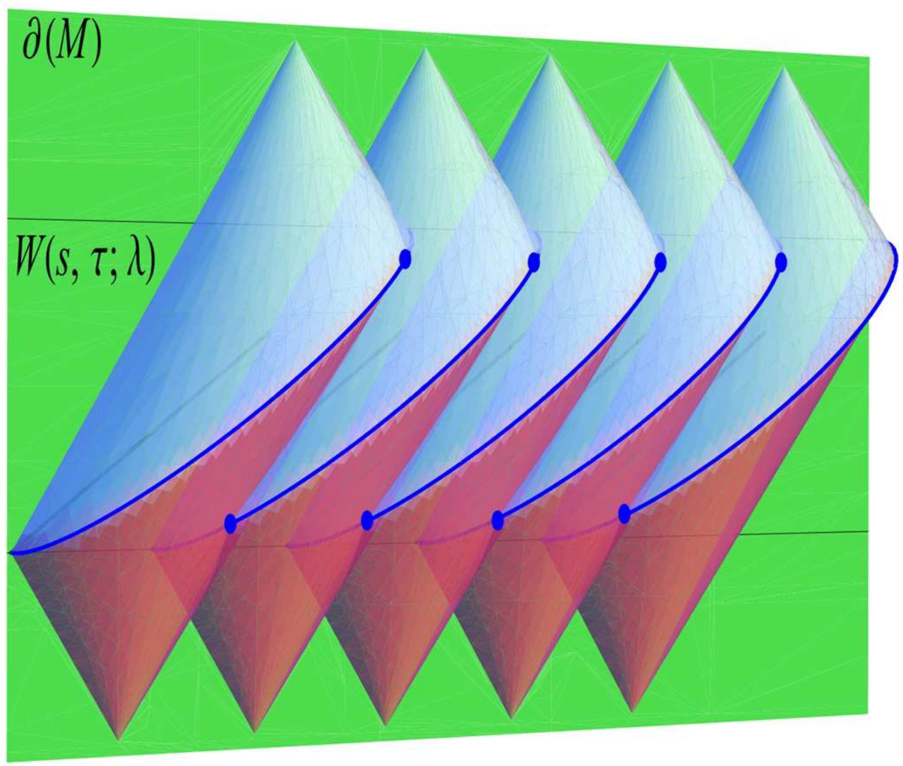

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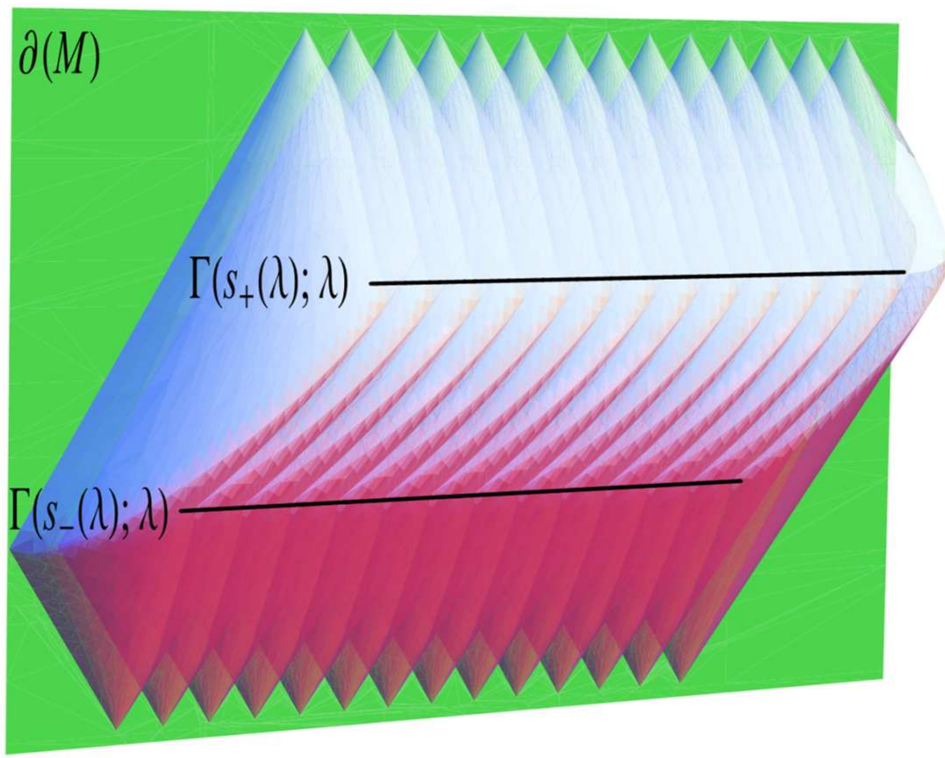

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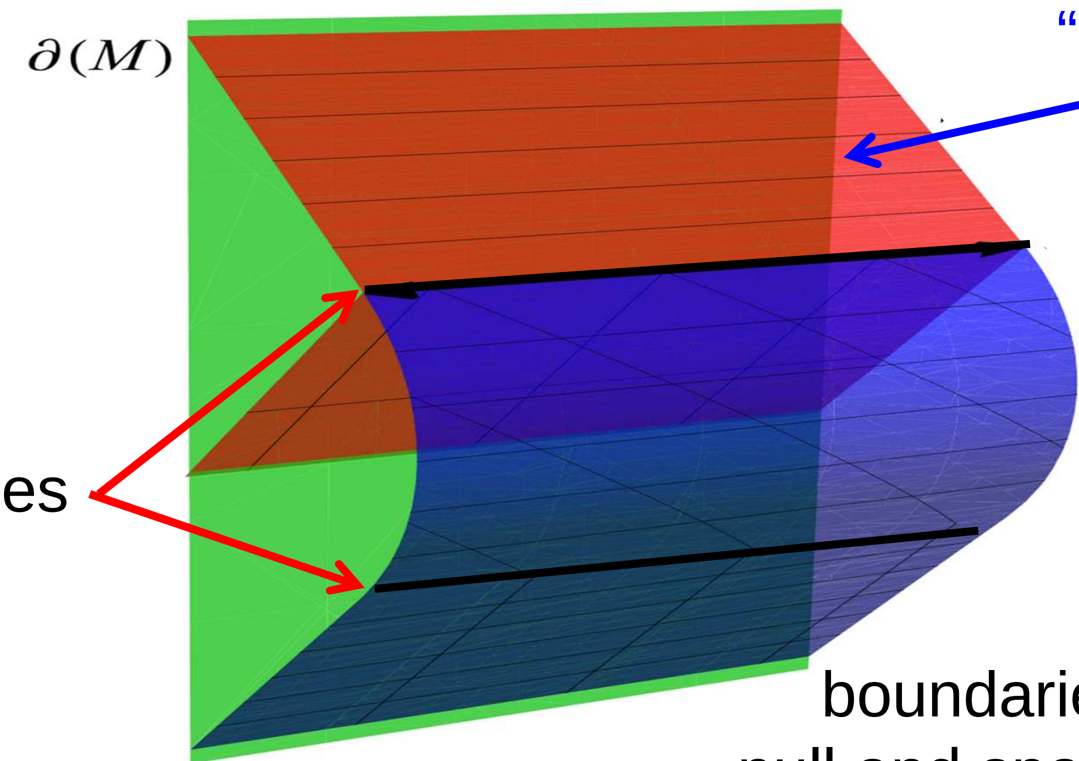


"enveloping surface"

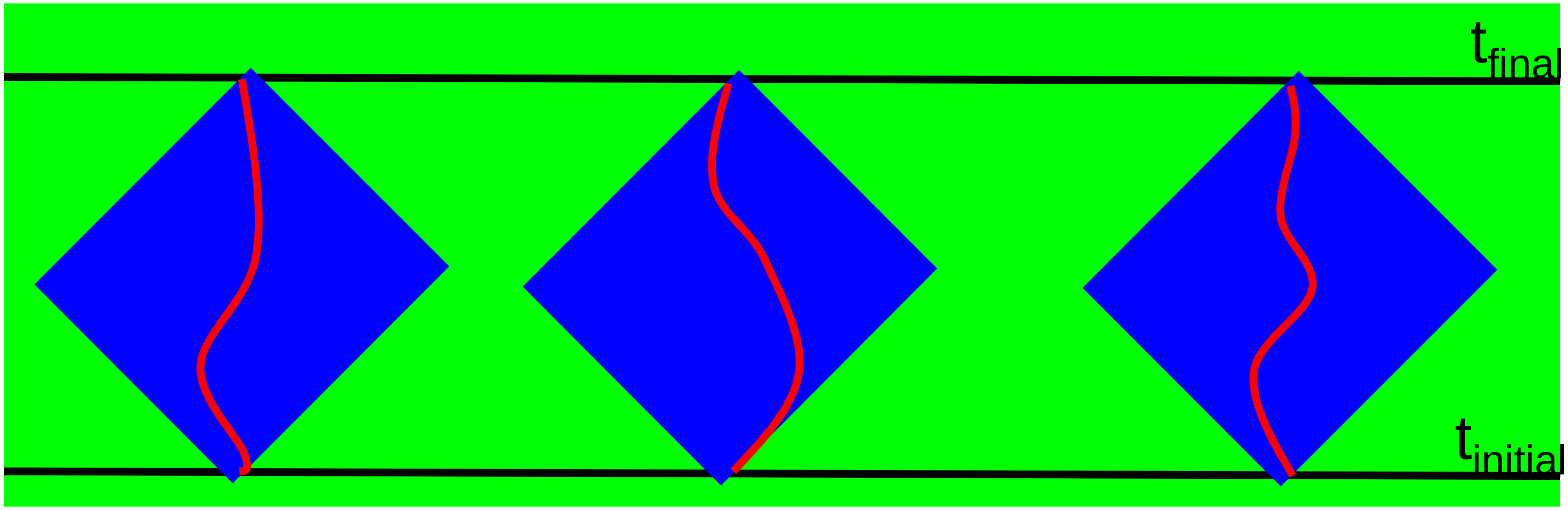
two surfaces

boundaries between null and space-like normal

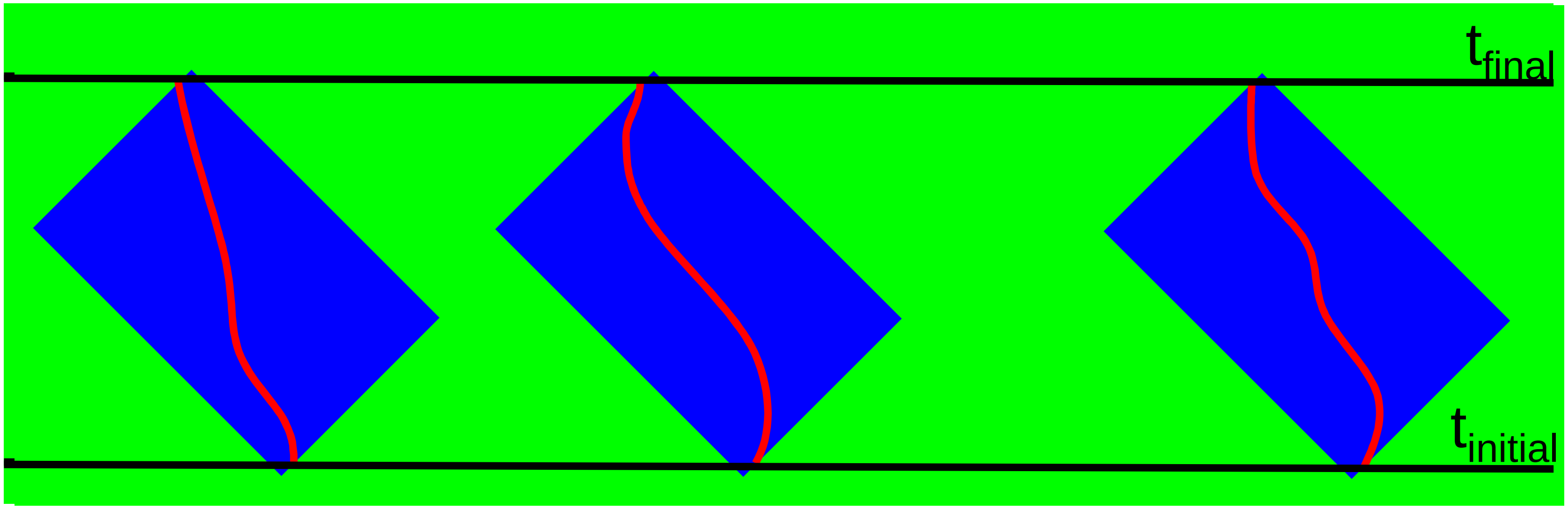
Time strips:

(Balasubramanian, Chowdhury, Czech, de Boer & Heller)

- observations limited to be within a finite "time strip"



- time strip alone does not fix "protocol" or differential entropy



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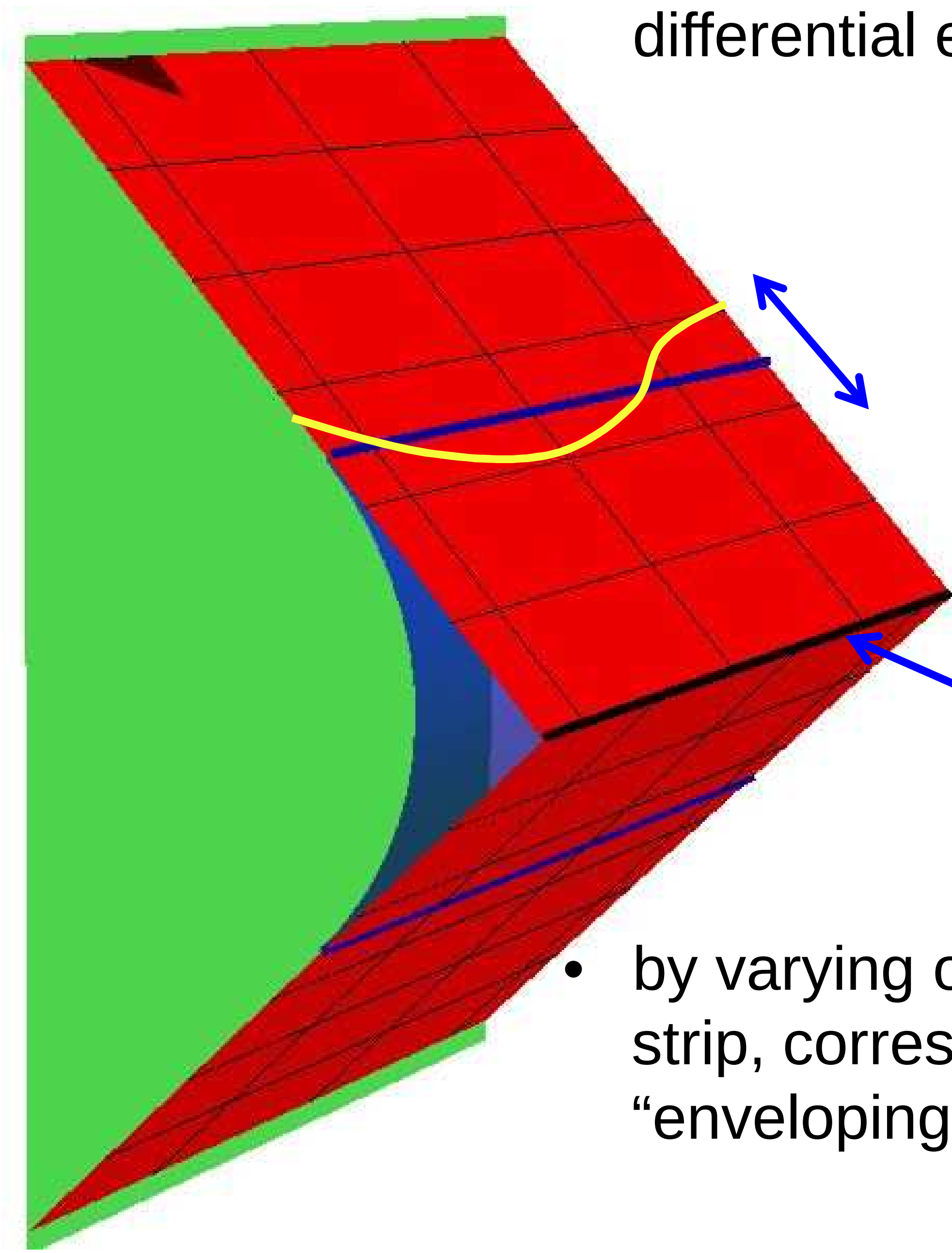
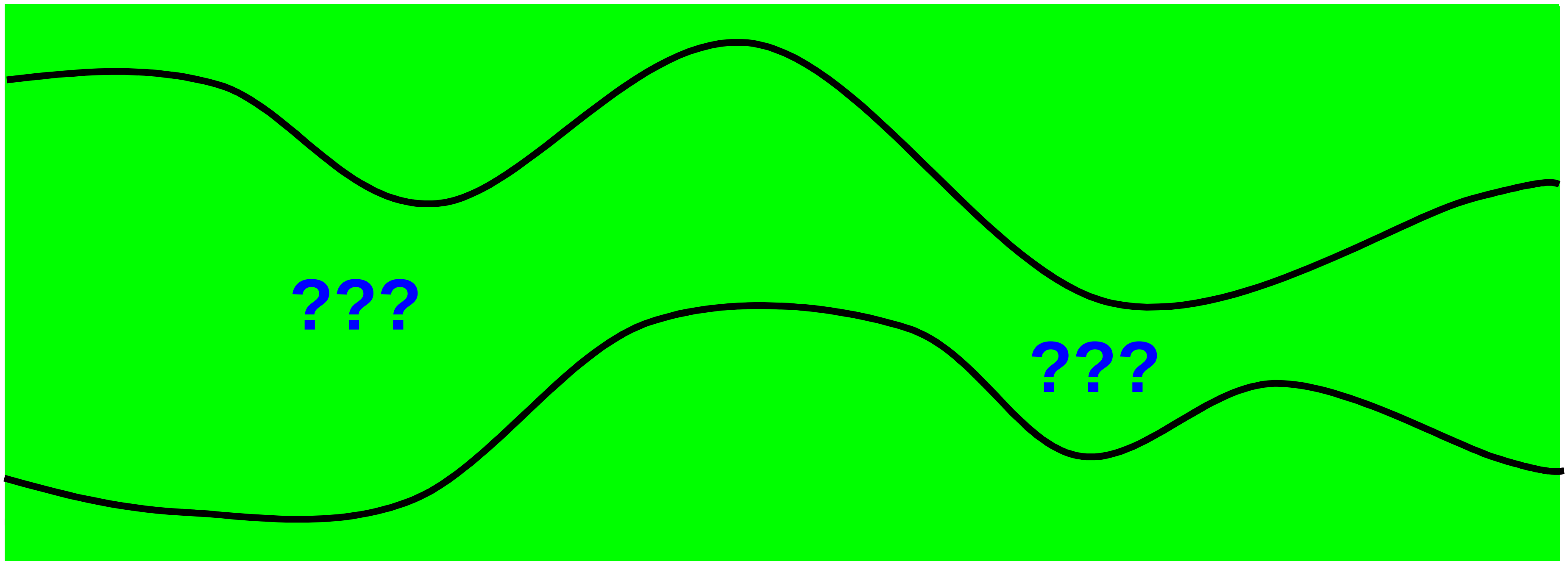


- by varying observers/intervals for fixed time strip, corresponding bulk curves explore the “enveloping surface” of minmal bulk surface

Time strips:

- observations limited to be within a finite "time strip"

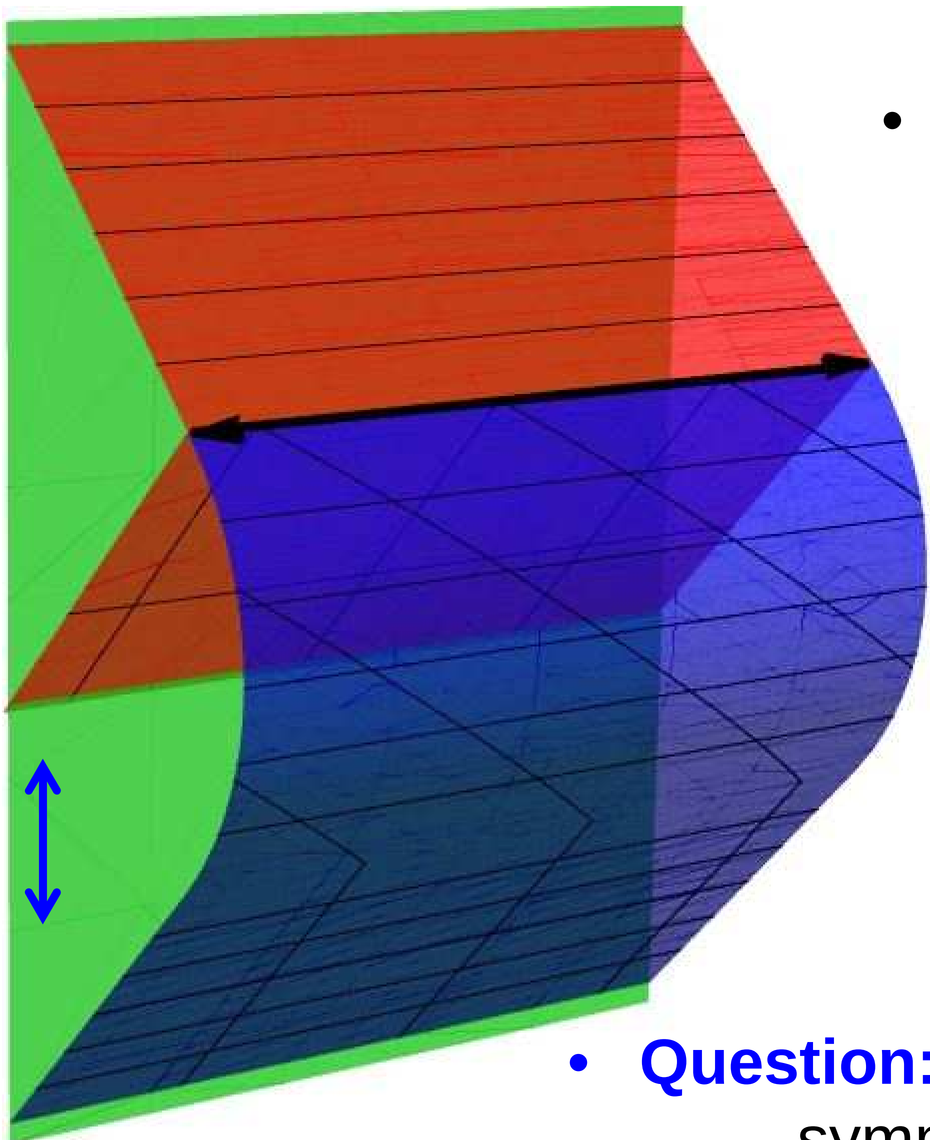


- many families of observers for the same time strip
- **Question**: what is most effective protocol to minimize the differential entropy for a given time strip?*

(* Hint: not maximum proper time protocol)

Time strips:

- time strip alone does not fix “protocol” or differential entropy



- alternatively, there are many different families of boundary intervals and time strips which will reconstruct the same bulk curve

- **Question:** is there a hidden “gauge” symmetry underlying this redundancy?

More questions:

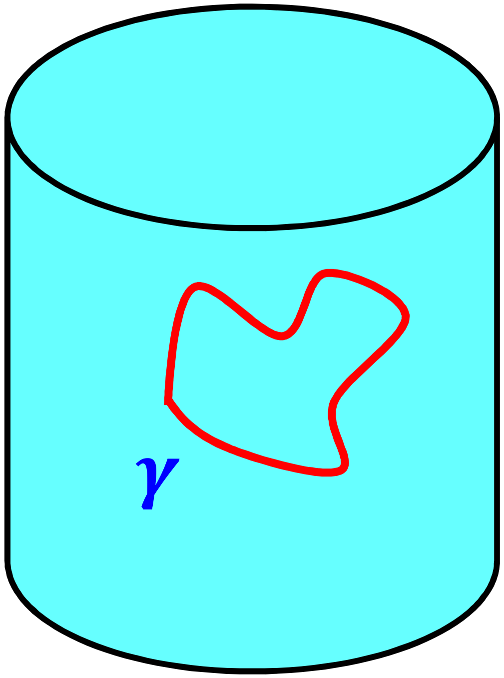
- why only consider entanglement entropy?

evaluating many holographic probes at leading order in large N involves extremizing some functional in bulk

⟶ same classical mechanics lemma applies

eg, reconstruct length of curve in bulk from two-point correlator of high dimension operator

$$\ell(\gamma) = \oint_0^1 d\lambda \, \frac{dq_L^a}{d\lambda} \, \frac{\partial_{q_L^a} \langle \mathcal{O}(q_L^a)\, \mathcal{O}(q_R^a)\rangle}{\Delta \langle \mathcal{O}(q_L^a)\, \mathcal{O}(q_R^a)\rangle}$$



(with D. Galante & J. Pedraza)

More questions:

- **Residual entropy:** what is the relation between differential entropy and residual entropy?

  see talk by Hayden

- **Minimal vs Extremal:** our proofs/discussions are local and so work with extremal surfaces but may not be "minimum area" surfaces which determine holographic entanglement entropy

  → is there a role for extremal but nonminimal surfaces?

  (V. Balasubramanian, B. Chowdhury, B. Czech, J. de Boer, arXiv:1406.5859)

- **Wandering surfaces:** in some cases, extremal surface may not reach boundary, eg, hit singularity or fall through horizon

  → is there a sensible story here?
  when do extremal surfaces reach singularity?

  in horizon case, (much of) story readily extends by purifying thermal state, ie, include other boundary (with J. Rao)

More questions:

- **Beyond generalized planar symmetry**:
  improved by approach of Czech, Dong & Sully

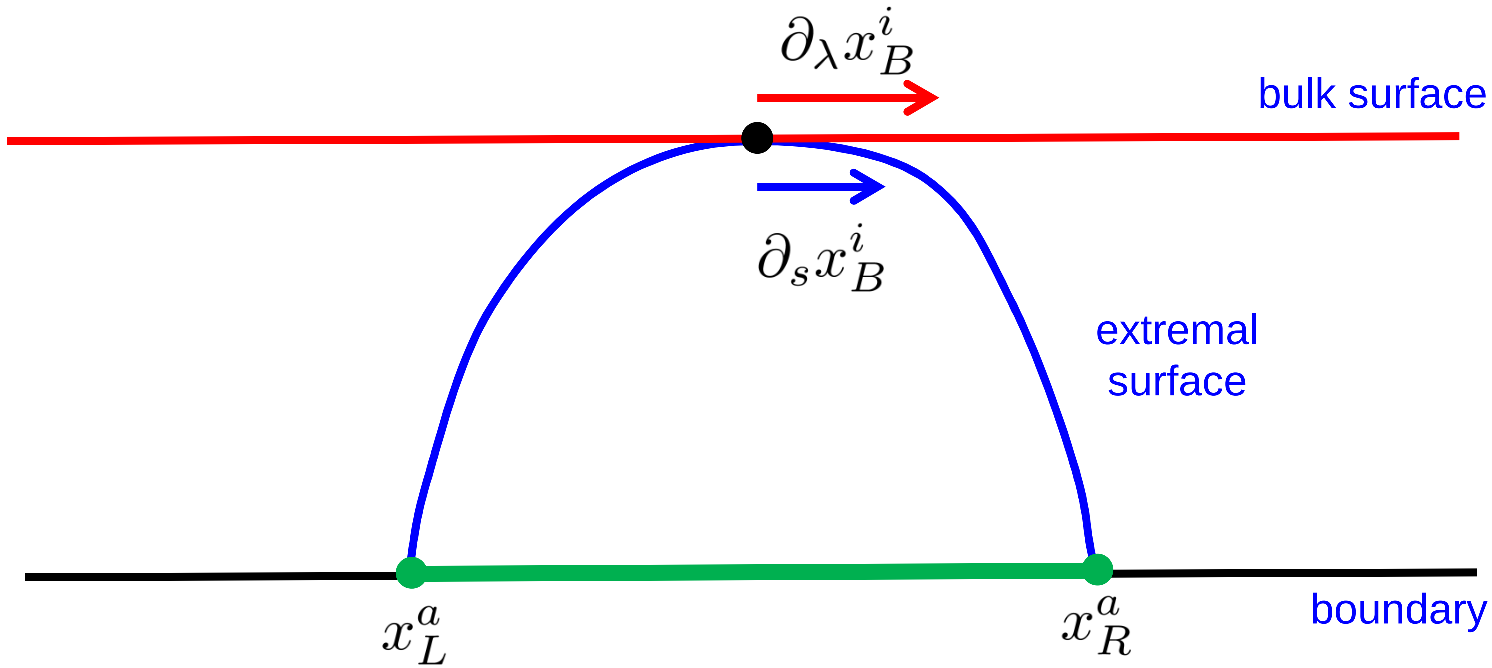
- covariant formulation of differential entropy?
- tiling boundary with finite regions?

see next talk by Sully

More challenges/questions:

- **Annoying signs:** sometimes tangent vectors are anti-aligned

→ recall tangent vector alignment: $$\frac{\partial_\lambda x_B^i}{|\partial_\lambda x_B|} = \frac{\partial_s x_B^i}{|\partial_s x_B|}$$



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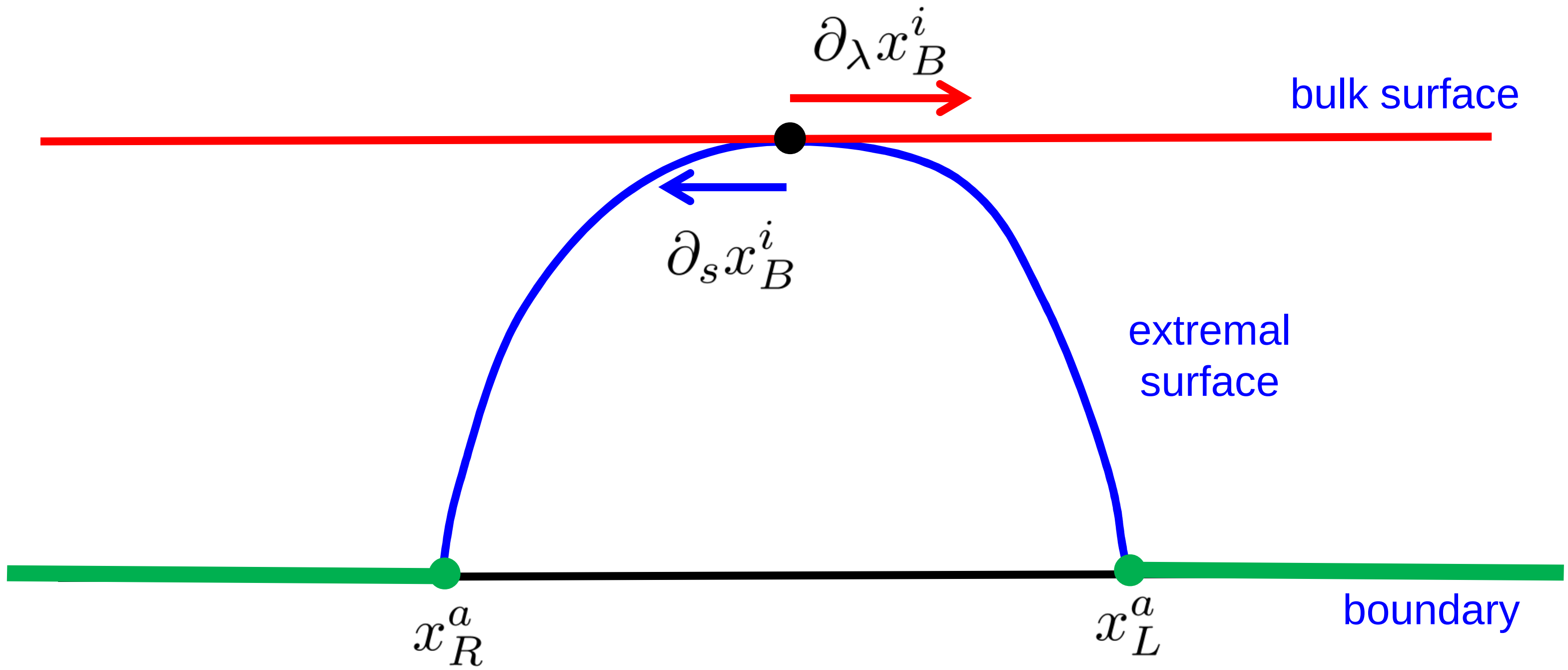


- take complementary regions; now have: $\frac{\partial_\lambda x^i_B}{|\partial_\lambda x_B|} = - \frac{\partial_s x^i_B}{|\partial_s x_B|}$

→ $S_{diff} = - S_{grav}$

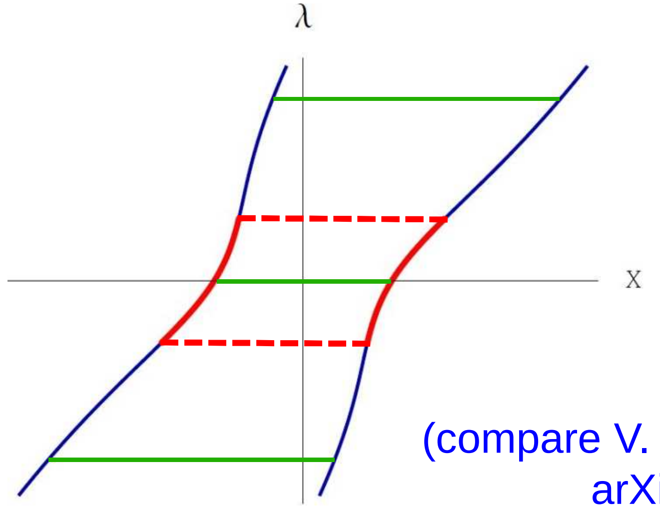
More challenges/questions:

- **Annoying signs:** sometimes tangent vectors are anti-aligned

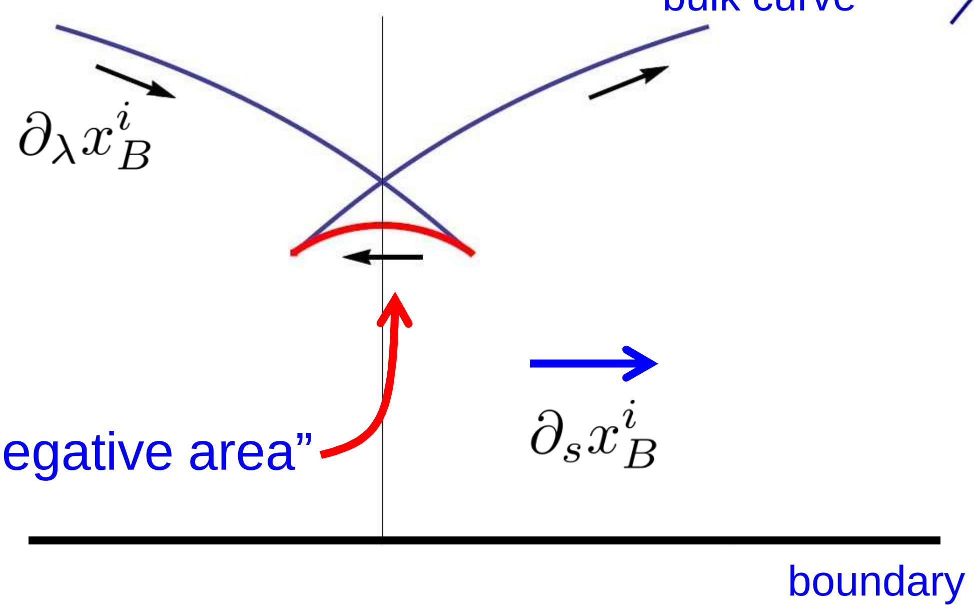
⟶ alignment of tangent vector can change at various points, eg, for constant t and AdS_3:

$$x_c = \frac{x_R + x_L}{2} = \lambda$$

$$x_R - x_L = 4 - \frac{2}{\lambda^2 + 1}$$



(compare V. Hubeny, arXiv:1406.4611)



- differential S yields "signed" area

$$S_{diff} = \int \alpha(\lambda)\, dA$$

???????

More challenges/questions:

- **Beyond leading order in N²:** first need to extend holographic entanglement entropy beyond saddle-point approximation

$$S(A) \; = \; \min_{\partial V = \Sigma} \left[\frac{\langle A_V \rangle}{4G_N} + S_{EE,\,1-loop} + \cdots \right]$$

(Faulkner, Lewkowycz & Maldacena, arXiv:1307.2892; Engelhardt & Wall, arXiv:1408.3203)

see talk by Wall

More challenges/questions:

- **Beyond leading order in N^2:** puzzle by Maldacena

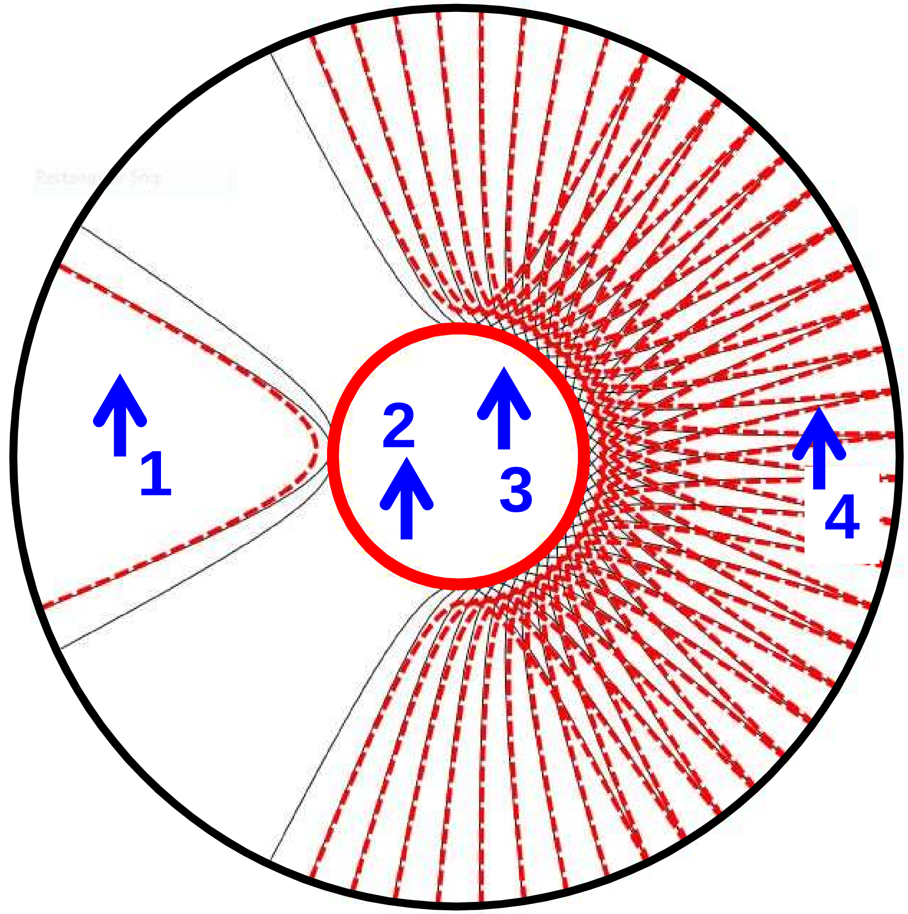
consider two states for spins:

$$|singlet\rangle_{1,2} \times |singlet\rangle_{3,4}$$

(extra entropy for hole)

$$|singlet\rangle_{1,4} \times |singlet\rangle_{2,3}$$

(not extra entropy for hole)



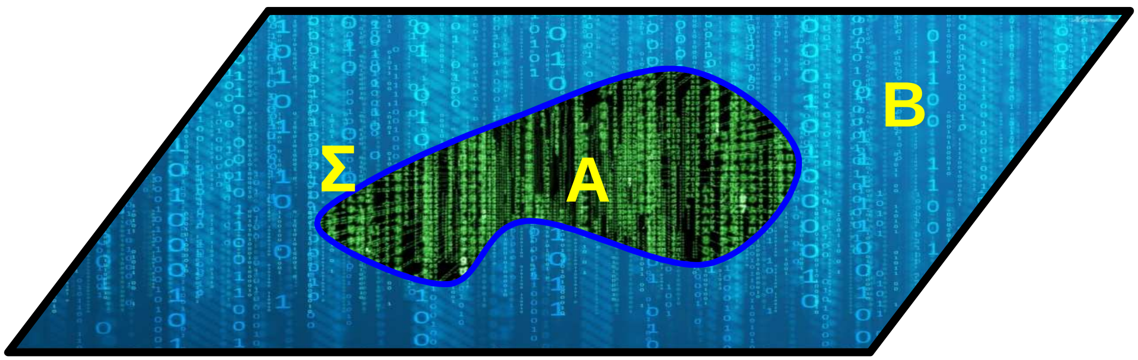
⟶ naïve extension of S_{diff} doesn’t see to distinguish two states

What extension of S_{diff} properly accounts for quantum corrections?

Conclusions:

• holographic S_{EE} suggests new perspectives

→ quantum information & entanglement may yield key insights to fundamental issues in quantum gravity



• spacetime entanglement: S_{BH} applies for generic large regions

• "hole-ography" (ie, gravitational entropy = differential entropy) points to a precise definition in AdS/CFT context

• "differential operators": new insights on quantum gravity in AdS

Lots to explore!