Holographic EPR pairs

Andreas Karch (University of Washington)

work with Kristan Jensen and Brandon Robinson.

talk at Quantum Information in Quantum Gravity Conference
UBC (Vancouver), August 22 2014

What is Entanglement?

We all recognize entangled states:

$$|\psi\rangle = |\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle$$

But wavefunction can not be directly observed.

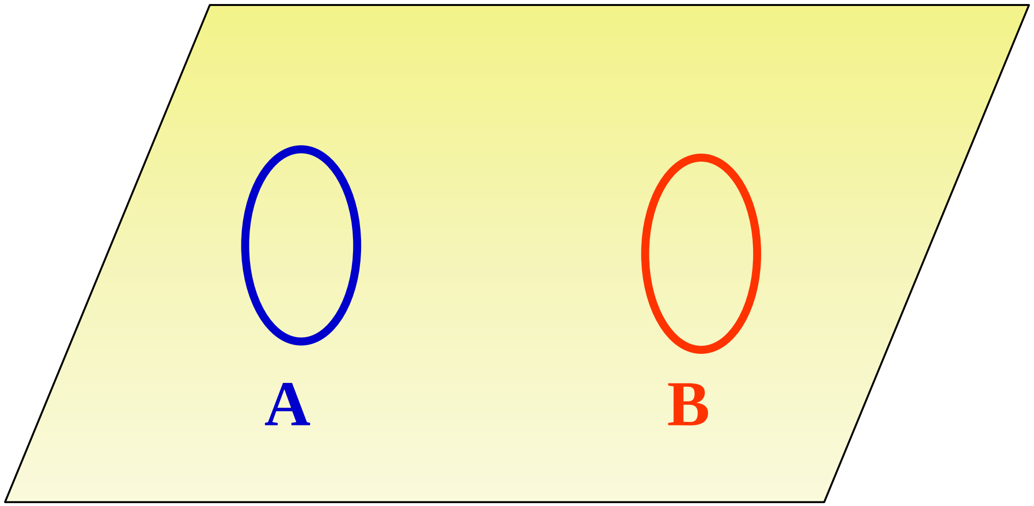
What is the observable characterizing entanglement?

Entanglement as Correlation

$$G = \langle\psi|O_A O_B|\psi\rangle - \langle\psi|O_A|\psi\rangle\langle\psi|O_B|\psi\rangle$$

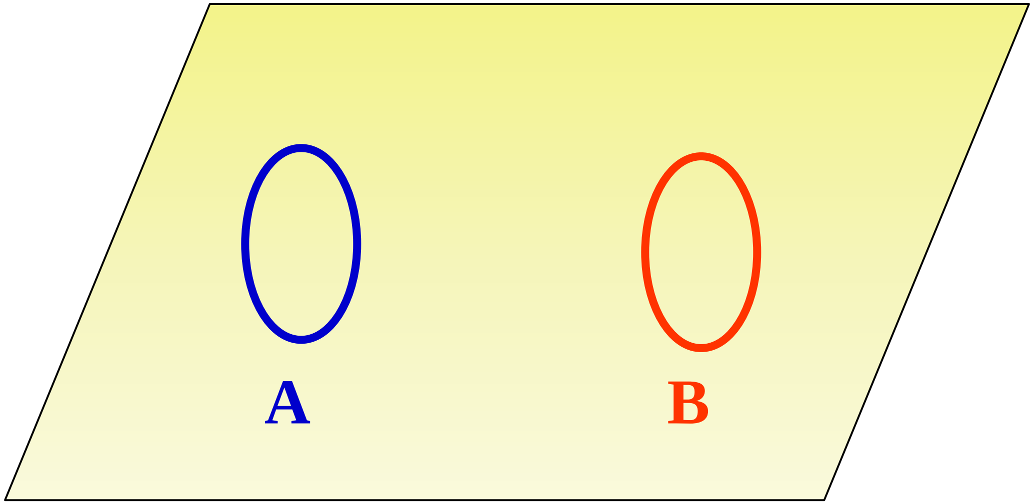
Observable measuring entanglement
are the disconnected correlation functions
for any pair of operators in subsystem A and B.

Entanglement in a local QFT



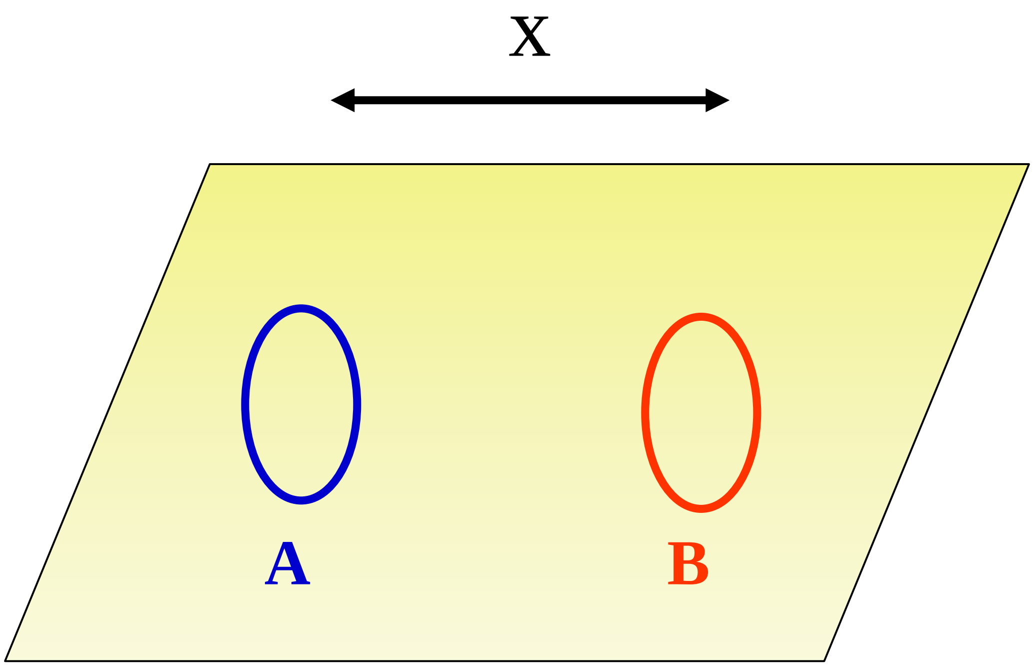
In a local quantum field theory we can identify Subsystems A and B by their location in space.

Entanglement in a local QFT



In vacuum modes of the electromagnetic field in nearby regions A and B are entangled!

Entanglement defines distance!

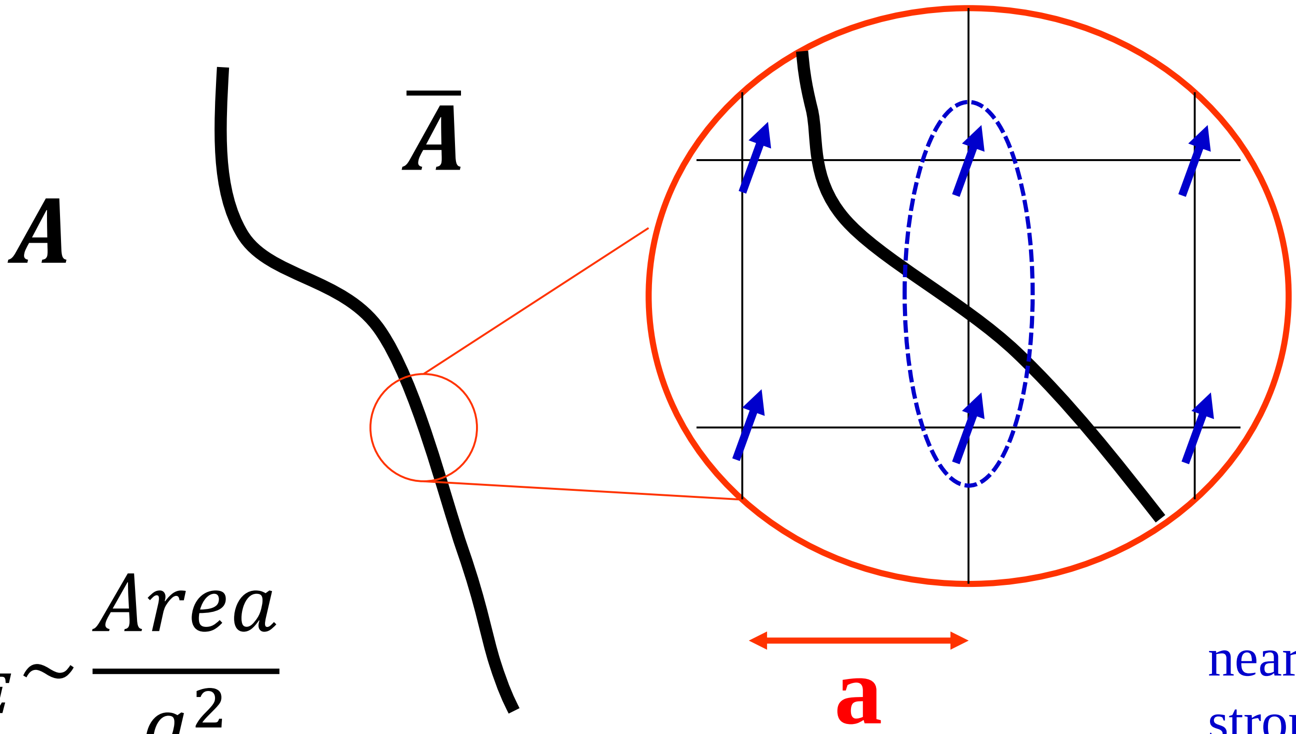


Entanglement decreases with distance!

$$\langle O_A O_B \rangle \sim {}^{1}/_{x}$$

Entanglement is short range

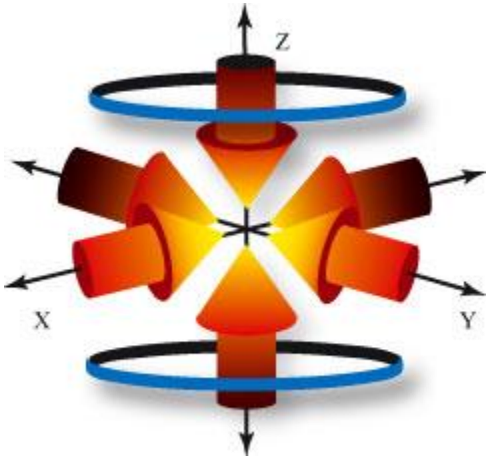
Clear Indication: Entanglement Entropy

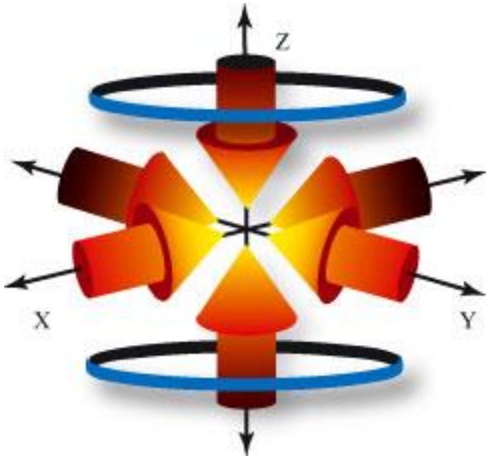


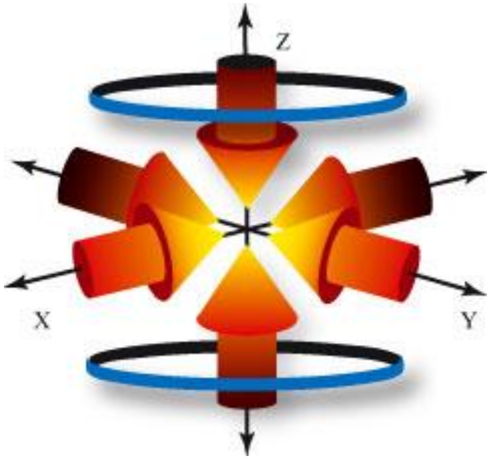
$$S_{EE} \sim \frac{Area}{a^2}$$

nearest neighbor pairs strongly entangled.

Entanglement = Connectedness

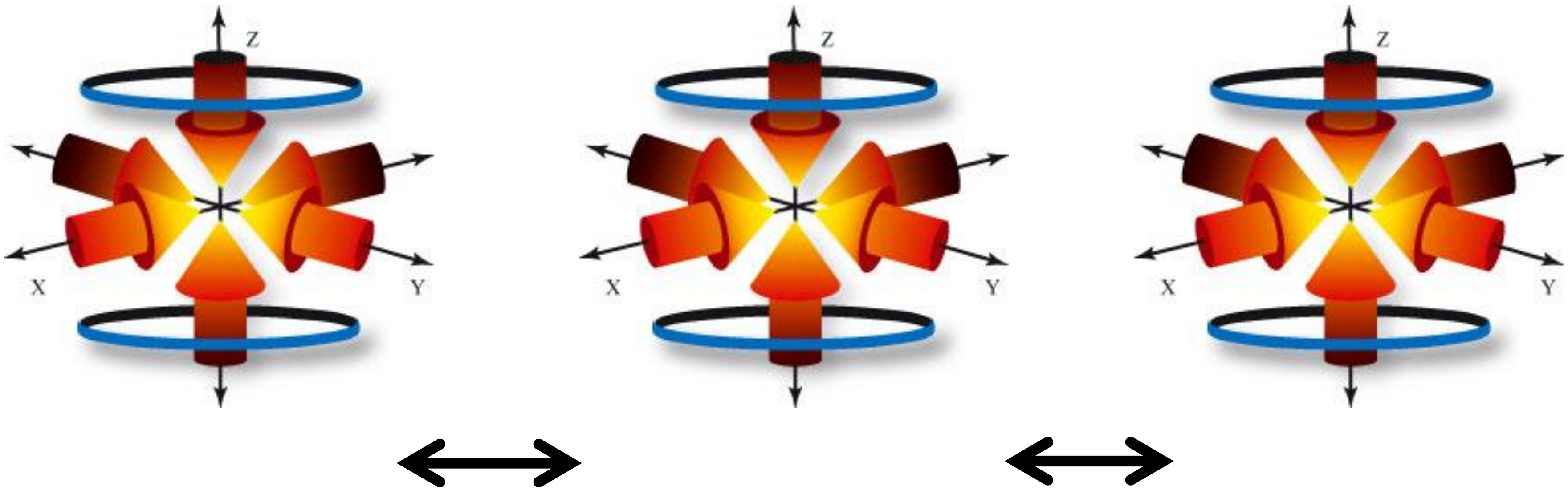






3 atoms in 3 traps; disconnected.

Entanglement = Connectedness



Entanglement “connects” the three into a line!

Entanglement defines notion of distance!

Entanglement and Holography

Emergent Dimensions

These ideas can be made very concrete in holography.

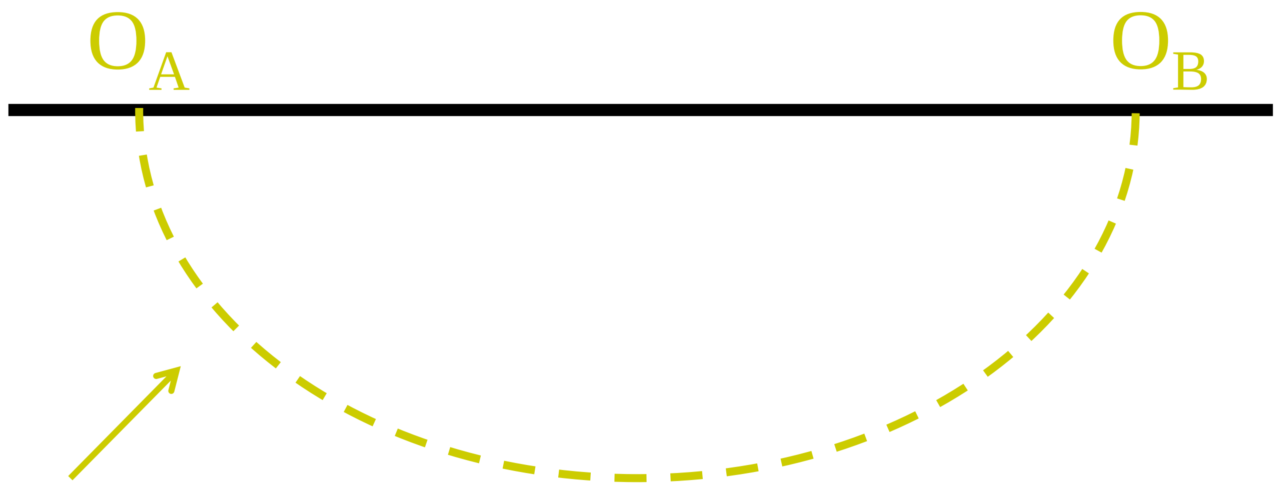
- Entanglement encodes bulk geometry

  (van Raamsdonk)

- Correlations are given by classical propagators. Finite Correlation implies finite spatial distance.

AdS/CFT = Classical Bulk

$$G = \langle\psi|O_A O_B|\psi\rangle - \langle\psi|O_A|\psi\rangle\langle\psi|O_B|\psi\rangle$$



classical bulk
Green's function

finite correlation
only possible for finite distance!

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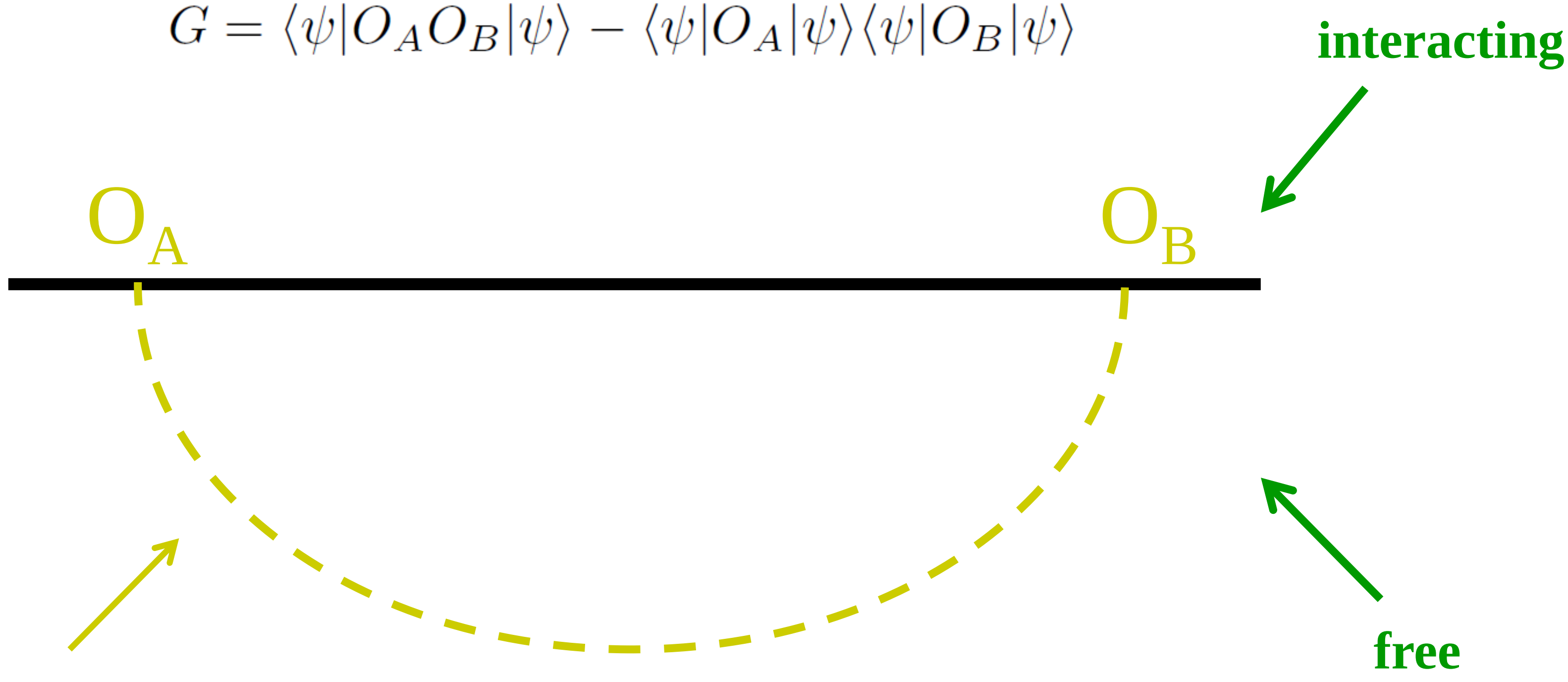

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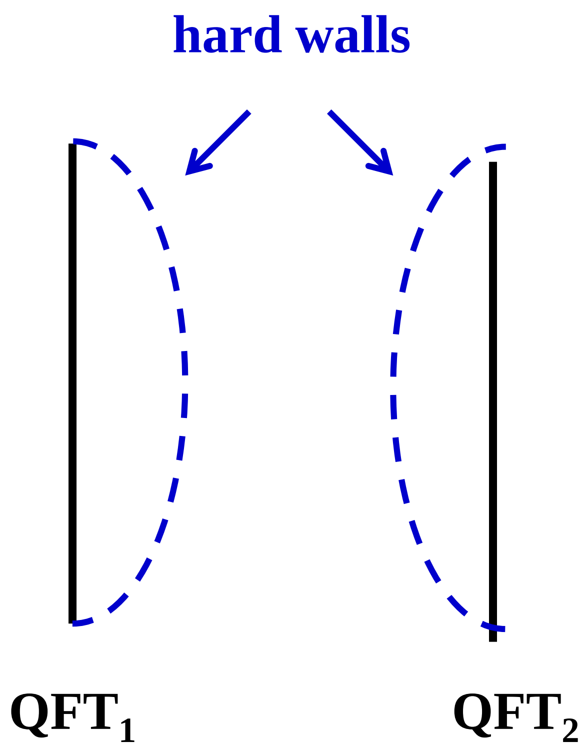
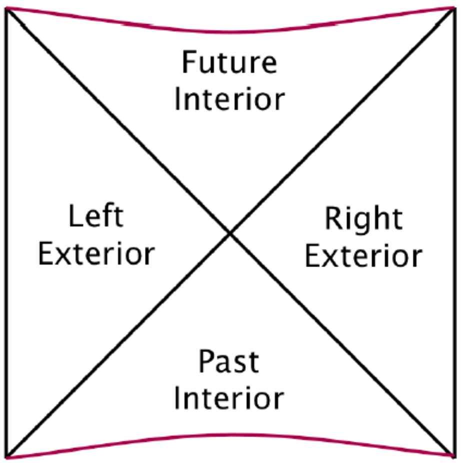
Example: 2 copies of a QFT



No Entanglement

Product State

Example: 2 copies of a QFT



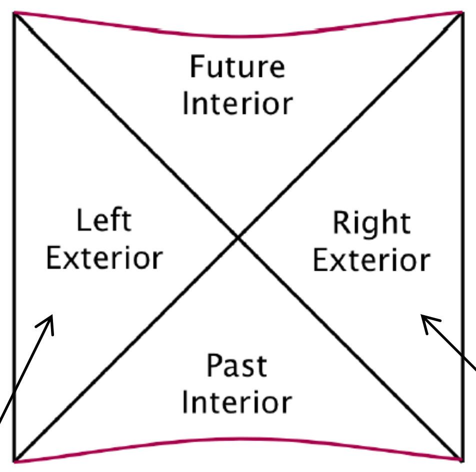
$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

Entangled state
(Thermofield Double)

Holographic dual: Eternal AdS black hole

(Maldacena)

Example: 2 copies of a QFT



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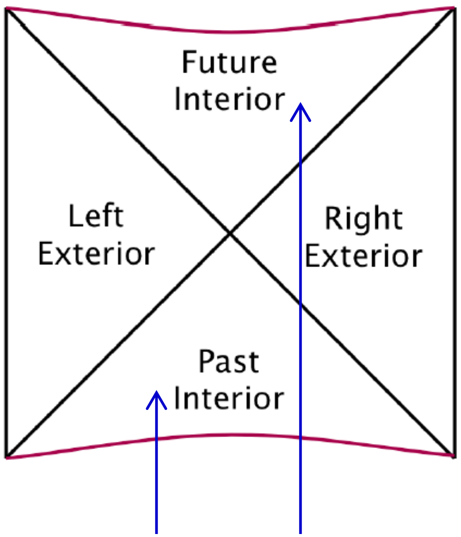
Entangled state (Thermofield Double)

QFT$_1$

QFT$_2$

Example: 2 copies of a QFT



$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

Entangled state
(Thermofield Double)

These two regions encode the correlation implied by the entanglement between the two QFTs.

The EPR pair

The EPR pair.

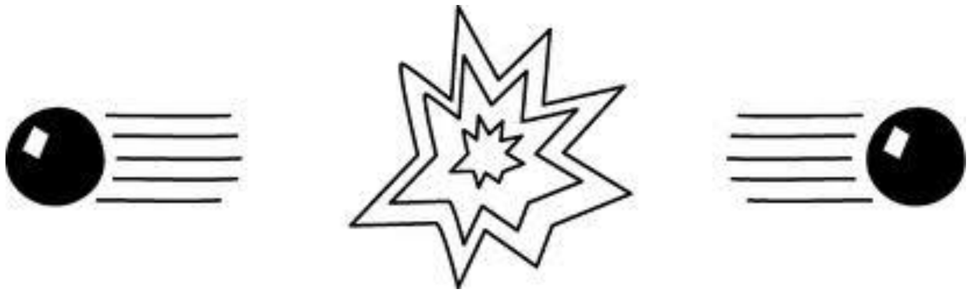
Extreme example of entanglement: **EPR pair**

E.g.: **anti-quark** **quark**

pair created in background electric field

The EPR pair:

spin 0 in initial state



We all know how to write the EPR wavefunction:

$$|\Psi> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow> - |\downarrow\uparrow>)$$

Special feature: no causal contact!

Operational definition EPR pair.

From now on we define an EPR pair to be an entangled pair not in causal contact.

Holographic EPR pair

EPR pair = non-vanishing correlators,
but no causal contact.

(Holography): Finite spatial distance between the two points in the bulk, but no causal contact.

Holographic EPR pair

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but no causal contact.

(Holography): Finite spatial distance between the two points in the bulk, but no causal contact.

This defines an ER bridge / wormhole!

ER=EPR

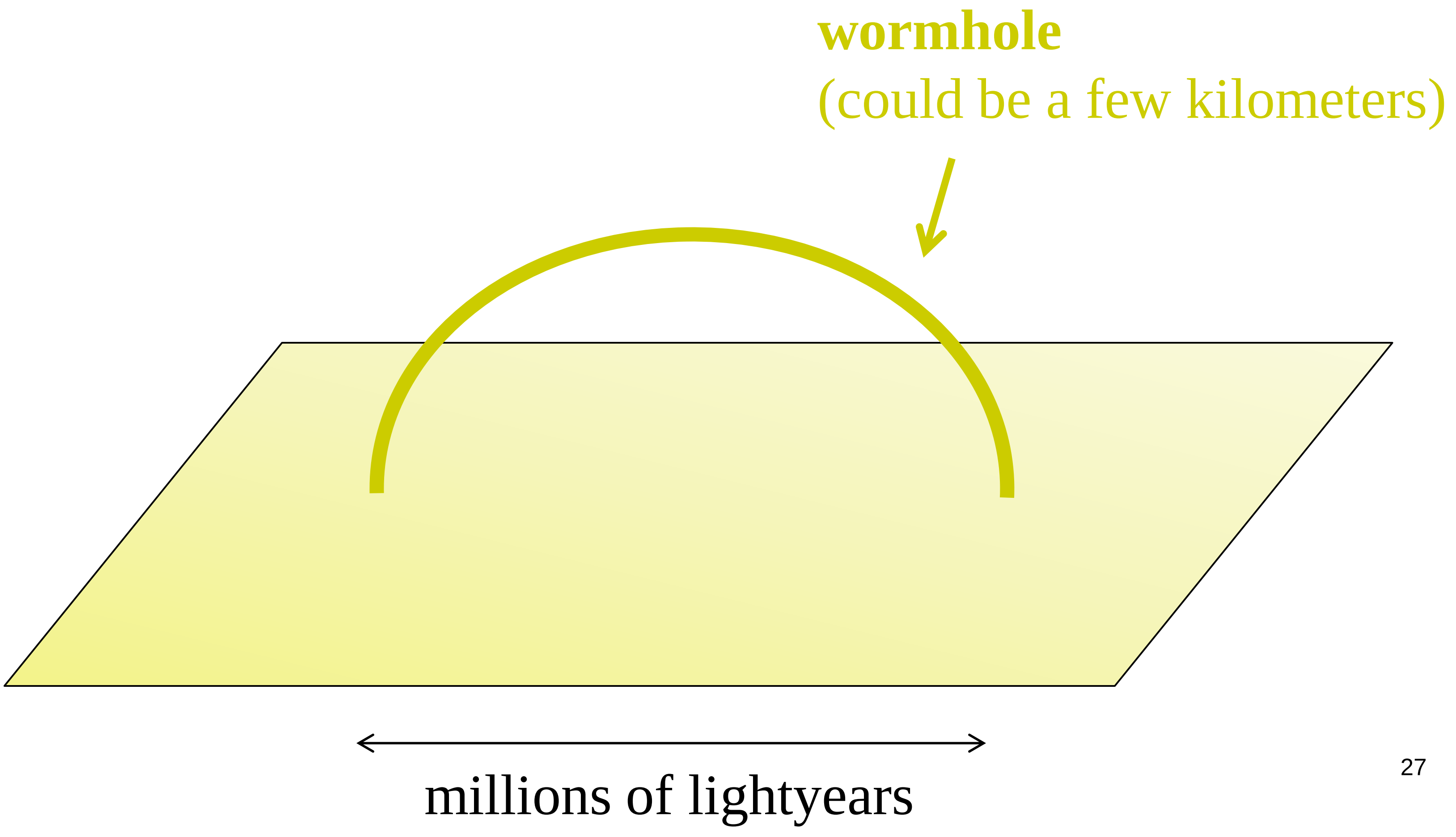
So almost by definition, the holographic dual of an EPR pair is a non-traversible ER wormhole.

(Maldacena-Susskind)

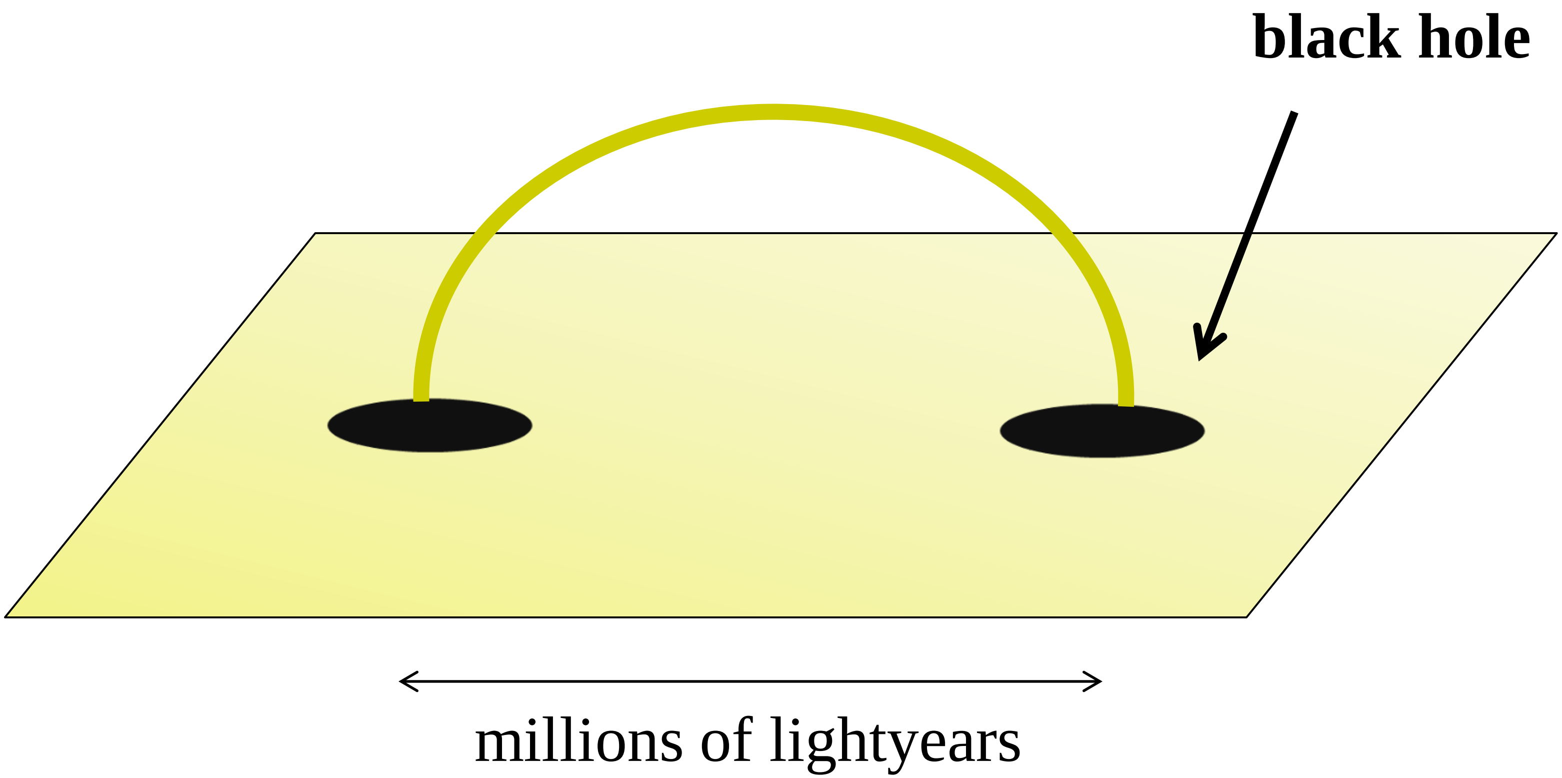
(MS make slightly stronger statement. EPR=ER. Crisp statement: ER and EPR give rise to identical physical consequences = correlators)

The ER Bridge

Einstein-Rosen Bridge

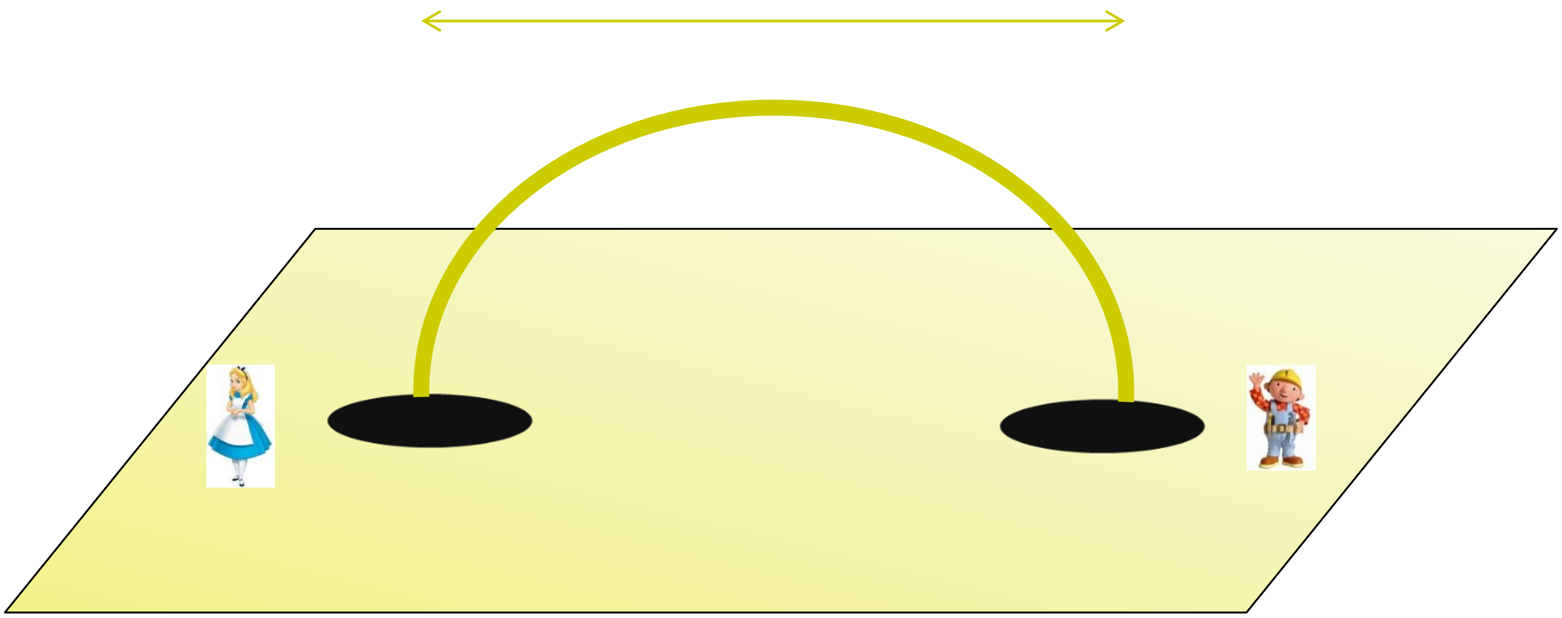


Einstein-Rosen Bridge



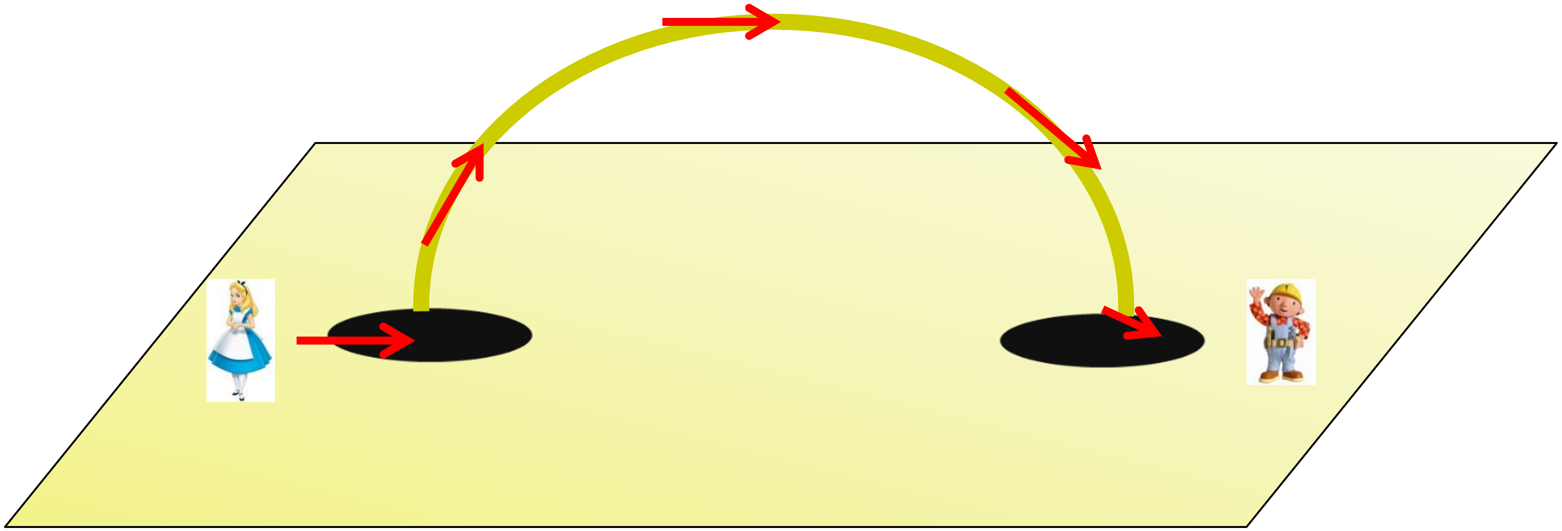
Einstein-Rosen Bridge

distance (through wormhole) just
a couple of kilometers
(finite spatial distance)



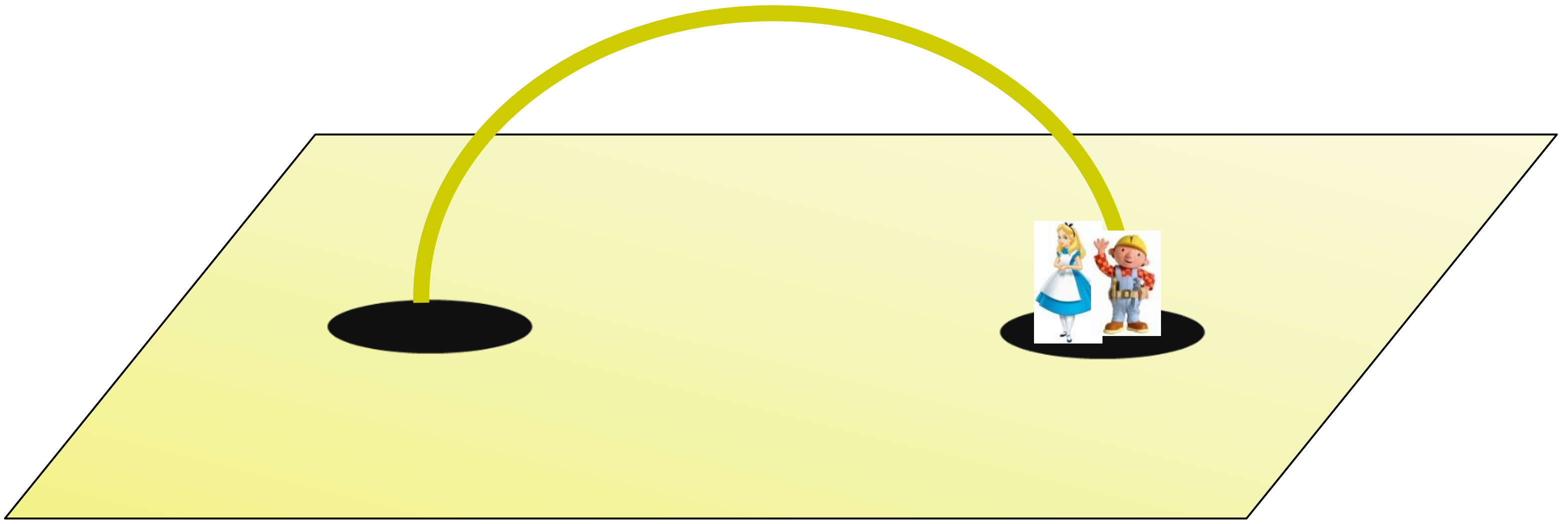
Einstein-Rosen Bridge

Alice can send signal into wormhole,
but signal can't reach Bob.
(Causally disconnected).

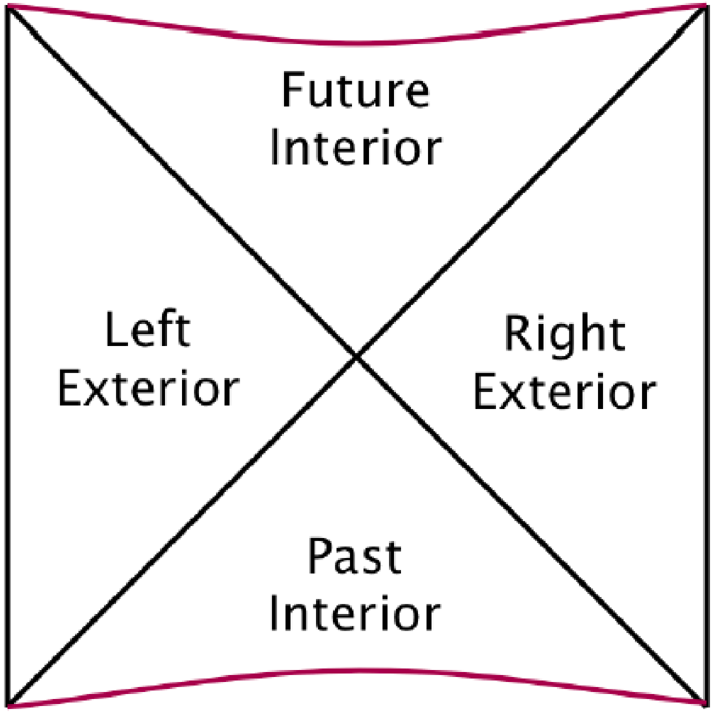


Einstein-Rosen Bridge

But if **both** jump into their respective black hole, they can easily meet (but never come back out)

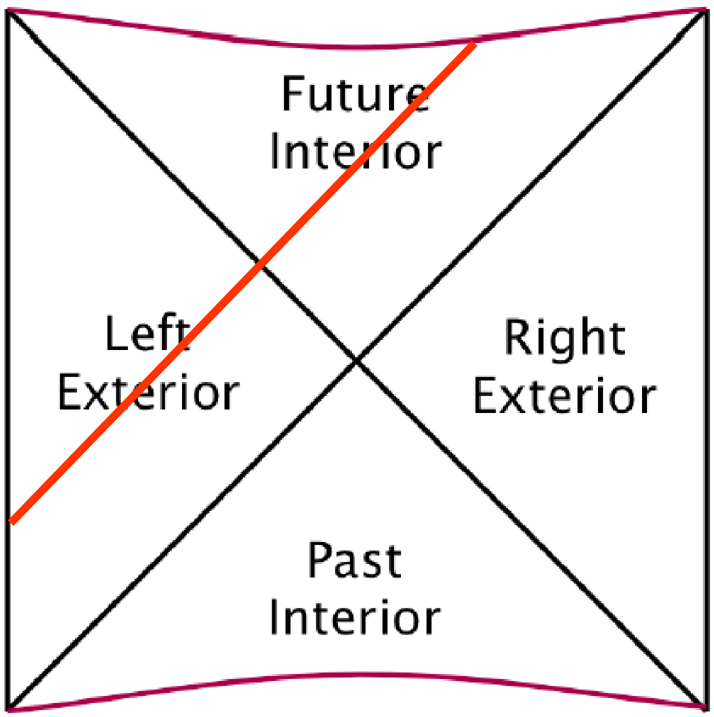


Example: Eternal Black Hole



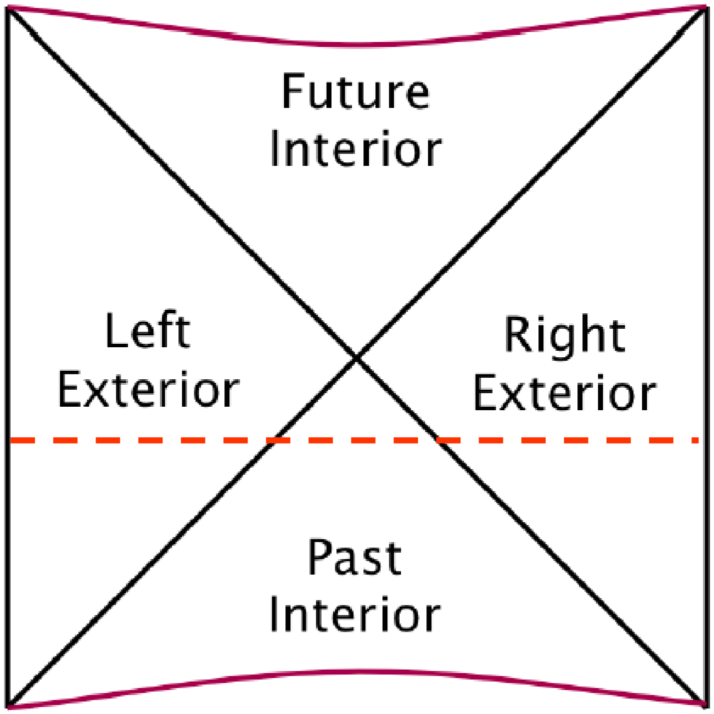
Think of this is describing two far away entangled black holes

Example: Eternal Black Hole



no causal contact between the two asymptotic regions…

Example: Eternal Black Hole

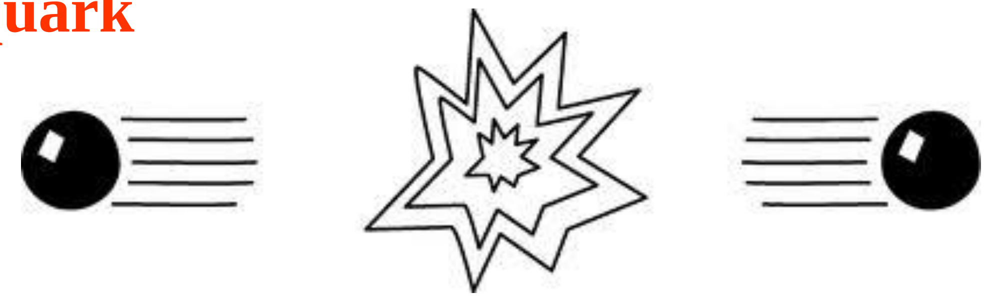


… but finite spatial distance.

Holographic EPR pair

Example: holographic EPR (Jensen-AK)

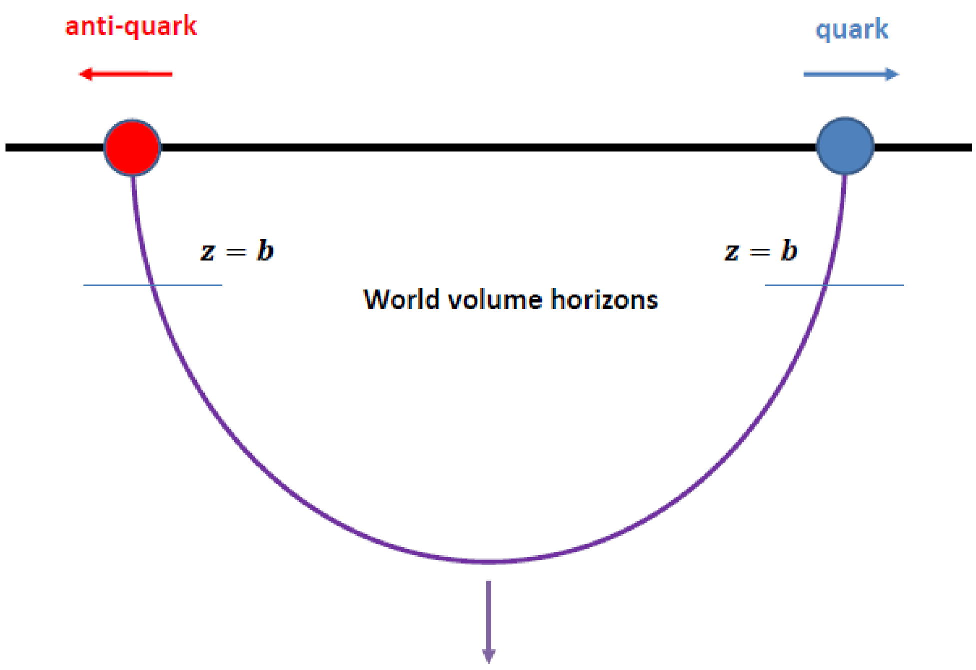
anti-quark **quark**



pair created in
background electric field

We can do this in N=4 SYM.
What is the holographic dual?

Holographic EPR

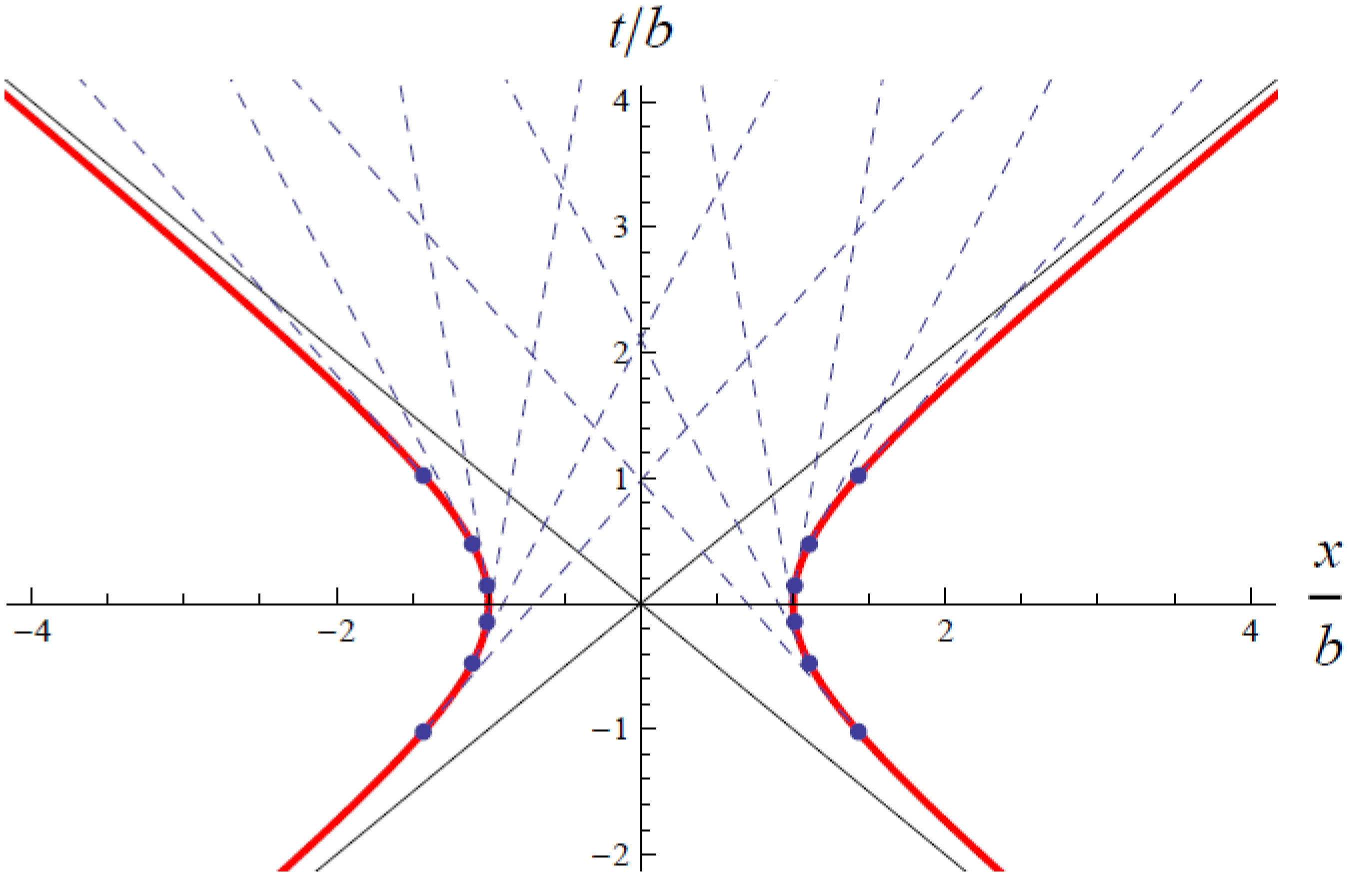


$$x^2 = t^2 + b^2 - z^2 \,.$$

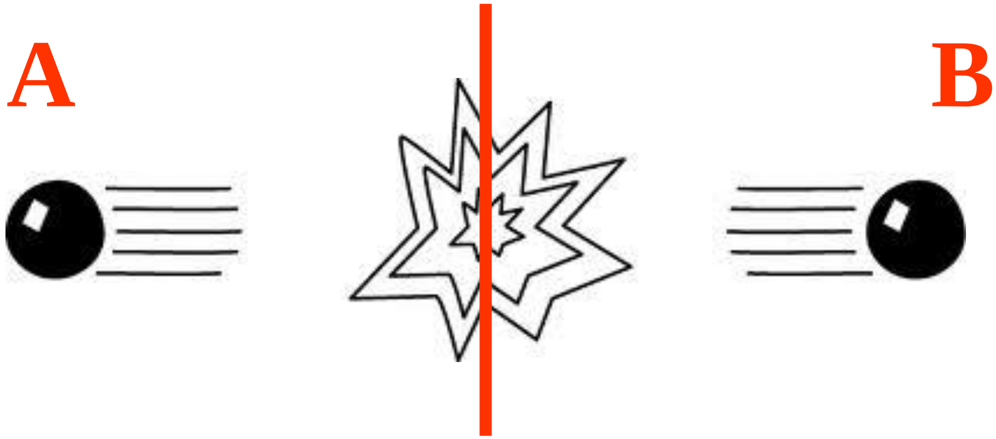
(Xiao)

Worldsheet = ER bridge (finite distance, no causal connection)

Holographic EPR: geodesics



S_{EE} for the holographic EPR pair



Finally: calculate EE for a probe brane!

$$S_{EE} = \sqrt{\lambda}/3$$

quark not just a single parton, $\sqrt{\lambda}$ gluons part of quasi-particle

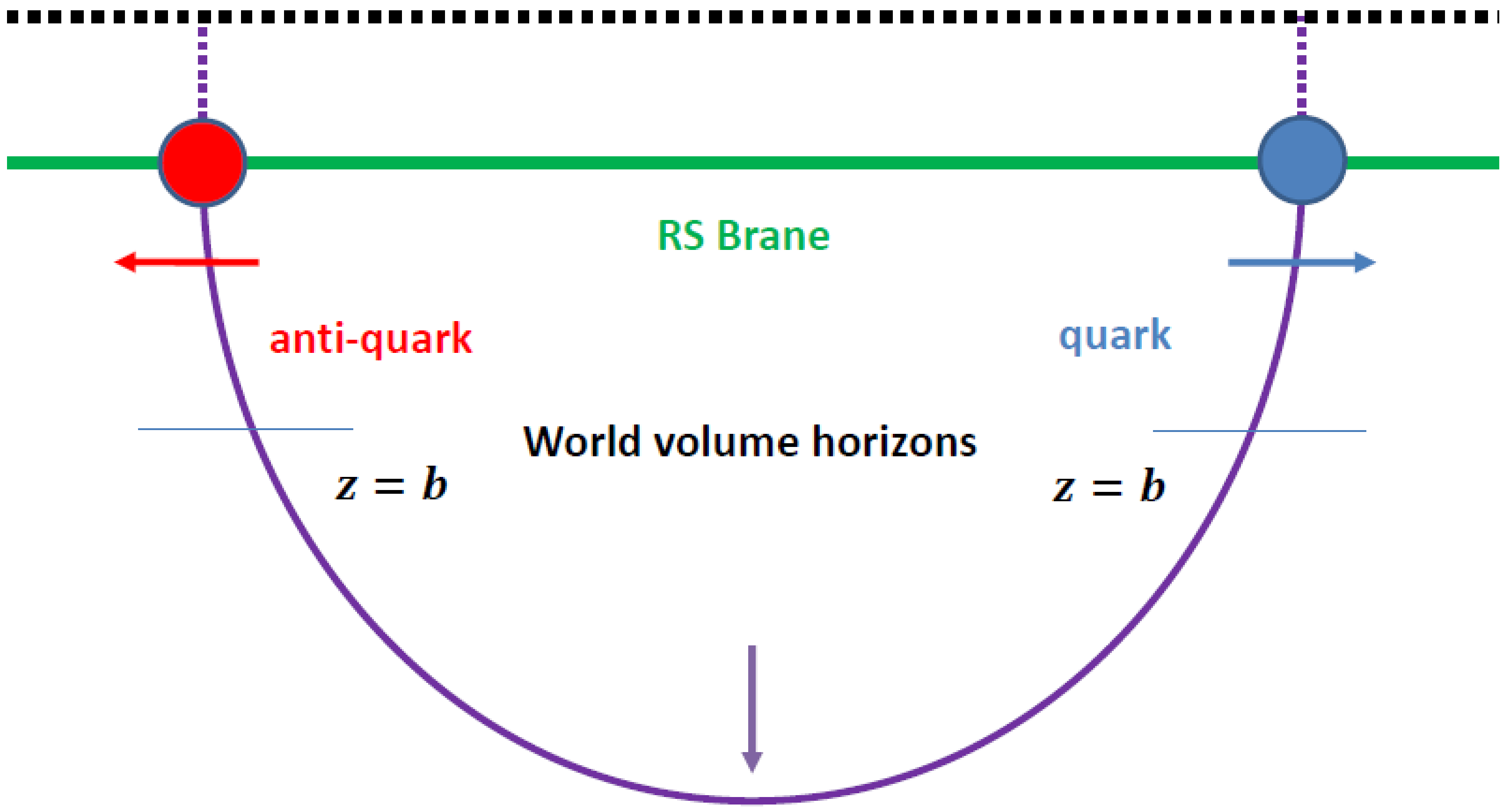
Lessons learned

ER=EPR appears to be special case and direct consequence of our standard understanding of holography for large N, large λ theories.

Generalizations

- Including Dynamical Gravity
- Holographic Hawking Pairs

1) Dynamical Gravity via RS



Bottom Line: No Change

RS-holography:

CFT with UV cutoff a
+ dynamical gravity with
G_N induced by matter loops

=

Brane located at fixed radial position; 5d fluctuations induce a localized 4d mode.

Only change: Horizon disappears when q-qbar separation is less than a.

2) Hawking Pairs

AdS/CFT allows us to study the CFT on any background metric g_0

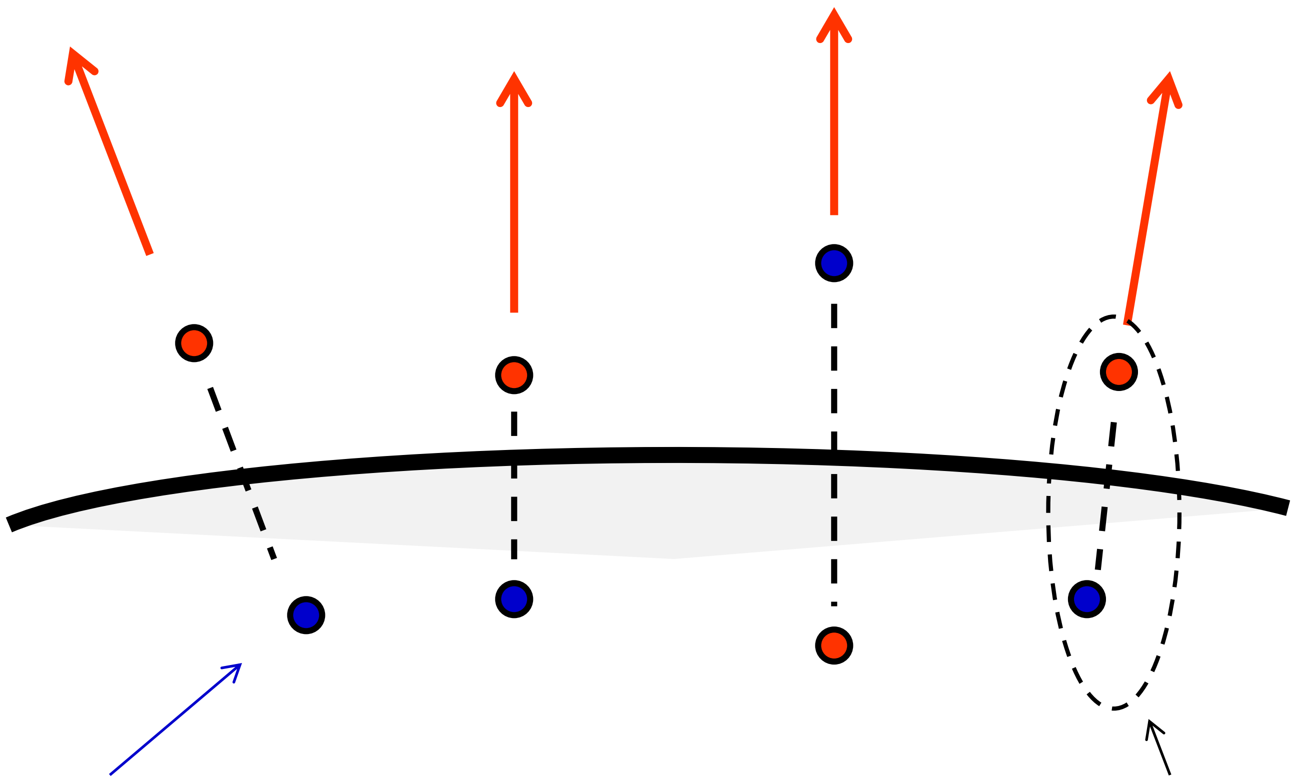
g_0 sets the near boundary behavior of the bulk metric. Need to solve Einstein equations with that boundary behavior (Fefferman-Graham).

If we chose g_0 to be a black hole, we can study a Hawking pair (= EPR pair separated by horizon).

Hawking Pairs

This will also allow us to address
a long standing problem about
Hawking radiation at strong coupling.

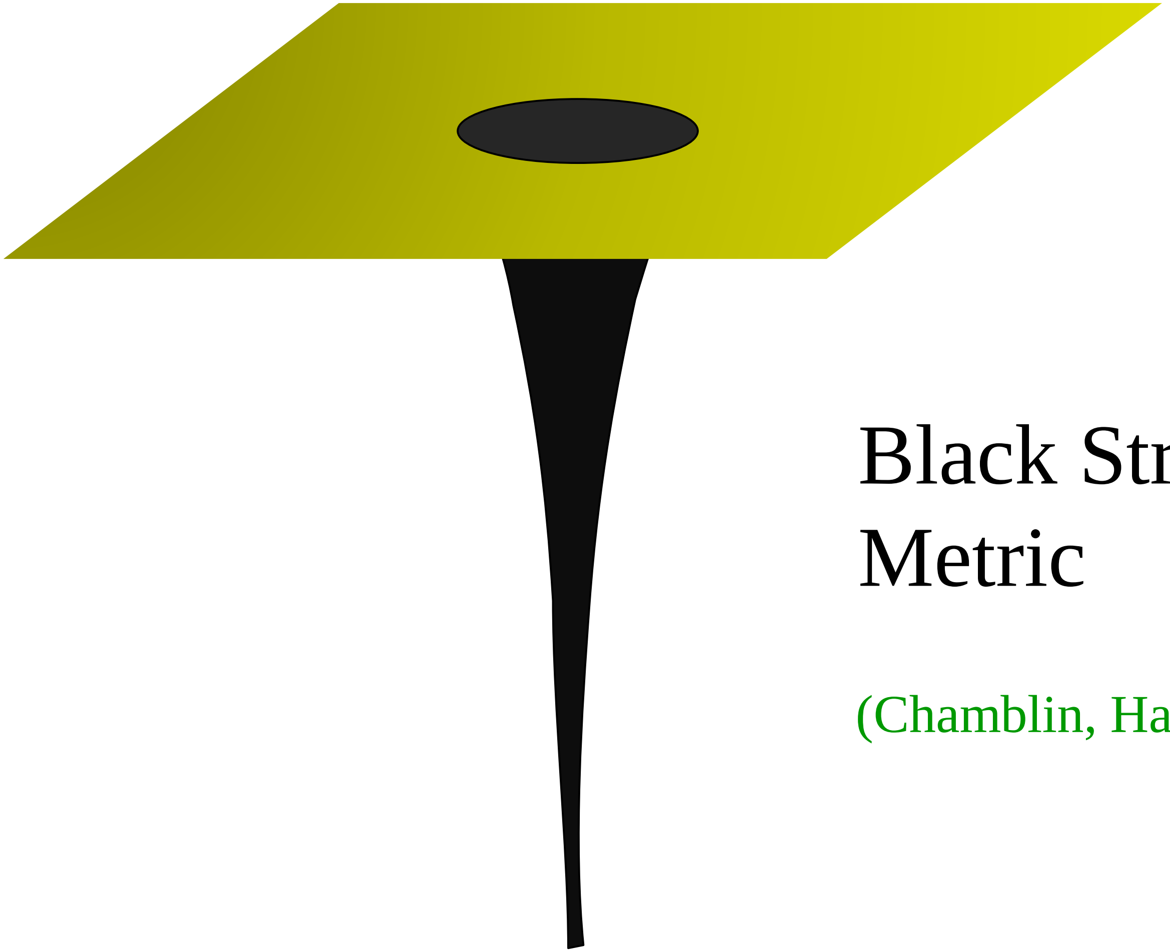
Hawking Radiation:



Quark in flavored N=4 SYM.

What's the holographic dual to a Hawking pair?

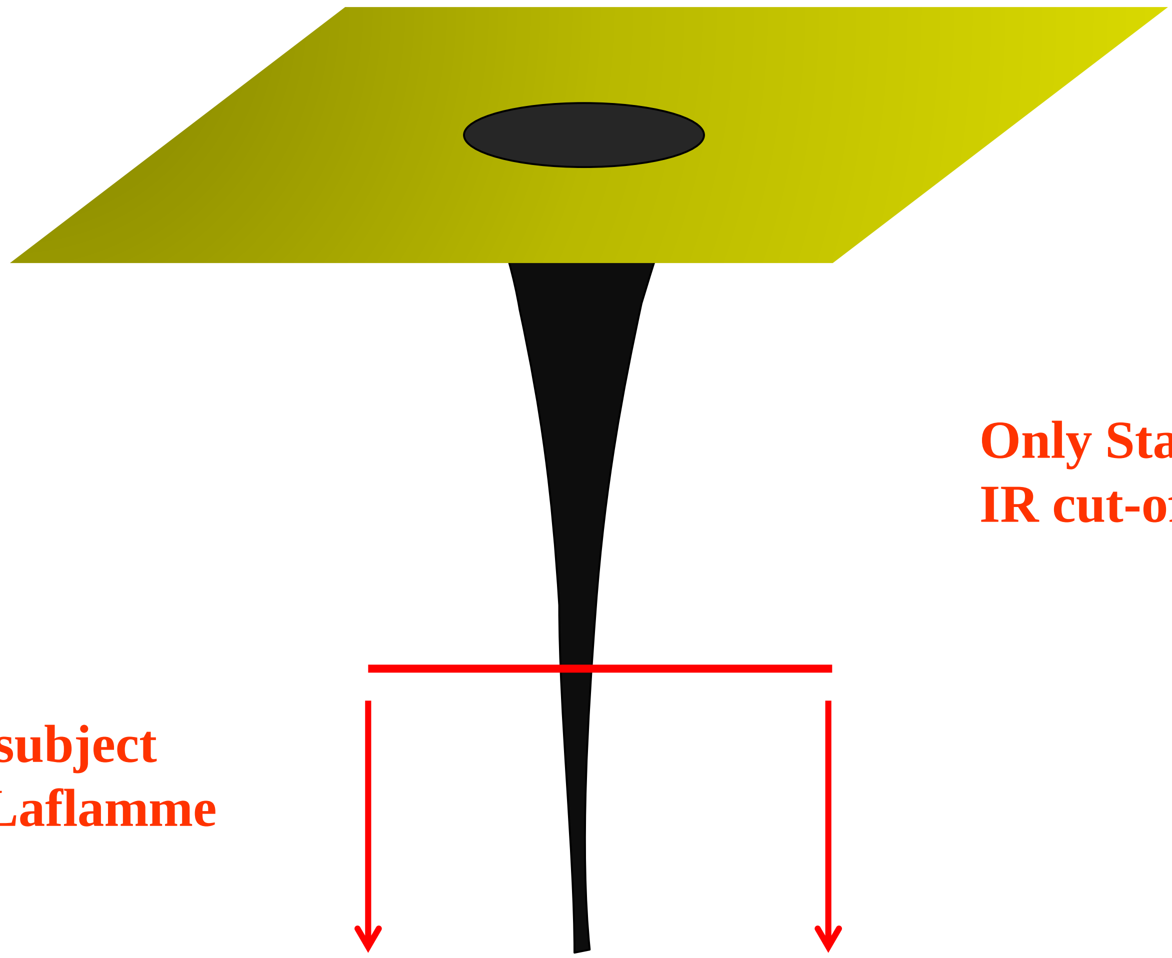
N=4 on Schwarzschild Black Hole



Black String Metric

(Chamblin, Hawking, Reall)

N=4 on Schwarzschild Black Hole



Only Stable with IR cut-off (hard wall).

Thin String subject to Gregory-Laflamme Instability!

Unstable vacuum on BH

The unstable state can still be studied.

Puzzle: No Hawking radiation!

$$\langle T_{\mu\nu} \rangle = 0$$

Potential Resolution:

at strong coupling, only color neutral Hawking radiation!

(Fitzpatrick, Randall, Wiseman)

Sharpening the Puzzle

Or maybe we just shouldn't over-interpret the unstable state?

Puzzle can be sharpened:

Black String for AdS_4-black hole is stable!

Still no Hawking Radiation.

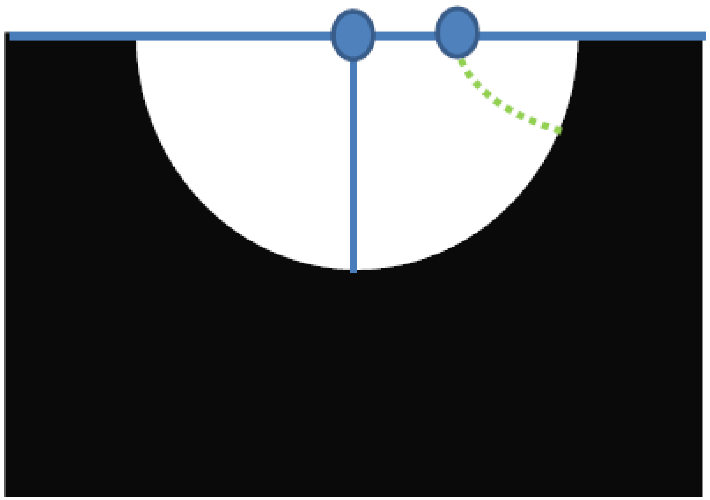
(Chamblin, AK; Gregory, Ross, Zegers)

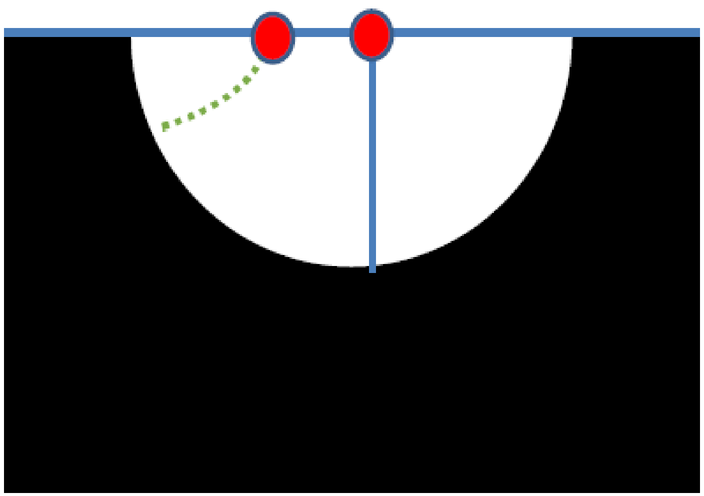
PS: More precise:

$$\langle T_{\mu\nu} \rangle = \# g_{\mu\nu}$$

- Very different from weak coupling (where one explicitly sees radiation term)
- Maybe subtle cancellation between vacuum polarization and radiation?
- Fischetti, Marolf and Santos give solution with two coupled AdS spaces and heat flow
- Our work however gives some evidence to support Fitzpatrick, Randall, Wiseman.

AdS_4 Black String Metric





mSYM on two joined copies of AdS_4 black holes

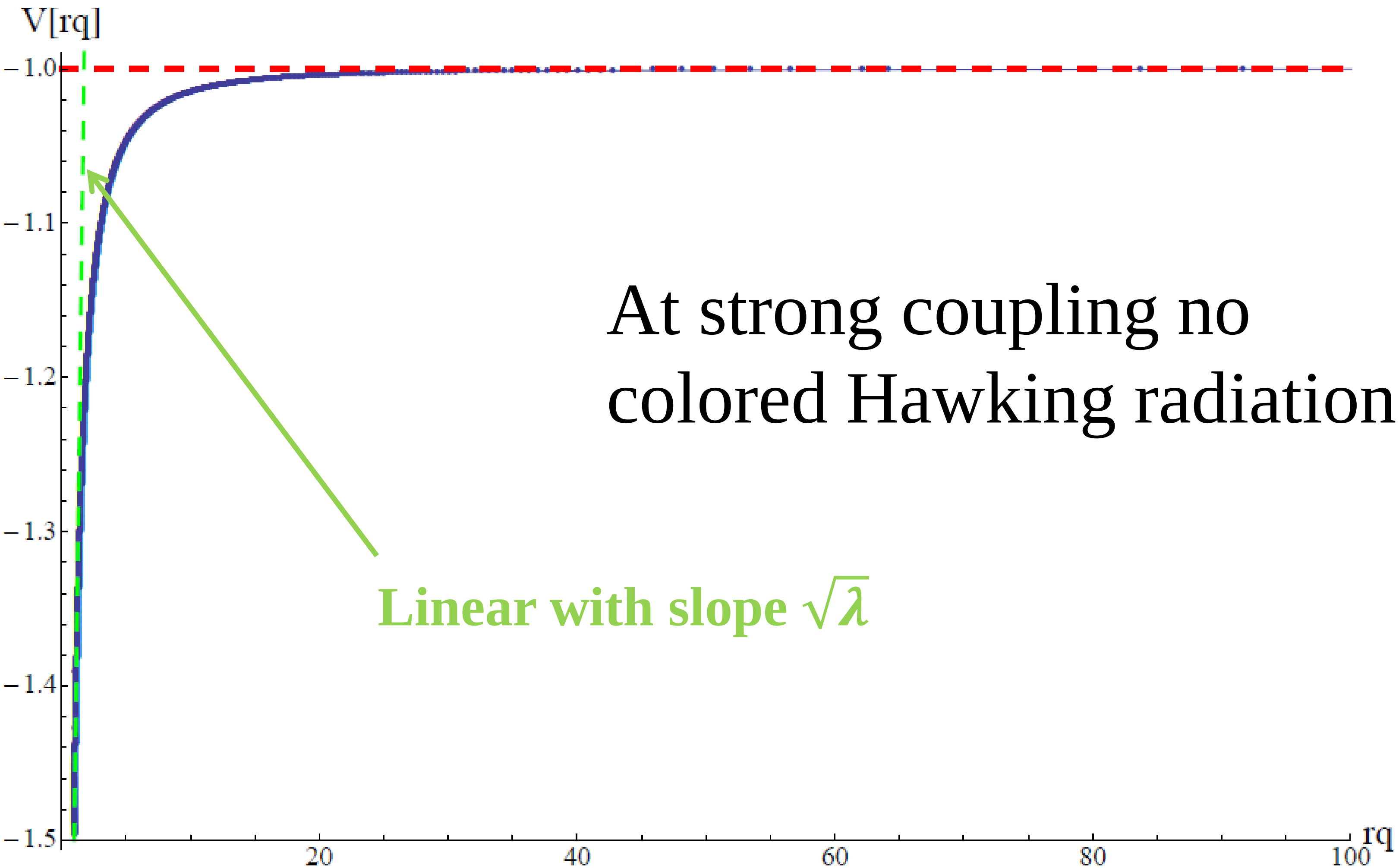
thermofield double

Hawking pair dual to ER

All our strings for all time
cross the bulk horizon through the
bifurcation point, they inherit
the causal structure of the bulk ER bridge.

Worldsheet metric = ER bridge.

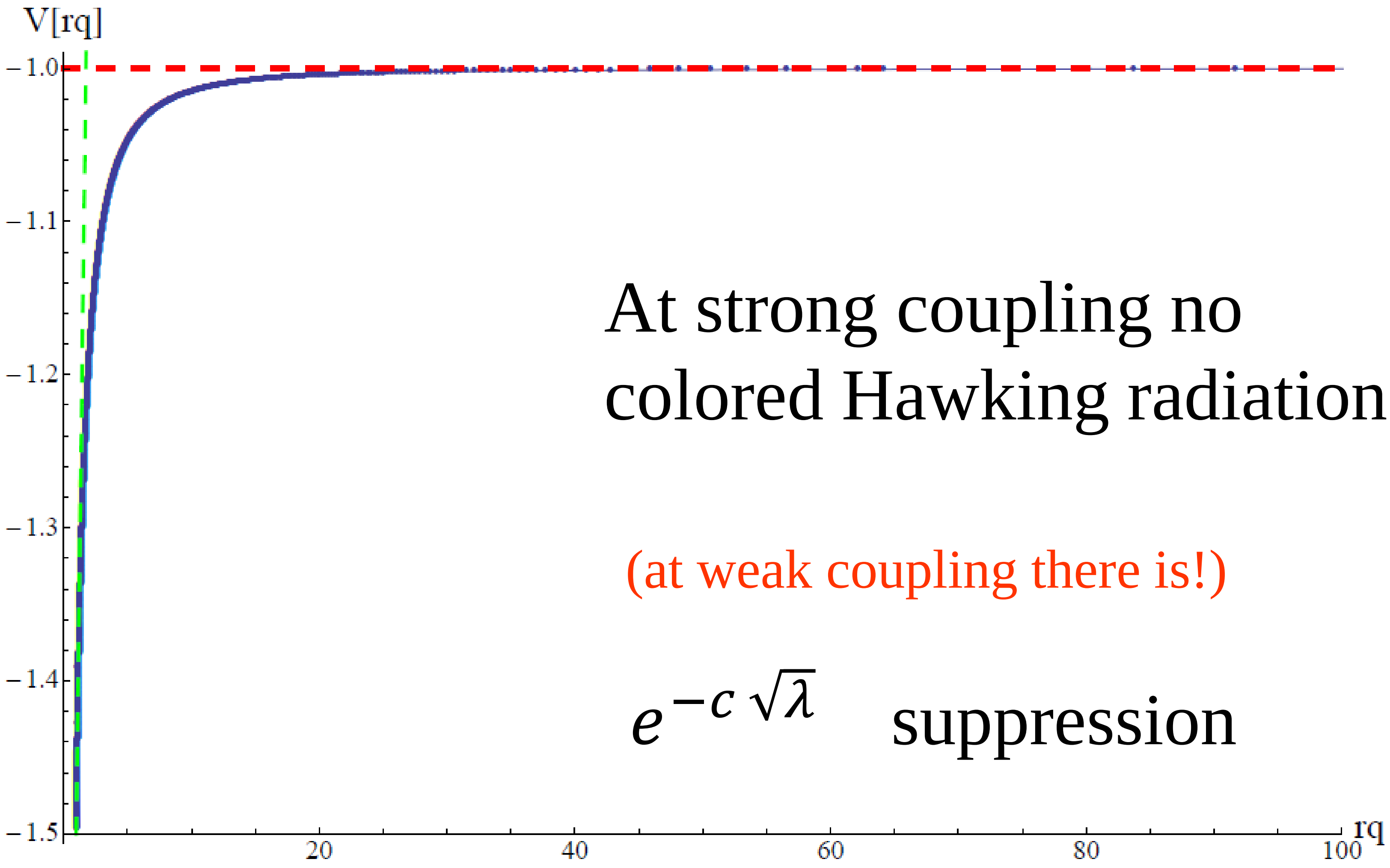
And Confinement!



At strong coupling no colored Hawking radiation!

Linear with slope $\sqrt{\lambda}$

And Confinement!



At strong coupling no colored Hawking radiation!

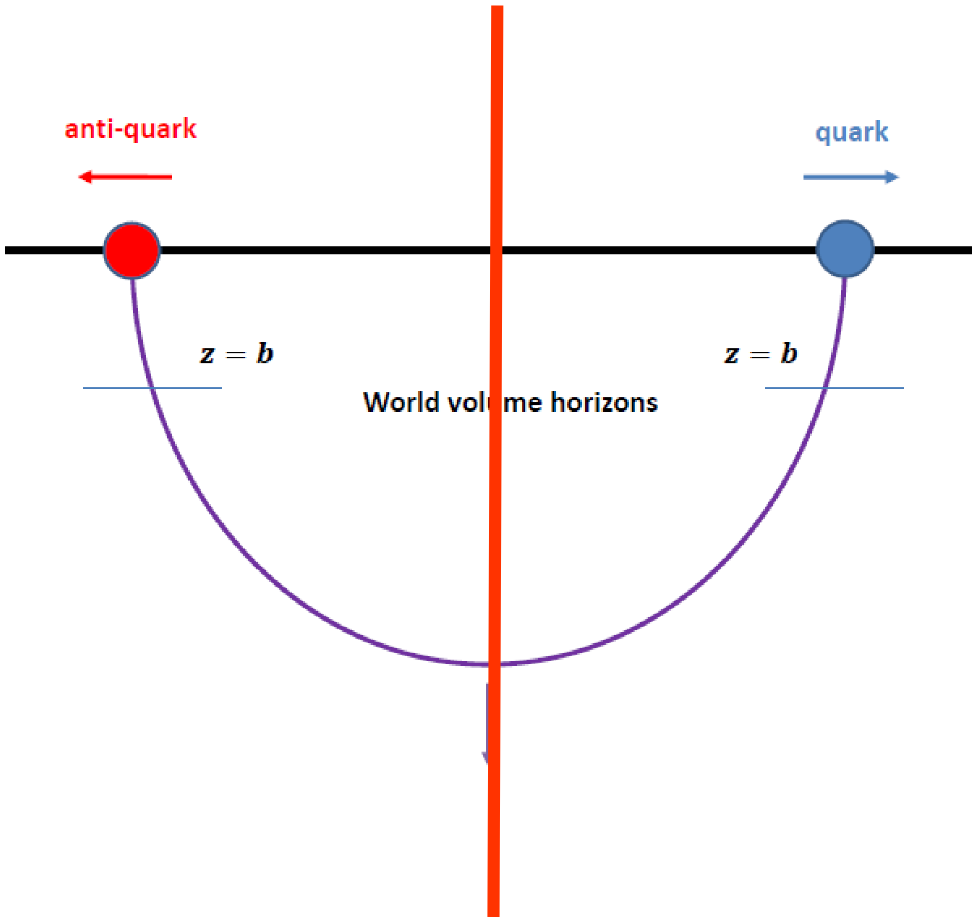
(at weak coupling there is!)

$e^{-c\sqrt{\lambda}}$ suppression

Outlook: EE for probes

work with Han-Chih Chang and Christoph Uhlemann

Holographic EPR



$$S_{EE} = \sqrt{\lambda}/3$$

Just one example of EE calculation for probe branes…..

Probe Branes

(AK, Katz)

d+1 dimensional
spacetime

n+1 dimensional
worldvolume

$$\frac{L^{d-1}}{G_N} \gg T_0 L^{n+1} \gg 1$$

Newton's constant

Probe brane tension **N**

N^2

L=curvature radius of background geometry

Probe Branes

(AK, Katz)

Gravity classical
dominant physics at $O(N^2)$
loops contribute at $O(1)$

"the glue"

$$\frac{L^{d-1}}{G_N} \gg 1$$

Newton's constant

N^2

L=curvature radius of background geometry

Probe Branes

(AK, Katz)

Worldvolume equations classical
dominant flavor physics at O(N)
loops contribute at O(1)

"the quarks"

$$T_0 L^{n+1} \gg 1$$

Probe brane tension N

L=curvature radius of background geometry

Probe Branes

(AK, Katz)

No gravitational backreaction from probe brane
$O(N^2)$ physics unaffected by order N physics
O(N) physics: wordvolume of flavor brane and leading order backreaction
at O(1) all hell breaks lose: 2nd order backreaction, glue loops, quark loops

$$\frac{L^{d-1}}{G_N} \gg T_0 L^{n+1} \gg 1$$

Newton's constant

Probe brane tension **N**

N^2

L=curvature radius of background geometry

Backreaction?

(see e.g. AK, O'Bannon, Thompson)

Generically, at order N backreaction matters.

$$S = N^2 S_{grav} + N\, S_{probe}$$

glue and quark sector

backreaction suppressed….

$$\delta h \propto G_N\, T_{\mu\nu} \propto {}^{1}\!/_{N}\;\; h$$

$$\boxed{\varepsilon = \; N^2 \varepsilon_{grav} + N\,(\varepsilon_{probe} + \frac{\delta \varepsilon_{grav}}{\delta g}\,\mathrm{h})}$$

$\varepsilon = {}^{\delta S}\!/_{\delta g_{00}}$ = energy density

… but enters at order N

Backreaction?

(see e.g. AK, O'Bannon, Thompson)

Exception: the free energy = on-shell action.

$$S = N^2 S_{grav} + N\, S_{probe}$$

glue and quark sector

backreaction suppressed

$$\delta h \propto G_N\, T_{\mu\nu} \propto {}^{1}\!/_{N}\;\; h$$

$$\boxed{\omega = N^2 S_{grav} + N\left(S_{probe} + \underbrace{\cancel{\frac{\delta S_{grav}}{\delta g}}}_{=0}\, \mathrm{h}\right)}$$

contribution due to backreaction vanishes due to equations of motion!

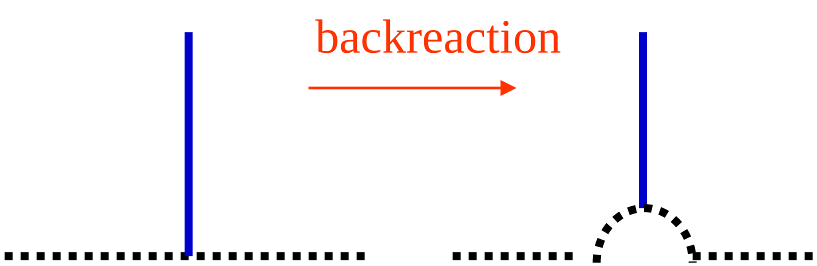
Gravity bad – DBI good

(see e.g. AK, O'Bannon, Thompson)

$$\boxed{\omega = N^2 S_{grav} + N\, S_{probe}}$$

good news! all equilibrium properties can be calculated without ever having to deal with backreaction!!!!

Including thermal entropy:



or

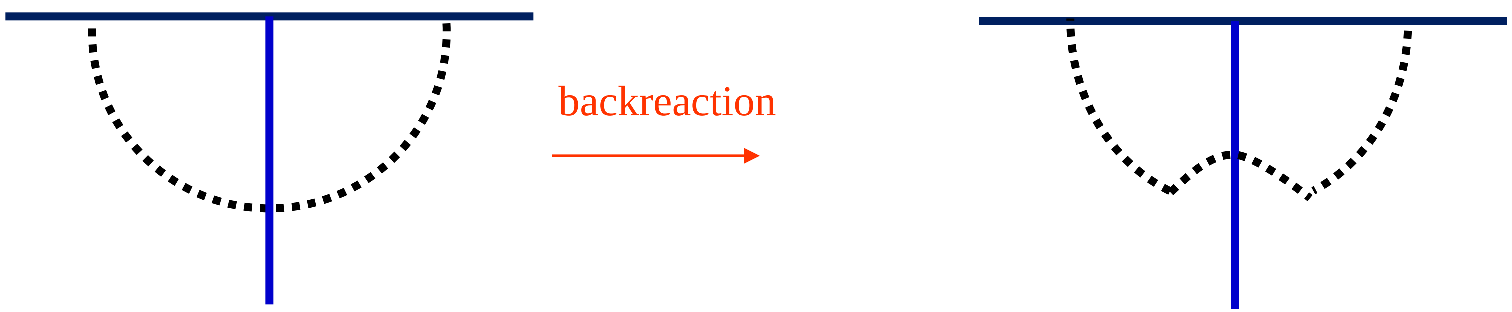
$$s = -\frac{\partial \omega}{\partial T}$$

but "cheat" works!

honest calculation:
calculate change in horizon area

DBI bad – Gravity good!!!

But for the EE we are stuck with needing leading order backreaction to even get order N contribution.



$$S^{EE} = \underbrace{S^{EE}_{grav}}_{O(N^2)} + \underbrace{\frac{\delta Area}{4G}}_{O(N)}$$

O(1/N)

O(N^2)

In this work we will give a general expression for δArea

Exceptions:

Casini, Huerta, Myers:

- spherical entangling surface
- conformal field theory

$$\text{EE} \xrightarrow{\text{conformal map}} \text{thermal entropy on hyperboloid}$$

Jensen, O'Bannon:

still applies for conformal flavors (massless quarks!) on conformal defect

In this case cheat still applies.

Limitations of Jensen – O'Bannon

- Massless flavors only
- No worldvolume gauge fields

not applicable to any of the systems we really want to be able to calculate the EE in!

Chang, AK: isolate aspects of backreaction that are important for EE.

- **internal space negligible**
- **often all fields but metric negligible**

tractable!

How about LM procedure?

(AK, Uhlemann)

LM: EE from on-shell action via replica trick.

But evaluating this is not trivial with probe.
2 choices how to proceed.

How about LM procedure?

(AK, Uhlemann)

LM: EE from on-shell action via replica trick.

1) Use EOMs to reduce on-shell action to surface term

This requires backreaction.
Reduces to RT in backreacted geometry.

How about LM procedure?

(AK, Uhlemann)

LM: EE from on-shell action via replica trick.

2) Do not backreact; can't use EOM.
   EE from bulk integral

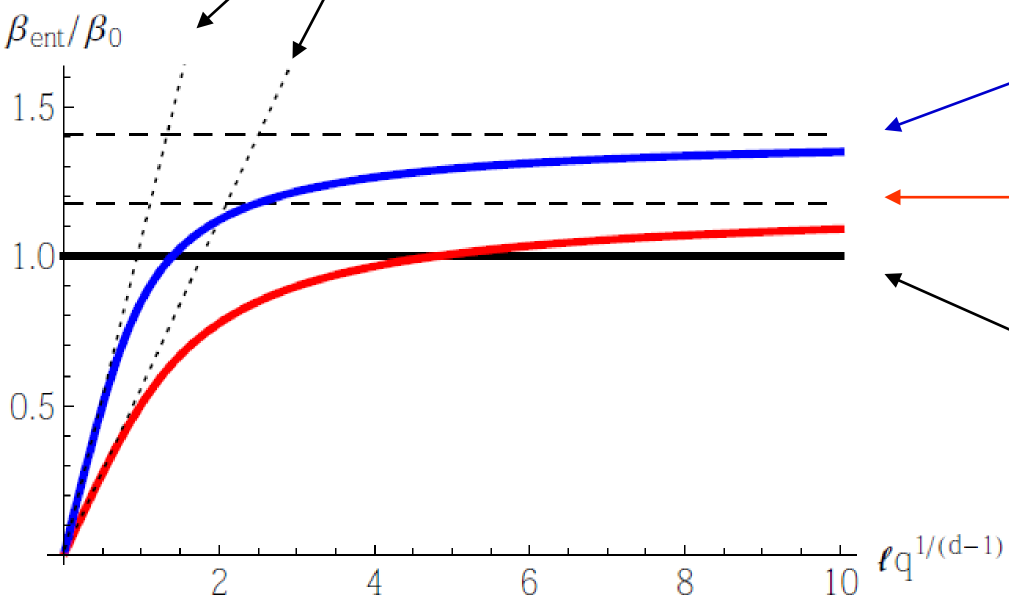
This is often even harder than including backreaction. Both methods agree (and give exact answer) for massive flavor.

Massless flavors at finite density:

(Chang, AK, Uhlemann)

EE ~ volume!?!

agrees with universal results for small l

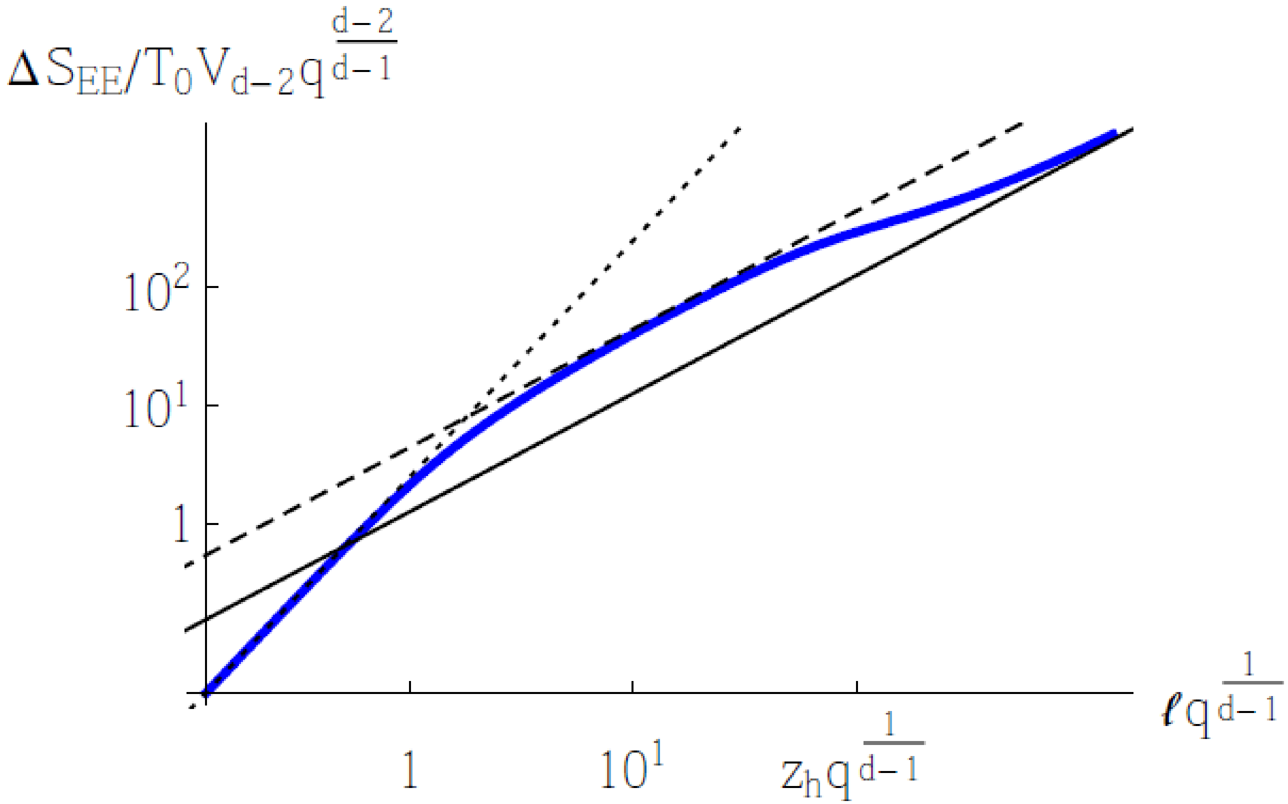


Strip

Disc

GS entropy density

Massless Flavors at finite density



small temp
as IR regulator

Thank you!